Control Flow

COMP 524: Programming Language Concepts
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The University of North Carolina at Chapel Hill

Based in part on slides and notes by S. Olivier, A. Block, N. Fisher, F. Hernandez-Campos, and D. Stotts.

Sequential Control Flow

Determines **what is computed**, and in **which order**.

Imperative PL: order mostly **explicit**.
Declarative PL: order mostly **implicit**

The Basis: Conditional Branching

Virtually all instructions sets support:

- **Unconditional branching** to a fixed address.
  - e.g., `jmp 0x123`: "jump to address 0x123"
- **Unconditional branching** to an address in a register, i.e, to an address determined at runtime.
  - e.g, `jmp (%eax)`: "jump to the address in the accumulator register."
- **Conditional branching** to a fixed address.
  - e.g., `jne 0x123`: "jump to address 0x123 if last two values that were compared were not equal"

This is sufficient to implement a universal programming language!

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  - e.g, **`jmp (%eax)`**: “jump to accumulator register.”
- **Conditional branching** to a fixed address.
  - e.g., **`jne 0x123`**: “jump to address 0x123 if last two values that were compared were not equal”

Any higher-level control flow abstraction can be realized in terms of these jumps.

This is sufficient to implement a universal programming language!

Sequencing

Sequencing: explicit control flow.

- Abstractions that **control the order of execution**.
- Crucial to imperative programming.

Levels of abstraction.

- **Unstructured control flow**
  - hardly any abstraction over jumps
  - **hard to reason about**
- **Structured control flow**
  - amendable to formal proofs
  - easier to understand
  - jumps are implicit

sequential composition:
do this first **;** *then this*

unstructured:
e.g., **`goto`**, **`break`**, **`redo`**, **`last`**, **`continue`**, **`continue`**, and **`return`** if used to "skip" over rest of subroutine

structured:
e.g. **`for`**, **`while`**, and **`if`**

Goto Considered Harmful

Title of a famous critique of unstructured control flow by Dijkstra, 1968.

Source: xkcd.com (go check it out!)

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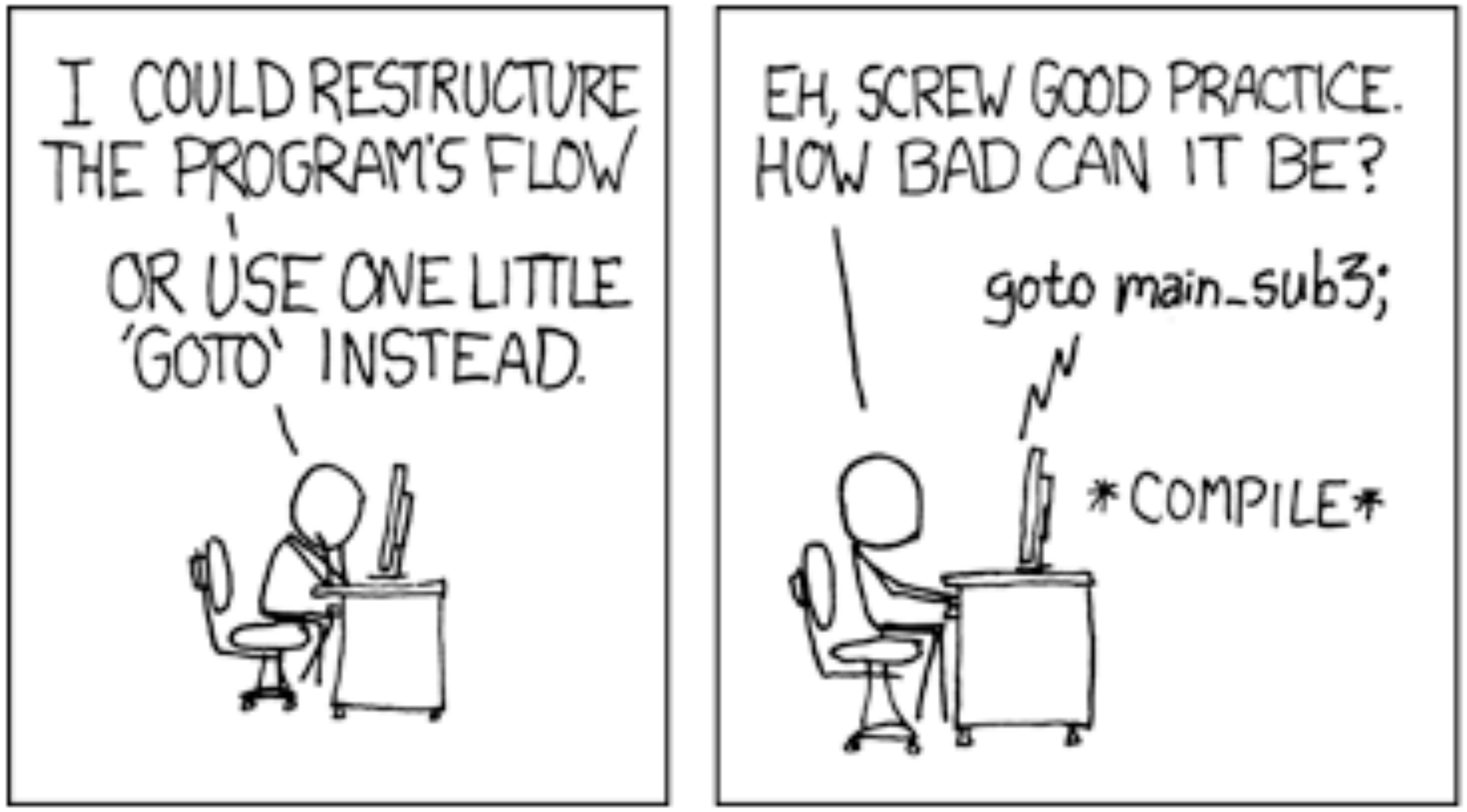

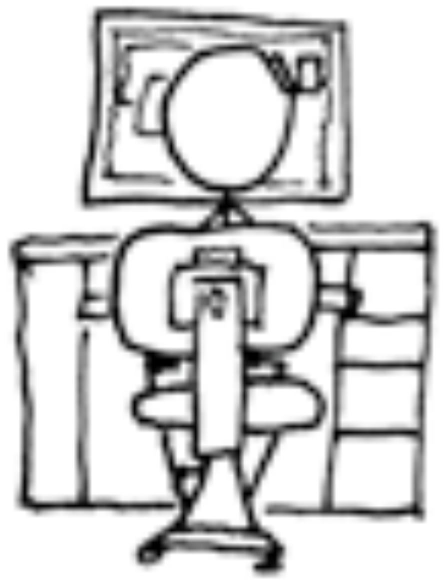

<u>Bohm & Jacopini, 1964</u>

every use of goto can be equivalently expressed with structured control flow constructs

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Goto Considered Harmful

Title of a famous critique of unstructured control flow by Dijkstra, 1968.

Bottom line: Don’t ever use `goto`. Try hard to avoid all other unstructured control flow constructs, too.

(Footnote: some very special settings can benefit from a goto, e.g., some kernel routines. However, this does not apply to 99% of all software, in particular business & web software.)

Bohm & Jacopini, 1964

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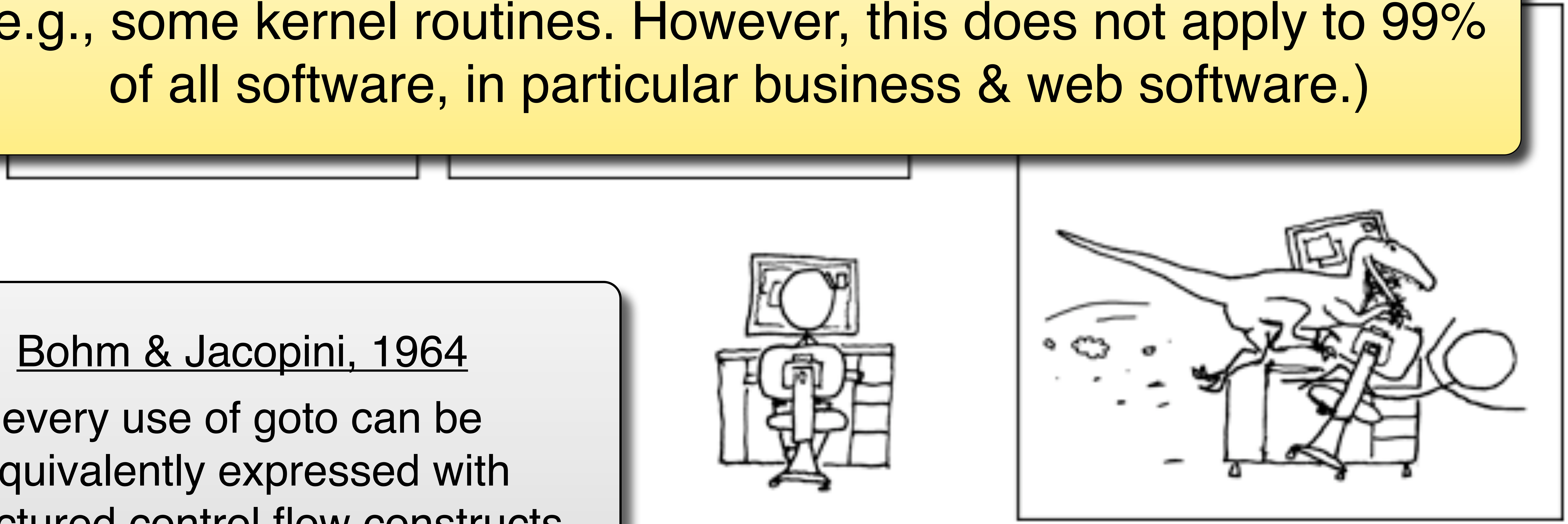

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Loops and Conditionals

Selection: execute one choice, but not the other(s).

- if-then-else
- if-then(-elsif)*-else
- switch, case statements
  - Implementation driven: exists to facilitate generation of efficient machine code.
  - This reason is somewhat obsolete with improved compilers.

Iteration: do something a pre-determined number of times.

- for (enumeration controlled)
  - from x to y; sometimes also from y downto x.
- for each (iterator)
  - executing a loop body for each element of a “collection.”
  - can be emulated with iterator pattern if not supported by language
    - hasNext() and next()

Logically-controlled loops…

Logically-controlled Loops

repeat something while a condition is satisfied

```
do
 {
 ...
 }
while i==true;
```

Post-test

```
for(;;){
 ...
 if i==true break;
 ...
}
```

Midtest

```
while (i==false)
{
 ...
}
```

Pre-test

Subroutines

subprograms, functions, procedures, methods,…

Control Flow Abstraction.

- ➡ Separate “**what it does**” from “**how it’s done**.”
  - ‣ API: subroutine as a service provider.
- ➡ Reuse **high-level** code.
  - ‣ DRY: write it only once.
  - ‣ Maintenance: fix bugs only once.
- ➡ Reuse **machine** code.
  - ‣ Usually, only one copy of a subroutine is included in final program.

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Instead of writing a concrete sequence of instructions, a subroutine is **parametrized** **sequence of instructions**.

Execution Context

A subroutine is executed in the context of the **(virtual) machine state** (global variables, device state, …).

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The "main effect" is the value that is computed (i.e., the return value).

Side Effect

A program fragment that alters the global context and thus indirectly affects the outcome of otherwise unrelated computations is said to have a **side effect**.

Function vs. Procedure

Pure Function.

- ➡ A pure function has **no side effects**.
- ➡ A pure function's **result only depends on its arguments**, and not on any global state; not affected by side effects.
- ➡ "Always the same" and "leaves no trace."

Pure Procedure.

- ➡ A pure procedure **returns no value**, and is only executed for its side effects.
- ➡ Java: any method with return type **void**.

Subroutine Parameters

```
define my_subroutine(X, Y, Z) {
   …
   print X;
   …
}

define getval() {
   return 42;
}

…
my_var = 4;
my_subroutine(my_var, 3 + 4, getval());
…
```

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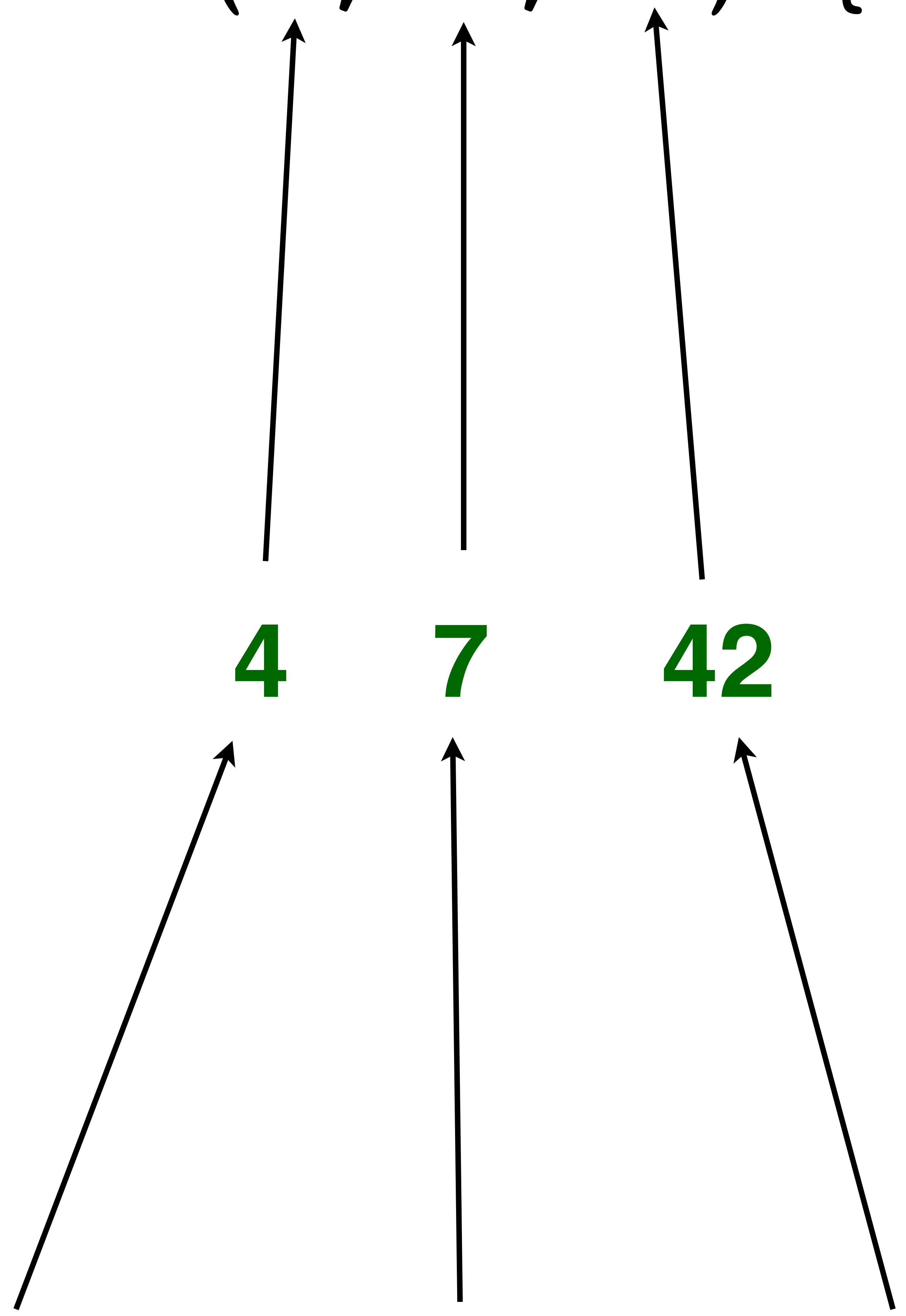

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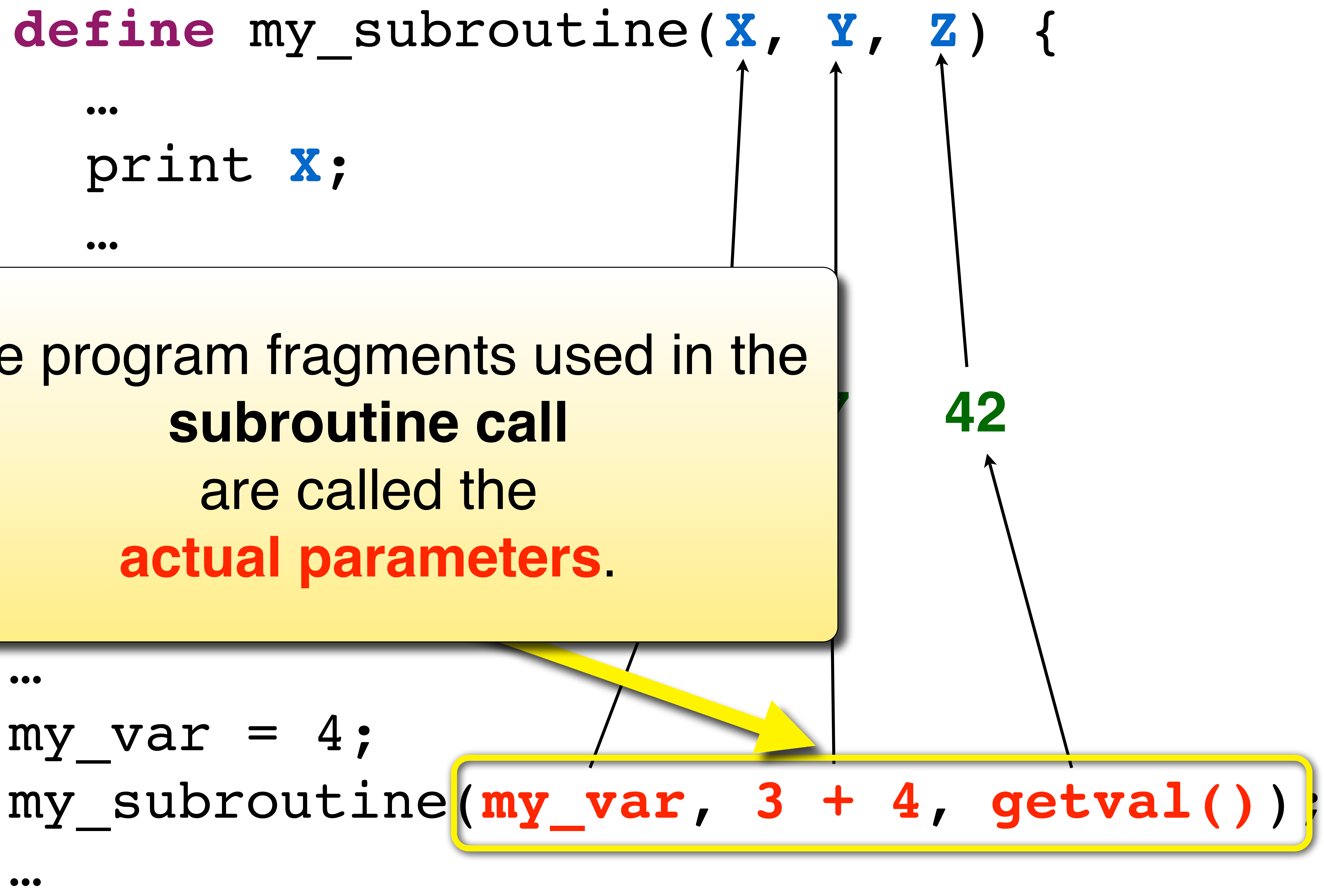
The names used in the **subroutine definition** are called the **formal parameters**.

Subroutine Parameters



```
define my_subroutine(X, Y, Z) {
   …
   print X;
   …
```

The program fragments used in the
subroutine call
are called the
actual parameters.

```
 …
 my_var = 4;
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```

The **values** resulting from the evaluation of the **actual parameters** are called the **arguments**.

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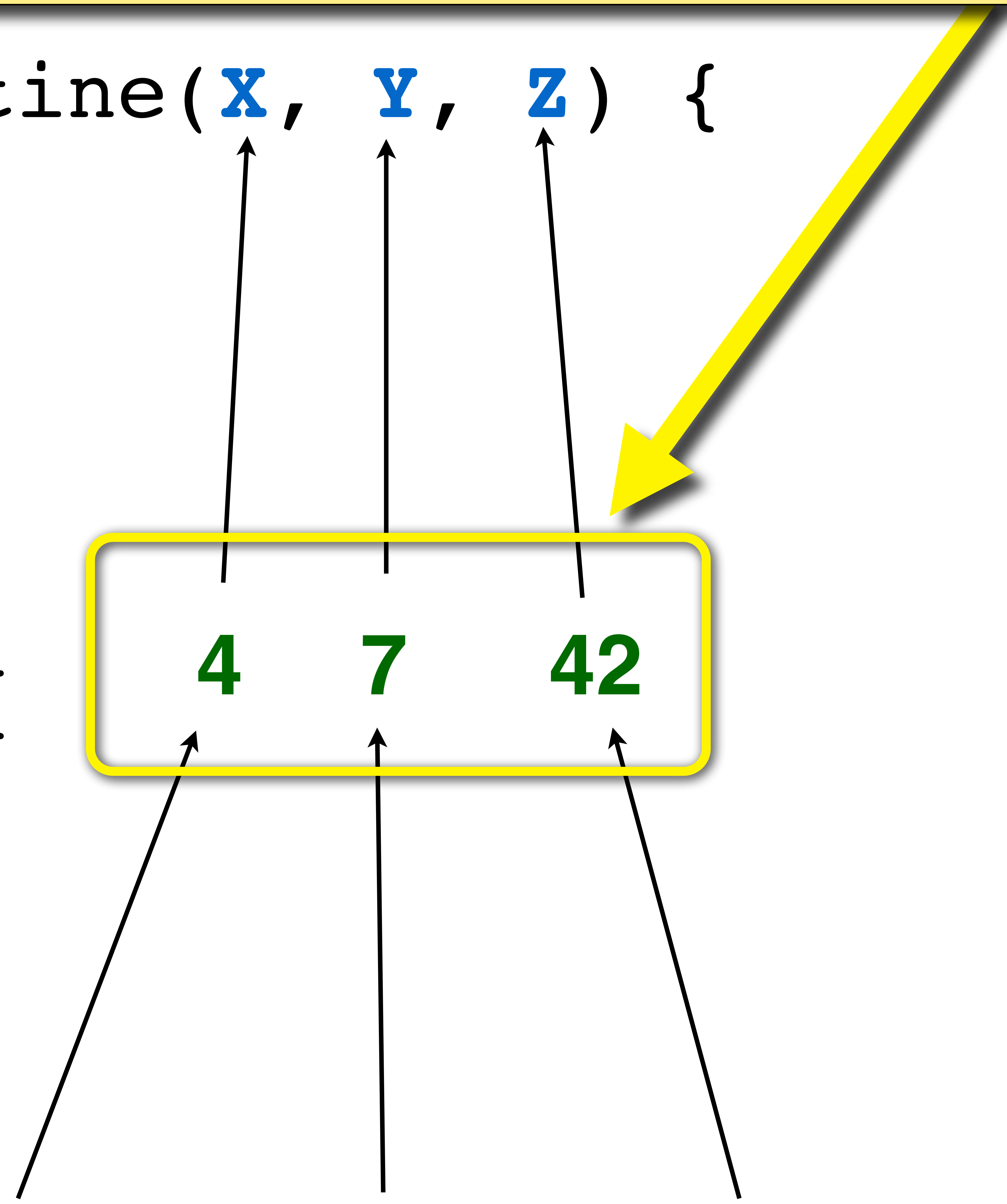

Parameter Passing

The formal parameters have to be bound to the arguments at some point during the subroutine call.

Actual Parameter Evaluation.

- ➡**When**? As soon as possible?
  - ‣Evaluate actual parameters before call?
  - ‣What if argument is not needed? **On demand?**
- ➡In **what order**?
  - ‣Left to right?
  - ‣Any order?
  - ‣Does it matter?
- ➡Are **updates by the callee** to the formal parameters "visible" to the caller?

Parameter Passing: Information Flow

In Parameters

Information/data provided by the caller;
(possibly) consumed by the callee.
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In-Out Parameters
Information/data provided by the caller;
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Any change by callee visible to caller.

Parameter Passing: Semantics

Pass-By-Value

Behaves as if arguments are copied
from the caller to the callee prior to the call.

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Behaves as if formal parameter is replaced by actual parameter in subroutine body; evaluated whenever needed.

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E al
para ded.

Example: Java
Scalar types (int, double, etc.) are in
parameters and passed-by-value,
whereas objects are passed-by-reference.

Parameter Passing: Semantics

Pass-By-Value

Example: C Preprocessor
Macro parameters are passed-by-name.

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Param

Usually implemented with actual copying, but details vary.

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Parameter Passing: Semantics

Pass-By-Value

Usually implemented by copying address, but sometimes more complex (e.g., Java RMI).

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Behaves as if formal parameter is replaced by actual parameter in subroutine body; evaluated whenever needed.

Parameter Evaluation Time

When to evaluate actual parameters to the obtain arguments.

Eager Evaluation.

- ➡ Evaluate all arguments before call.
- ➡ **Easy to implement**.
- ➡ But can be **problematic**.
  - ‣ What if not needed?
  - ‣ What if error might occur?

Normal-order evaluation.

- ➡ Evaluate every time when argument needed.
- ➡ But **only if needed**.
- ➡ i.e., call-by-name.
- ➡ May be **not very efficient**; hard to implement.

Lazy evaluation.

- ➡ Actual parameter evaluated **once** when the argument is used.
- ➡ Result cached.

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Mainly used in **purely-functional languages**: requires that time of evaluation does not impact result.

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Positional Parameters

How are actual parameters and the resulting arguments matched to formal parameters?

Matched one-to-one, based on index.

- Order of formal parameters determines the order in which actual parameters must occur.
- **Simple** to understand and implement.
- Sometimes too **inflexible** or **inconvenient**.
  - Infrequently used options must always be specified.
  - Rigid order required; can be tedious for many parameters.

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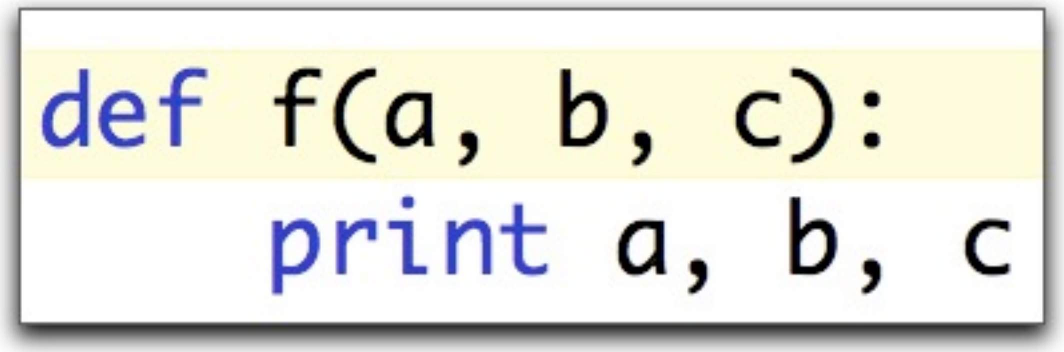

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def f(a, b, c):
    print a, b, c
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Python Shell Output:

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>>> f(1, 2, 3)
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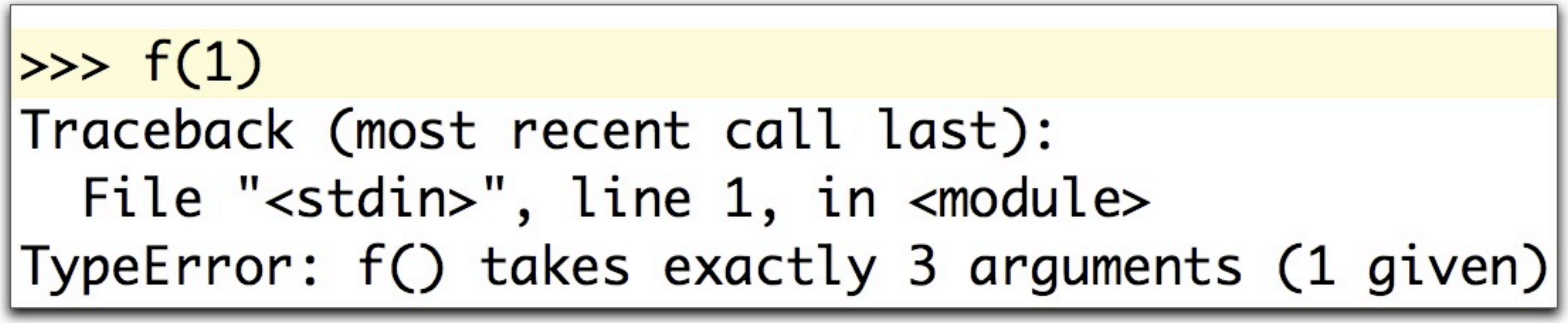

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Specifying **too few or too many** actual parameters results in error.

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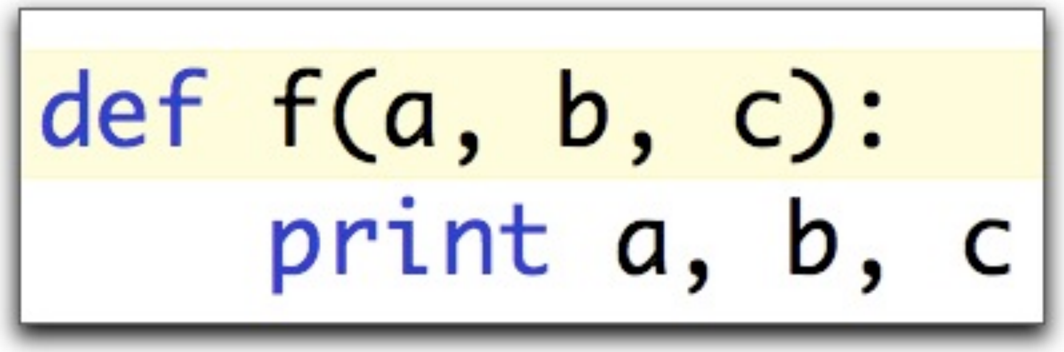

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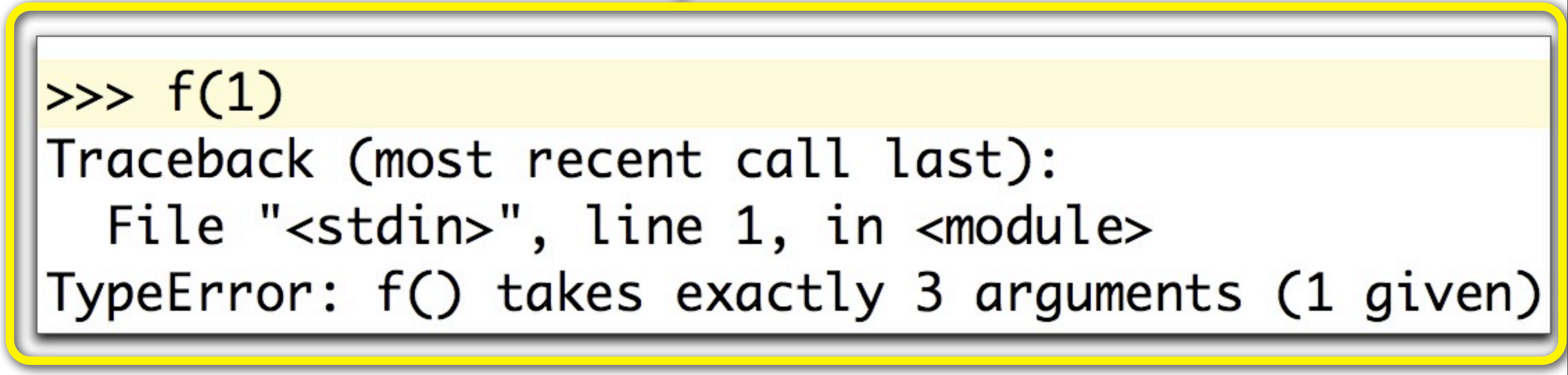

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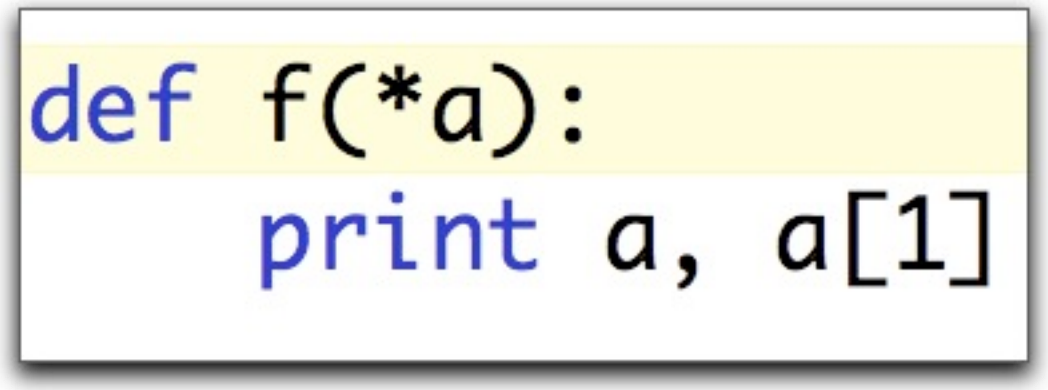

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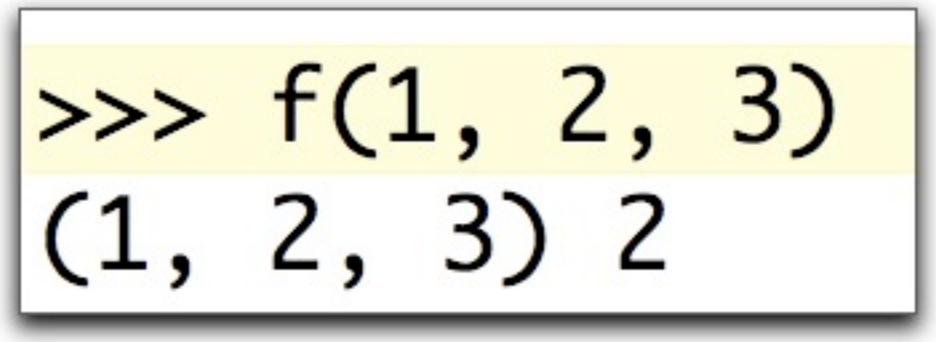

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(1, 2, 3) 2
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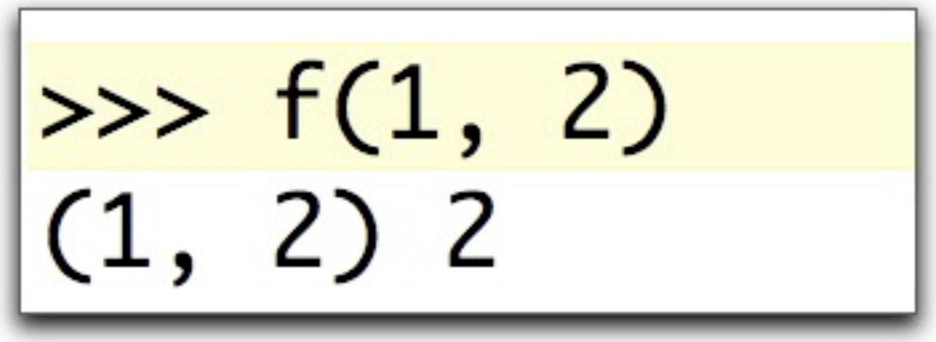

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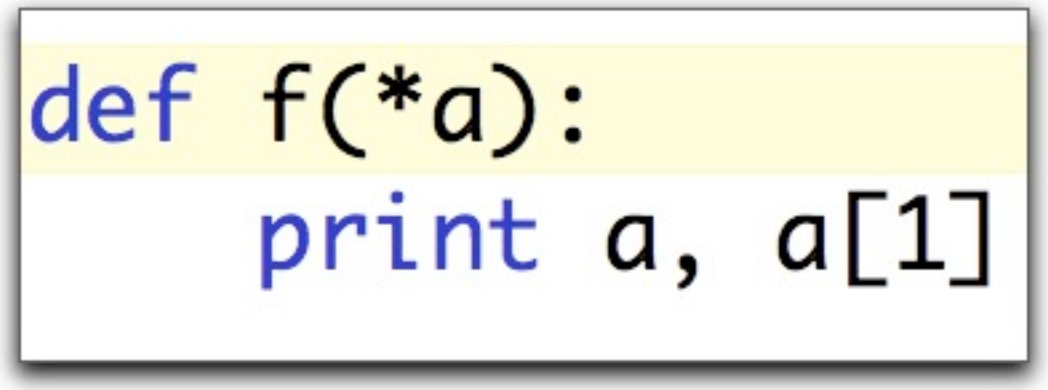

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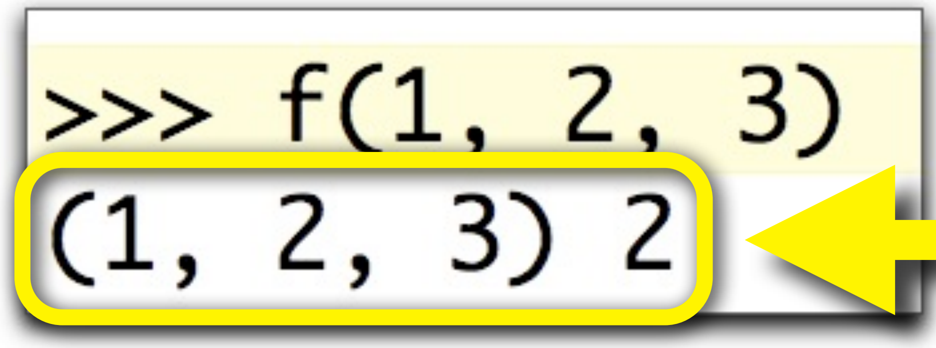

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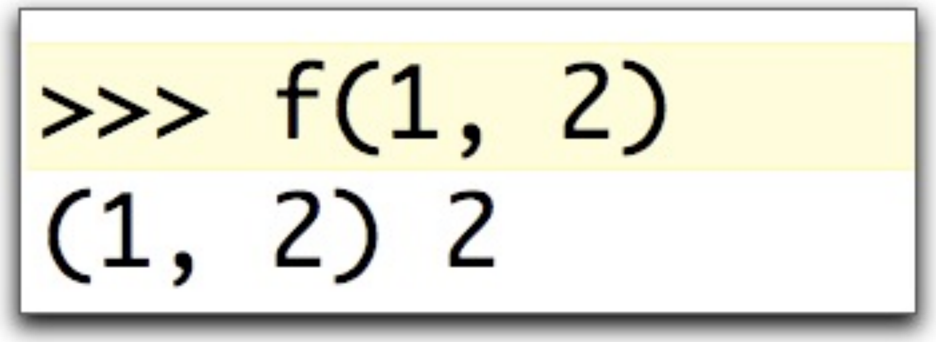

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In Python, all arguments are available as a tuple.

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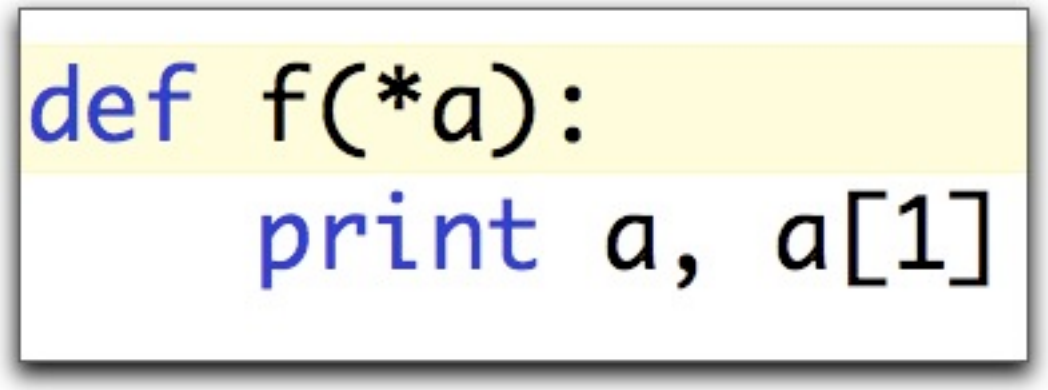

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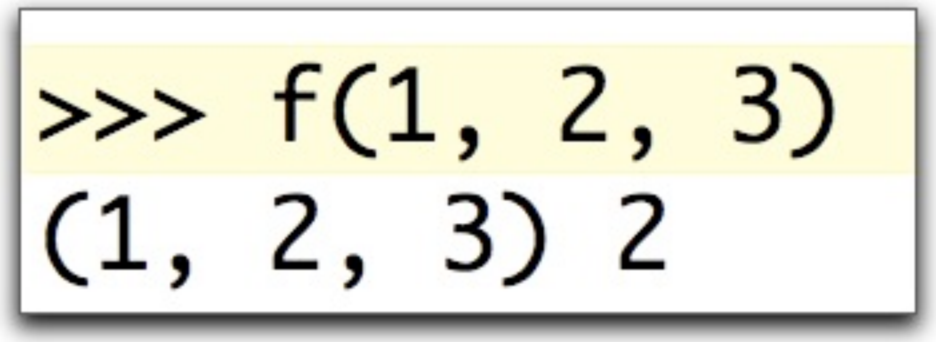

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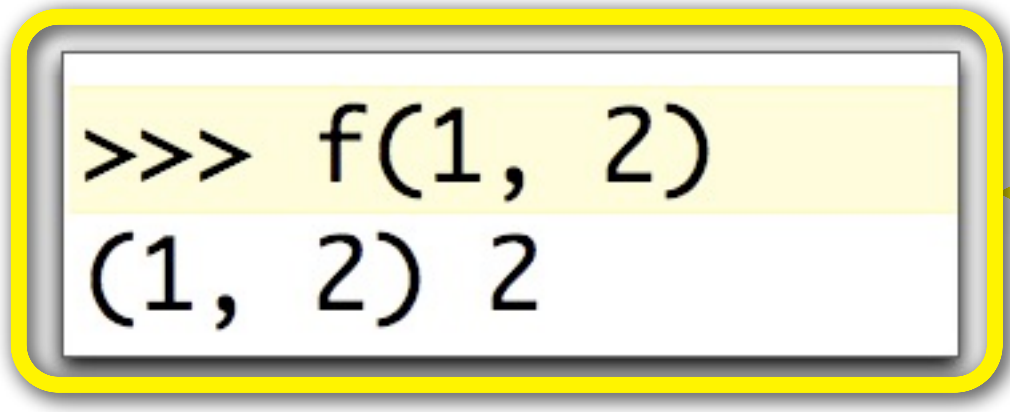

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The number of required parameters not fixed.

Keyword Parameters

How are actual parameters and the resulting arguments matched to formal parameters?

Matched one-to-one, either by position or keyword.

- ➡ Parameter can occur **out of order**.
- ➡ If default value is provided, then parameter **can be omitted**, too.
- ➡ Some languages (e.g., C++) allow only default values, but not keyword parameters.
  - ‣ Result: can be omitted, but not provided out of order.

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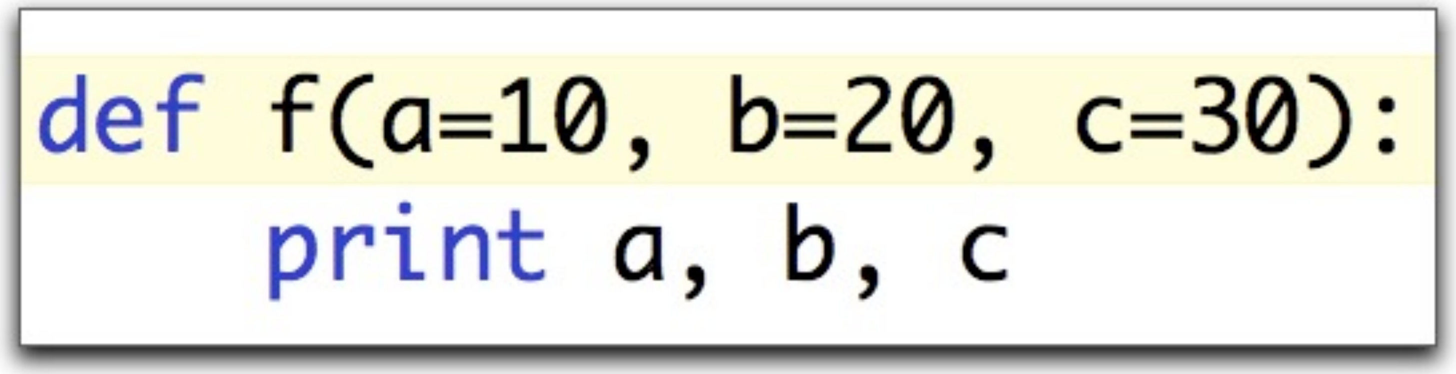
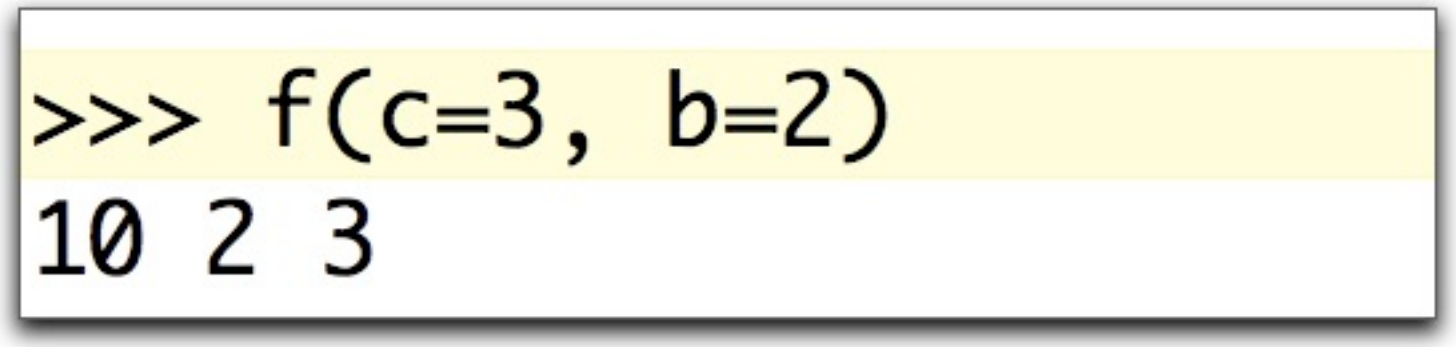


```
def f(a=10, b=20, c=30):
    print a, b, c
```

Python Shell Output:



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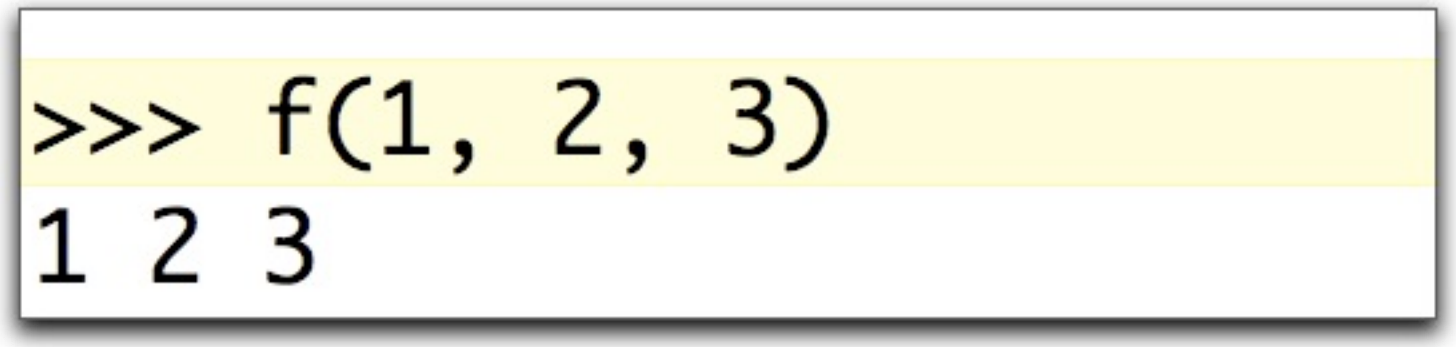

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Keyword Parameters

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Parameters can be **provided as needed**; by naming their **keyword**, they can occur in any order.

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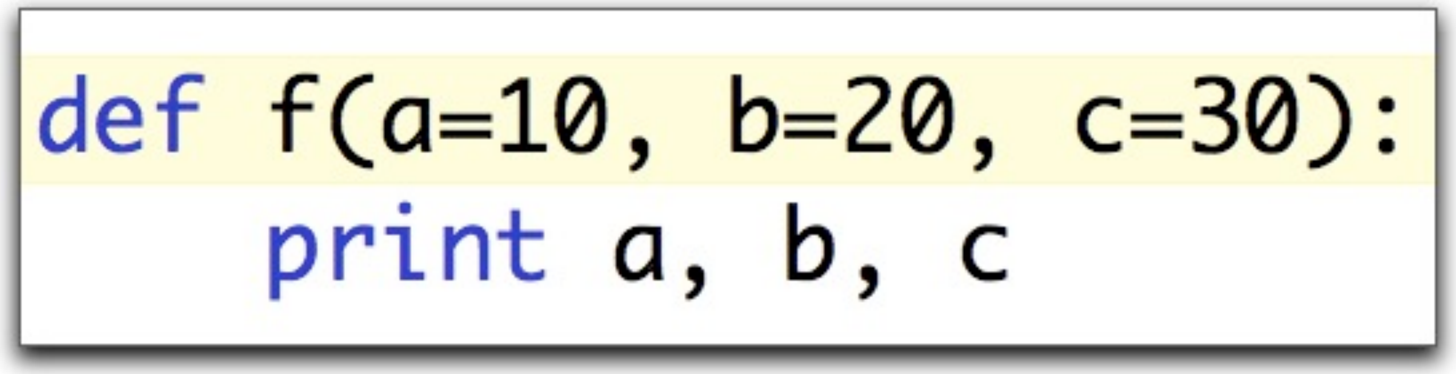

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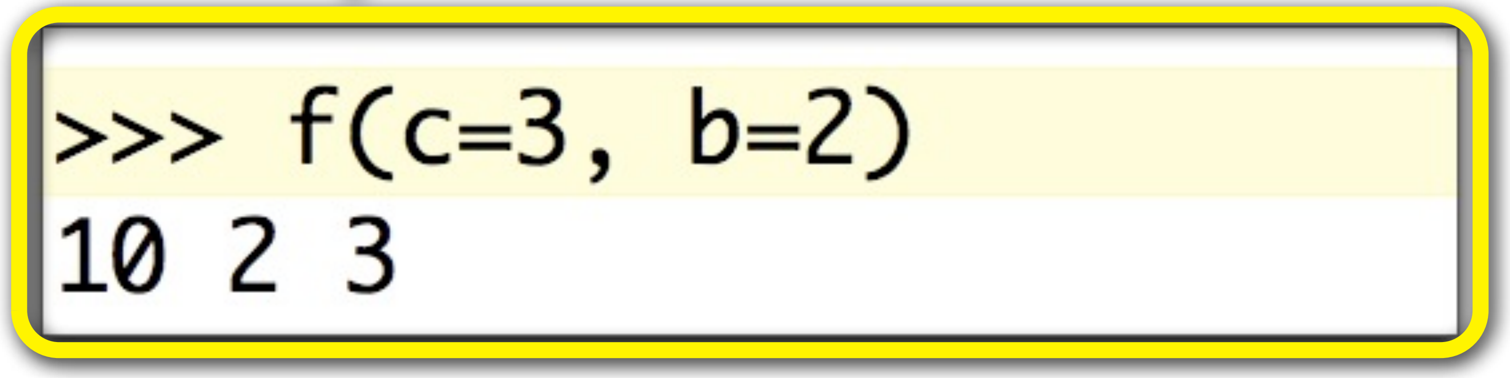

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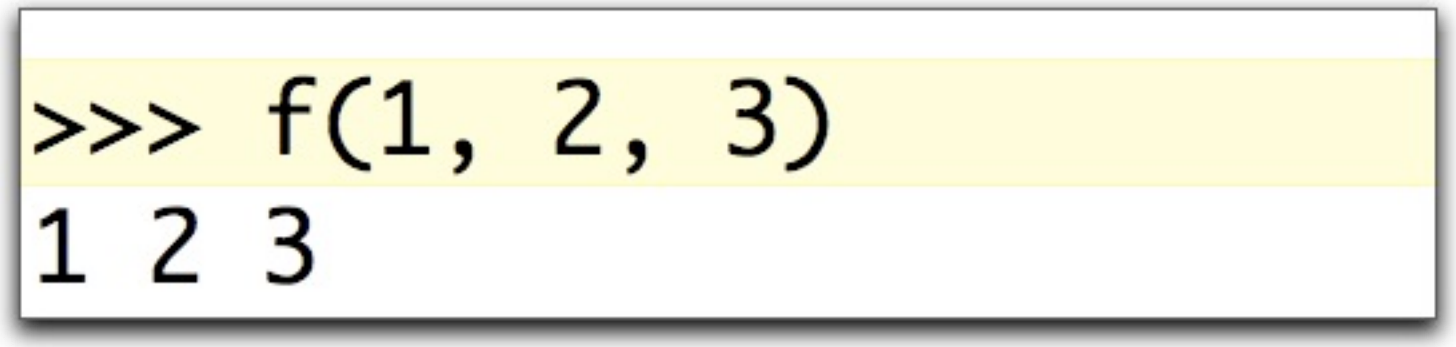

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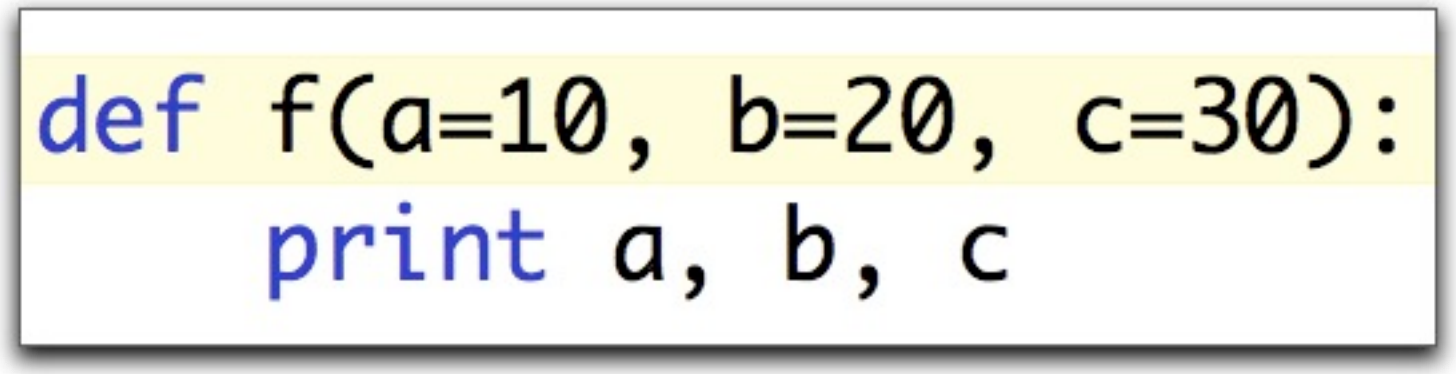
- ➡ If default value is provided, then parameter **can be omitted**, too.
- ➡
  - Result: can be omitted, but not provided out of order.

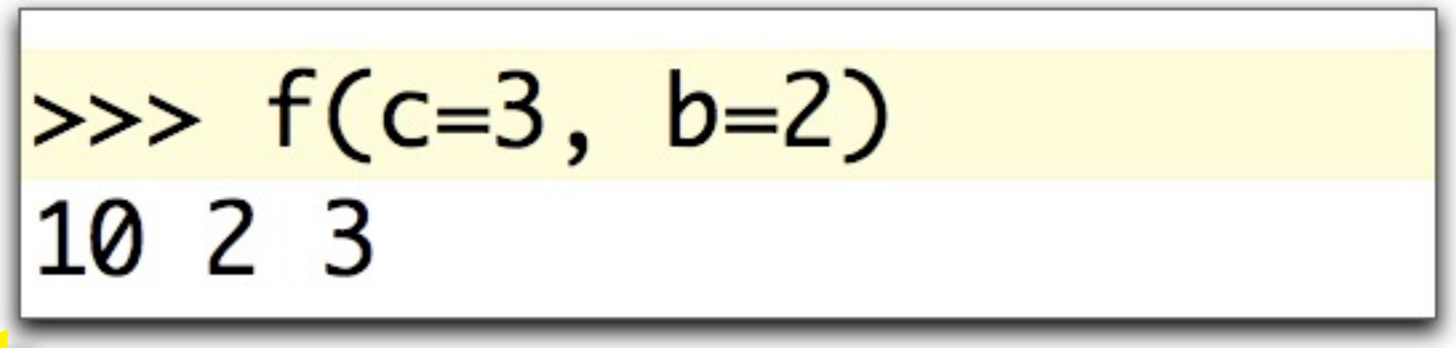
Function can still be called with **positional** parameters.

Python Function Definition

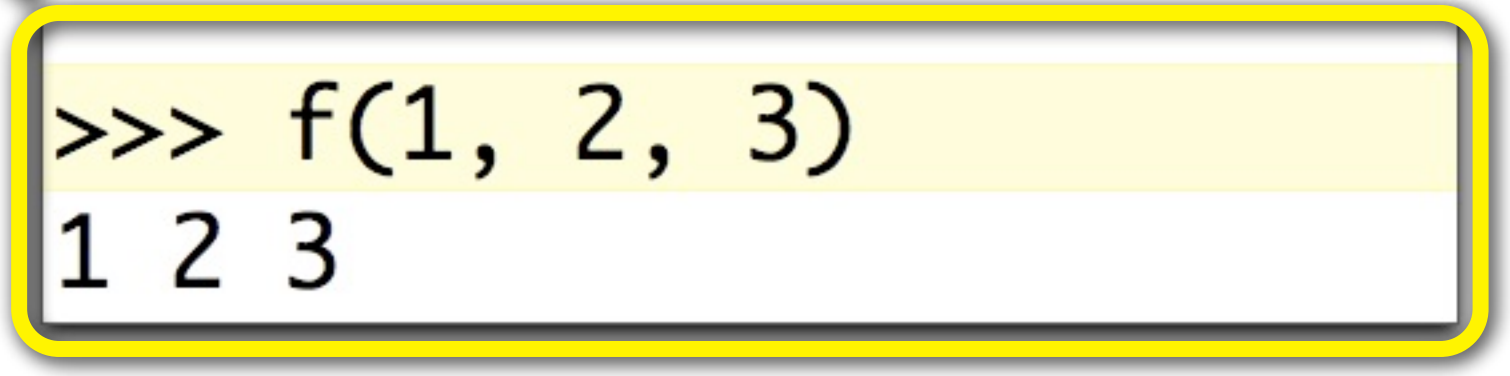


```
def f(a=10, b=20, c=30):
    print a, b, c
```

Python Shell Output:



```
>>> f(c=3, b=2)
10 2 3
```



```
>>> f(1, 2, 3)
1 2 3
```

Parameter Passing: Efficiency

Compile-time.

- ➡ Parameters with **default values** and **keyword parameters** do not necessarily incur additional runtime overheads.
- ➡ Can be **automatically translated** to regular positional parameters.

Run-time.

- ➡ Support for **variable number of parameters** ("varargs") requires **construction of list-like structure** and **iteration**.
- ➡ However, the added flexibility is usually a good tradeoff.

Parameter Passing: Efficiency

Compile-time.

- Parameters with **default values** and **keyword parameters** do

Example: C

On x86, most **positional parameters** are passed through **registers** (**fast**), but varargs must be passed via the **stack** (**slower**).

R

- However, the added flexibility is usually a good tradeoff.

Recursion

$$fib(n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ fib(n-1) + fib(n-2) & \text{otherwise} \end{cases}$$

Definition of the Fibonacci Sequence for n ≥ 0.

A subroutine that calls itself.

- ➡ Either directly or indirectly.
- ➡ Requires runtime stack.

Repetition without loops.

- ➡ Natural fit for “divide-and-conquer” algorithms.
  - E.g., Quicksort.
- ➡ From a math point of view:
  - **recursion is natural**;
  - **loops can be difficult to reason about**.

```
naive_fib(0, 0) :- !.
naive_fib(1, 1) :- !.
naive_fib(X, N) :-
  N_1 is N - 1,
  N_2 is N - 2,
  naive_fib(A, N_1),
  naive_fib(B, N_2),
  X is A + B.
```

Naive, recursive computation of Fibonacci numbers in Prolog.

Recursion

$$fib(n) = \begin{cases} 0 & \text{if } n = 0 \\ 1 & \text{if } n = 1 \\ fib(n-1) + fib(n-2) & \text{otherwise} \end{cases}$$

This causes exponential runtime complexity!

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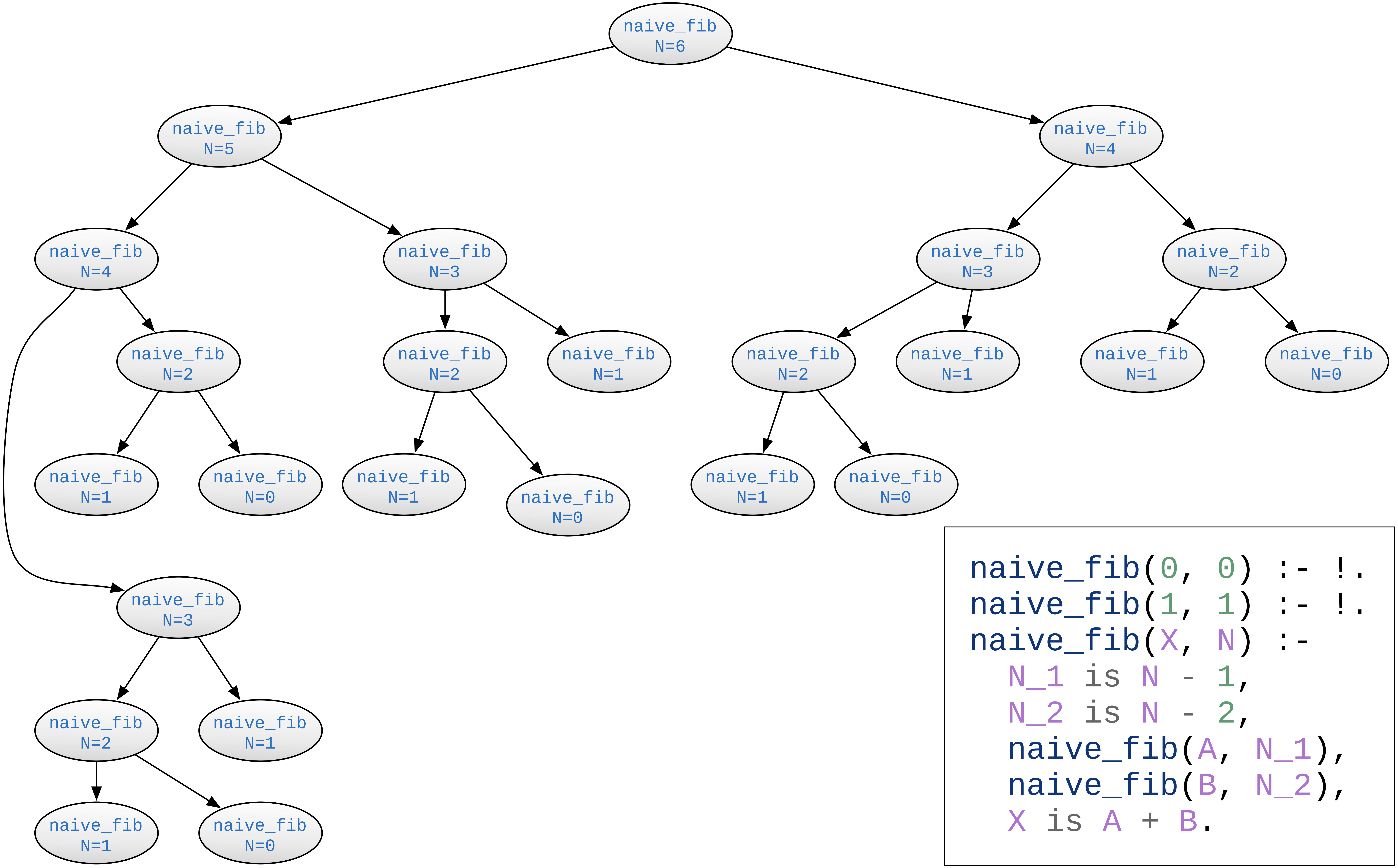
Naive, recursive computation of Fibonacci numbers in Prolog.

Exponential Call Tree for `naive_fib(X, 6)`



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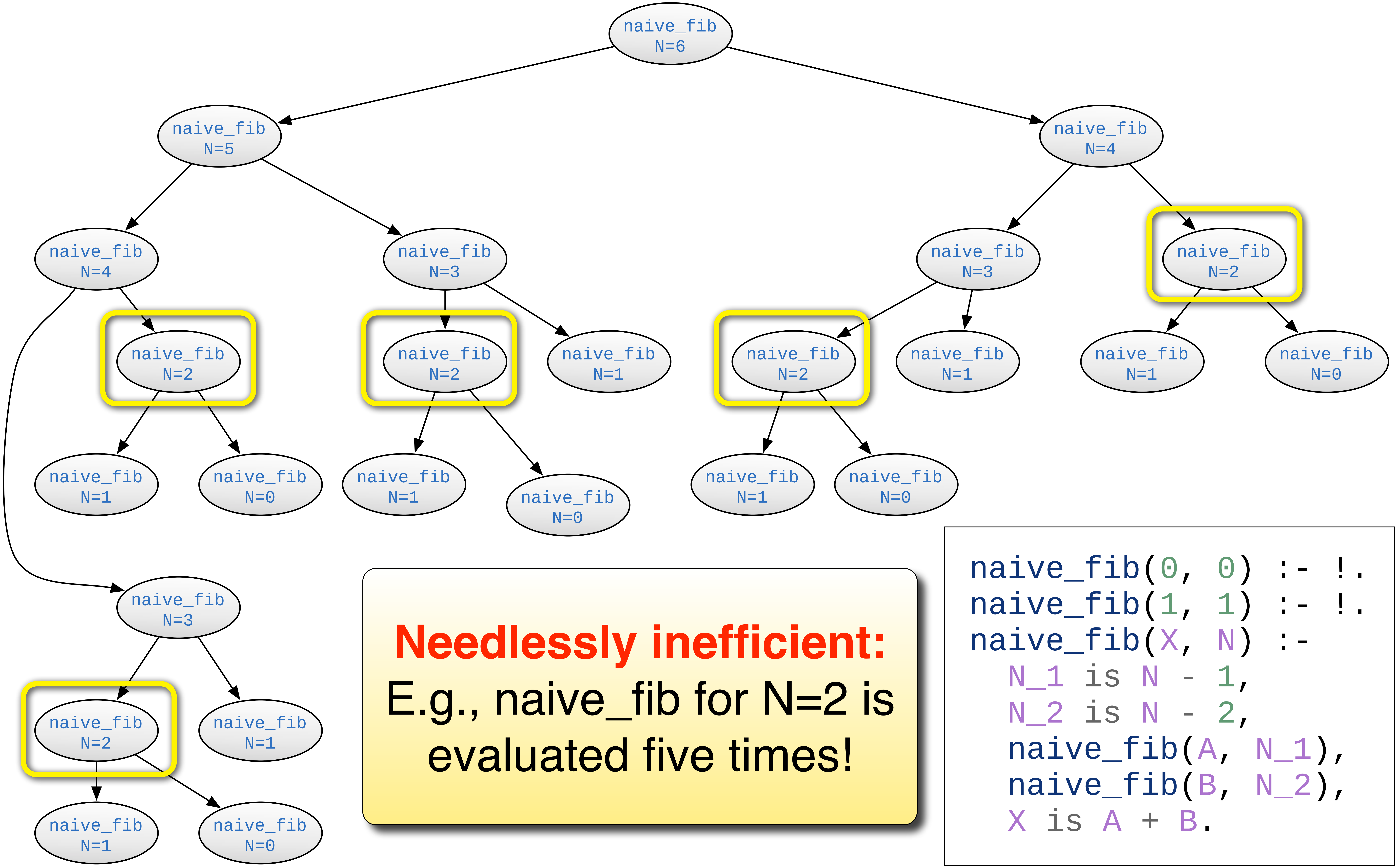

Needlessly inefficient:
E.g., naive_fib for N=2 is evaluated five times!

```
naive_fib(0, 0) :- !.
naive_fib(1, 1) :- !.
naive_fib(X, N) :-
    N_1 is N - 1,
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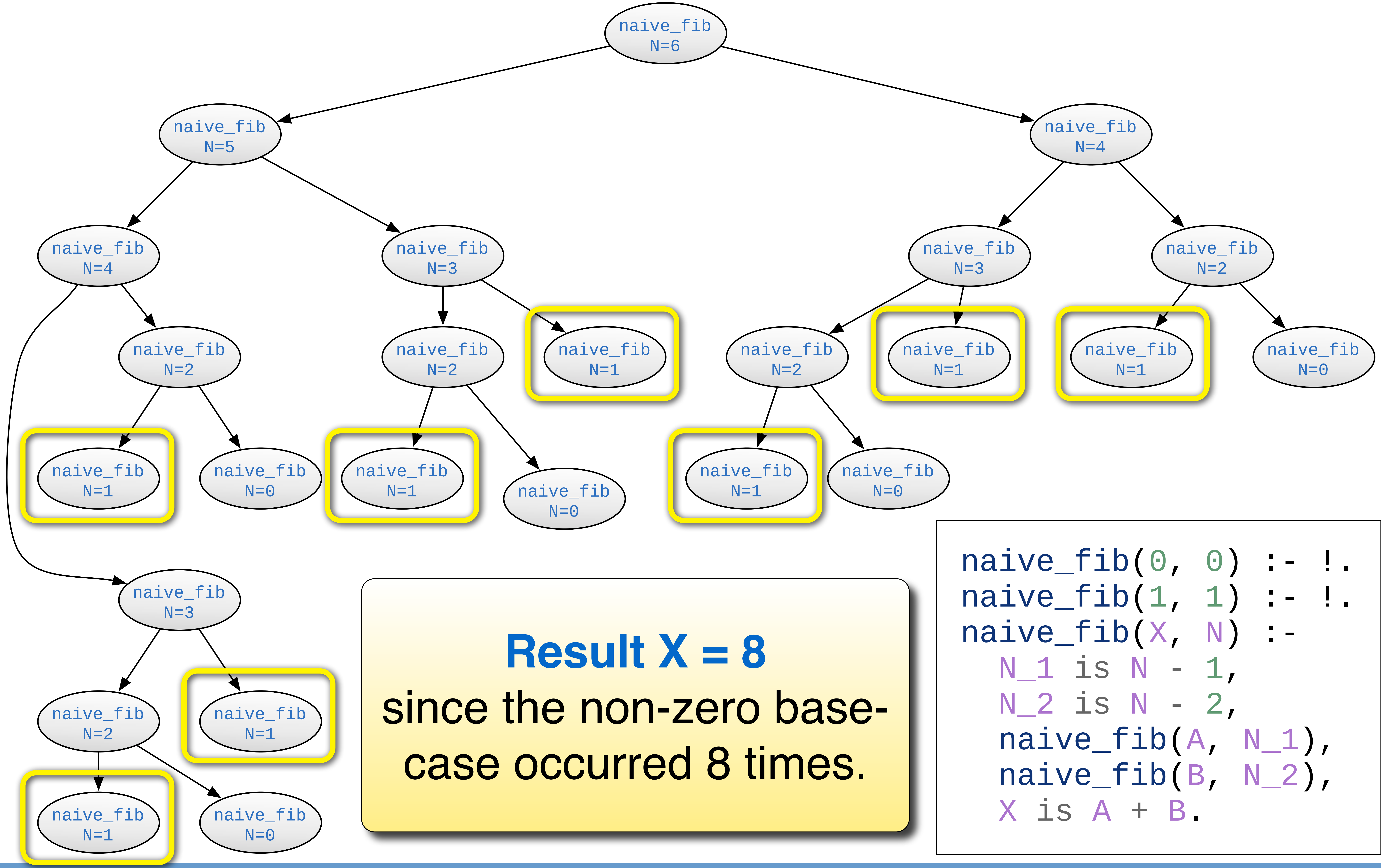

Result X = 8
since the non-zero base-case occurred 8 times.

```
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   N_1 is N - 1,
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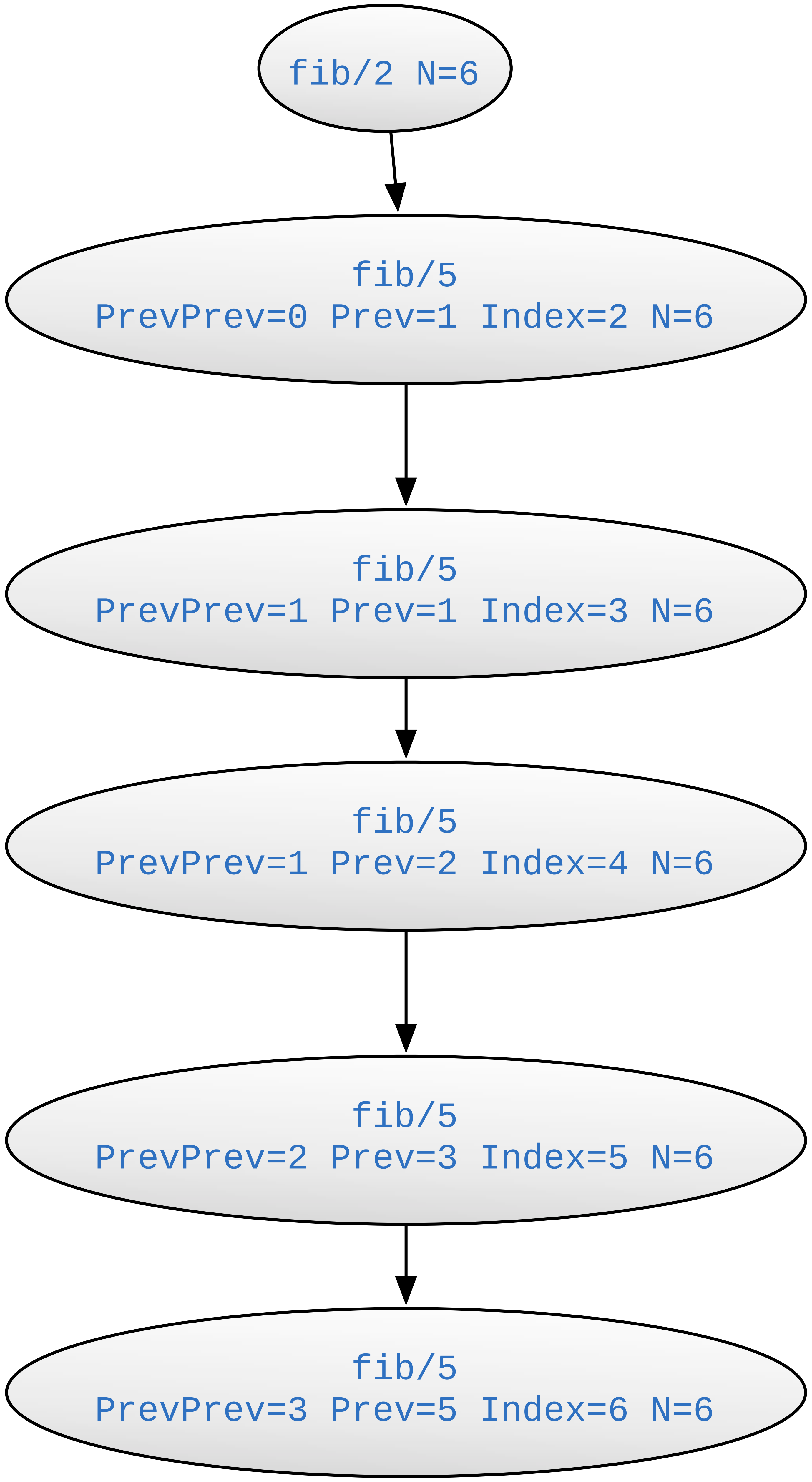
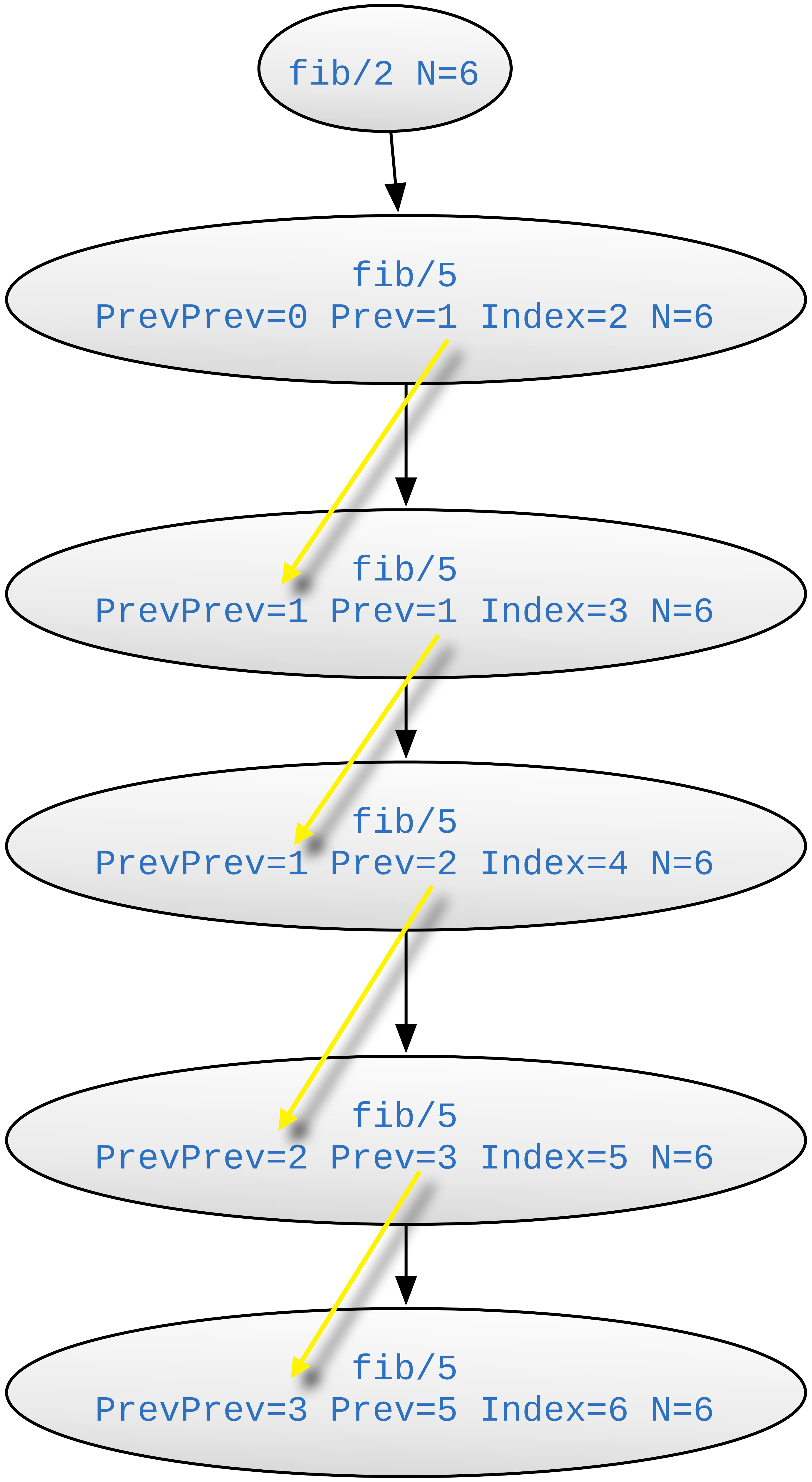
Linear Recursion



```
% fib/2 --- compute the Nth Fibonacci
number.
% Two trivial cases first.
fib(0, 0) :- !.
fib(1, 1) :- !.
% Cases that actually require iteration.
fib(X, N) :-
  fib(0, 1, 2, N, X).

% fib/5 --- Fibonacci helper clause;
%           does the actual iteration.
% Base case: have reached end of iteration.
fib(PrevPrev, Prev, Index, Stop, Res) :-
  Index = Stop, !,
  Res is PrevPrev + Prev.
% Recursive case: have not yet reached the
% desired index.
fib(PrevPrev, Prev, Index, Stop, Res) :-
  Index < Stop,
  Cur  is PrevPrev + Prev,
  Next is Index + 1,
  fib(Prev, Cur, Next, Stop, Res).
```

Linear Recursi



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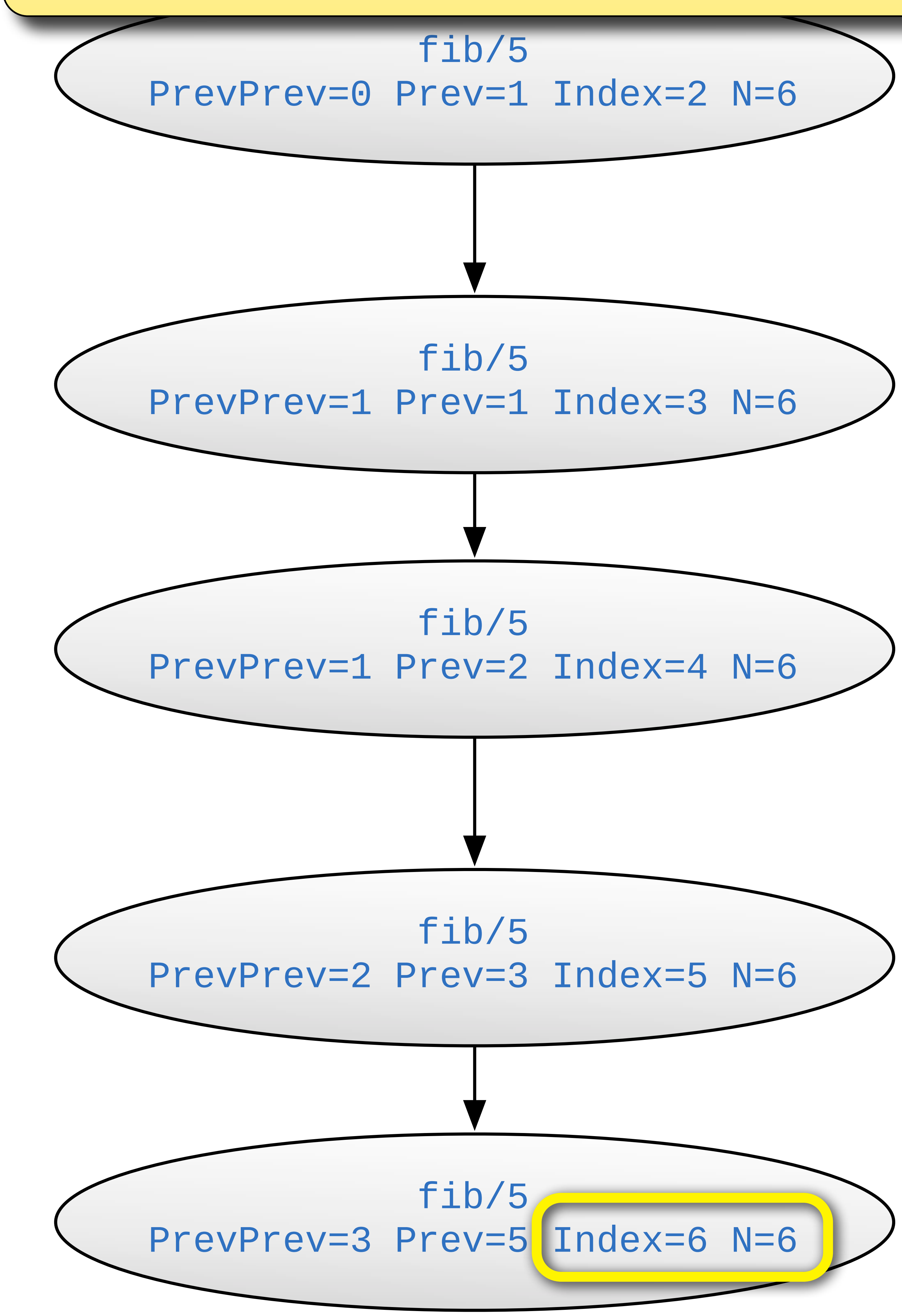
```

Auxiliary values are kept for the iterations where they are needed.

Iteration ends when desired index is reached.
At this point, computing the result is simple since
both previous Fibonacci numbers are known.

X = Res = 8



fib/5
PrevPrev=0 Prev=1 Index=2 N=6

fib/5
PrevPrev=1 Prev=1 Index=3 N=6

fib/5
PrevPrev=1 Prev=2 Index=4 N=6

fib/5
PrevPrev=2 Prev=3 Index=5 N=6

fib/5
PrevPrev=3 Prev=5 Index=6 N=6

```
onacci

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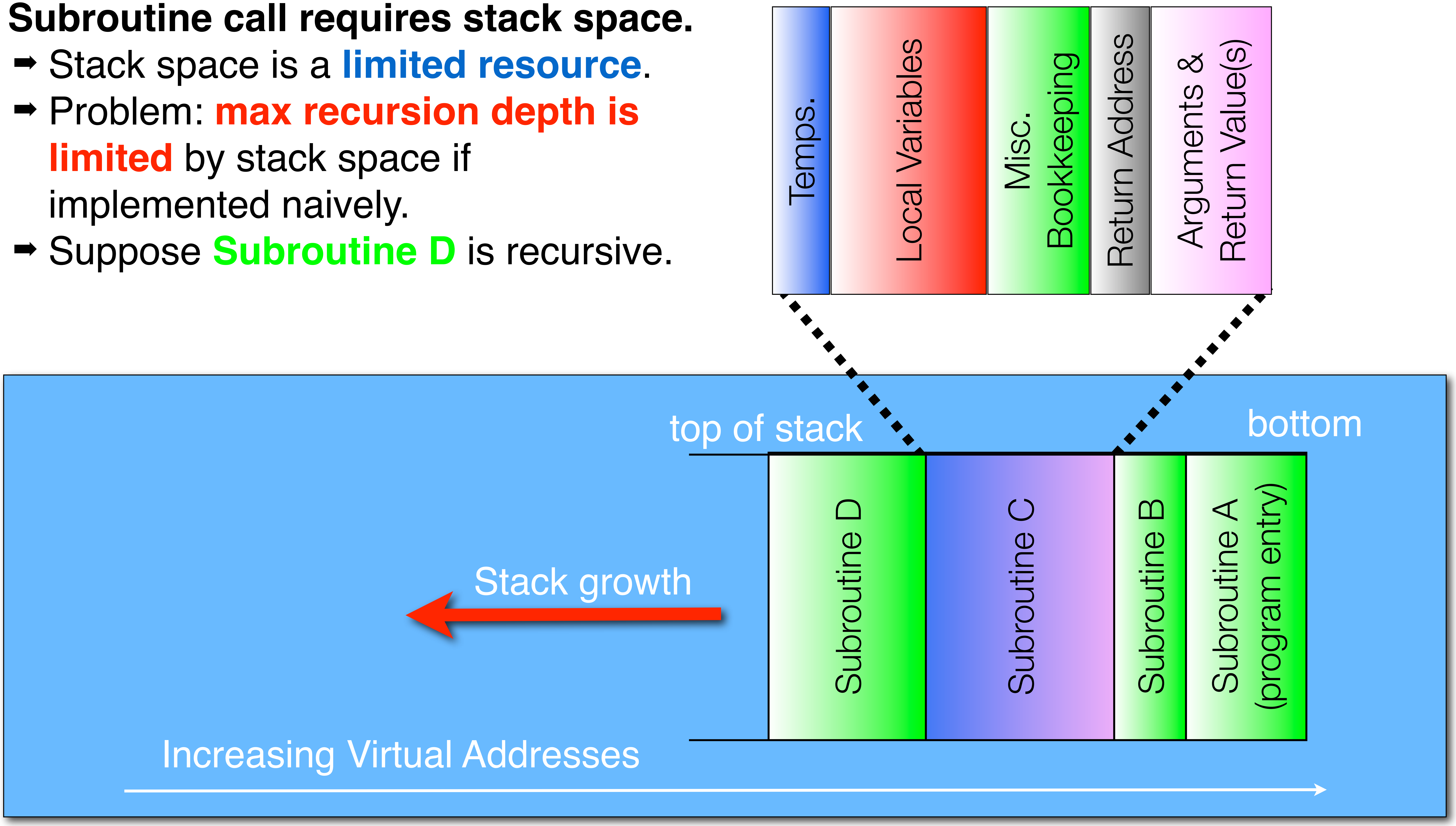
  fib(Prev, Cur, Next, Stop, Res).
```

Stack Overflow

Subroutine call requires stack space.

- Stack space is a **limited resource**.
- Problem: **max recursion depth is limited** by stack space if implemented naively.
- Suppose **Subroutine D** is recursive.

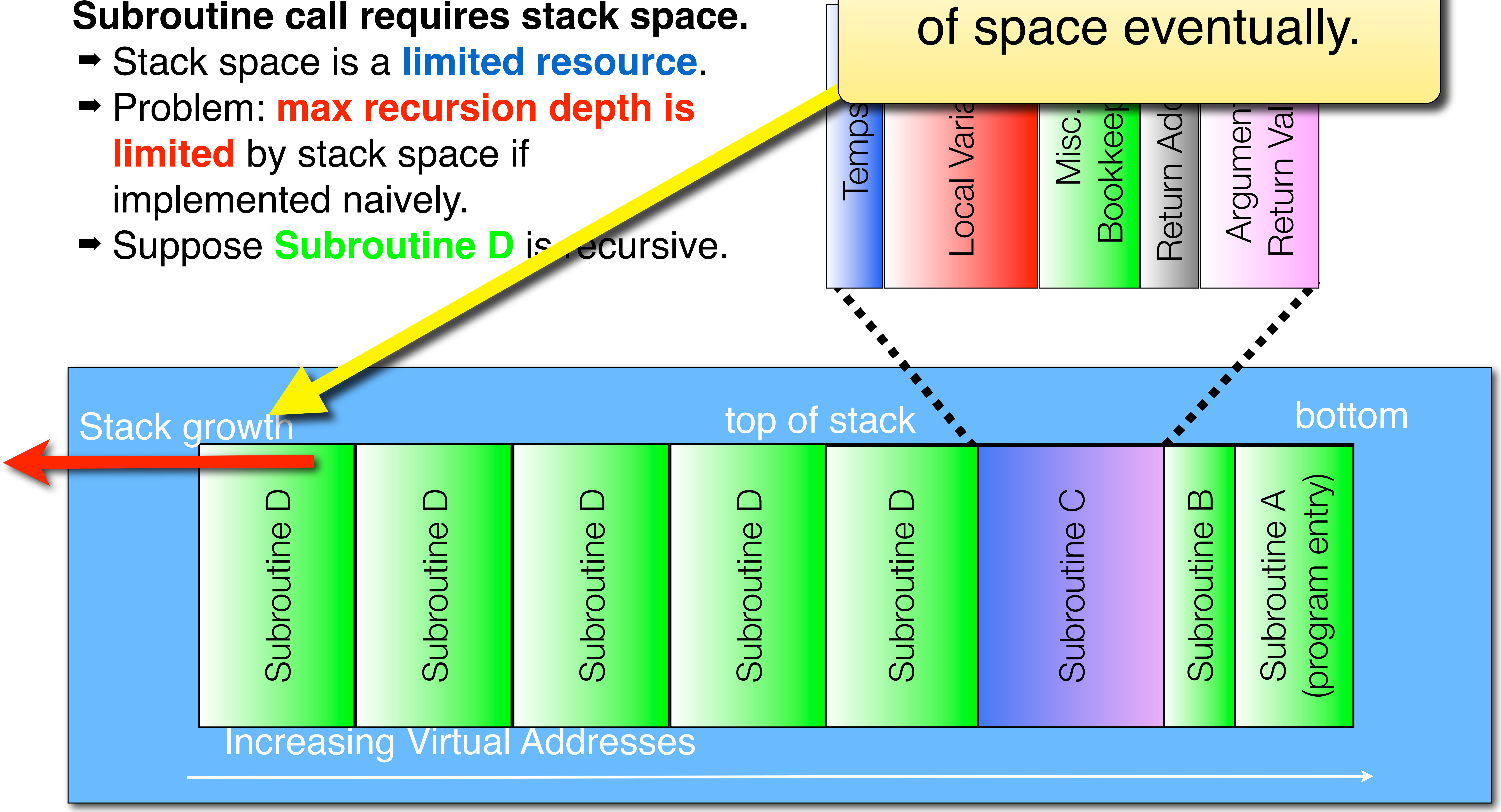


Stack Overflow

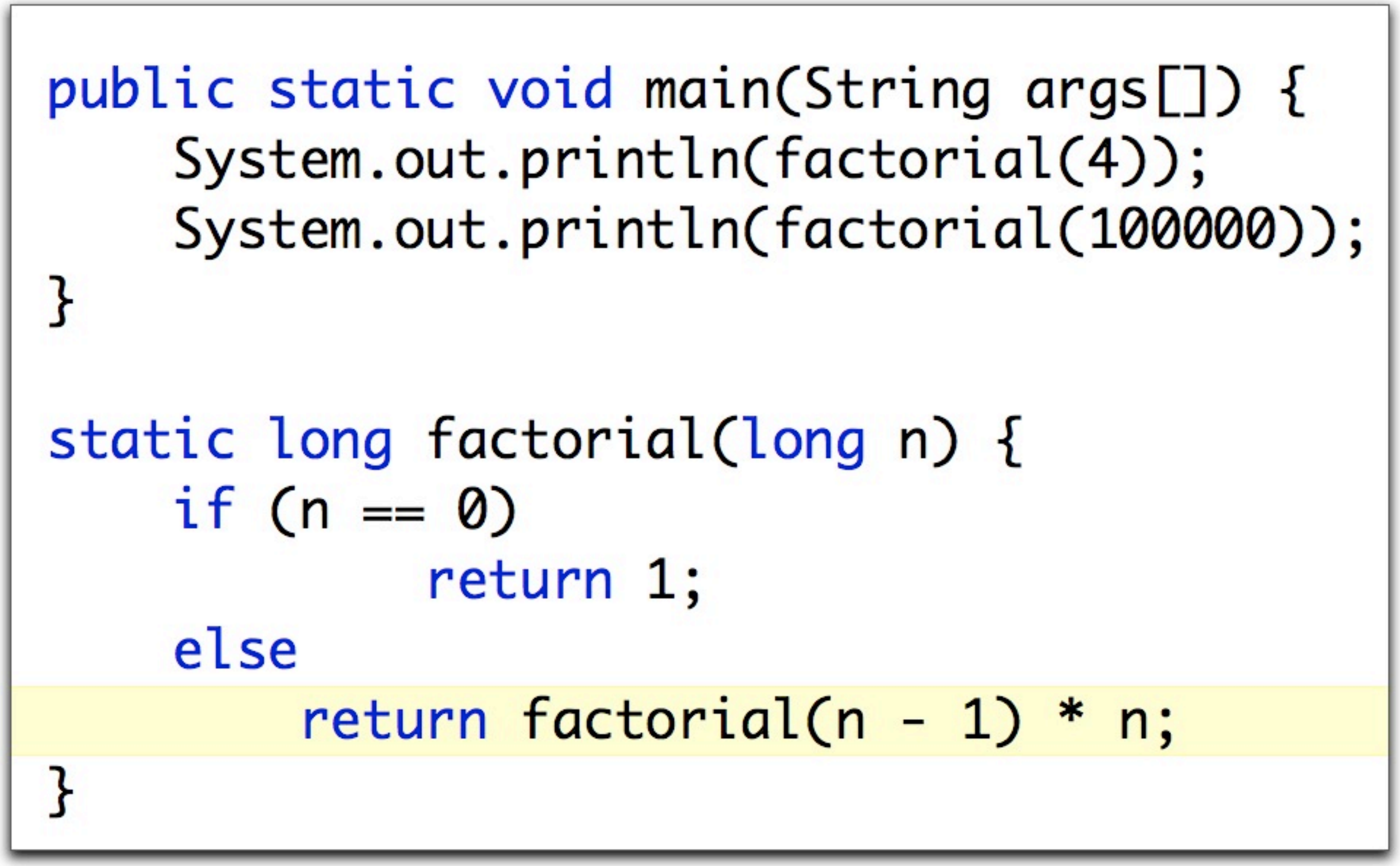
Subroutine call requires stack space.

- Stack space is a **limited resource**.
- Problem: **max recursion depth is limited** by stack space if implemented naively.
- Suppose **Subroutine D** is recursive.

The recursion will run out of space eventually.



Stack Overflow Example



```
public static void main(String args[]) {
    System.out.println(factorial(4));
    System.out.println(factorial(100000));
}

static long factorial(long n) {
    if (n == 0)
            return 1;
    else
        return factorial(n - 1) * n;
}
```

Output:

```
24
Exception in thread "main"
java.lang.StackOverflowError
  at Factorial.factorial(Factorial.java:18)
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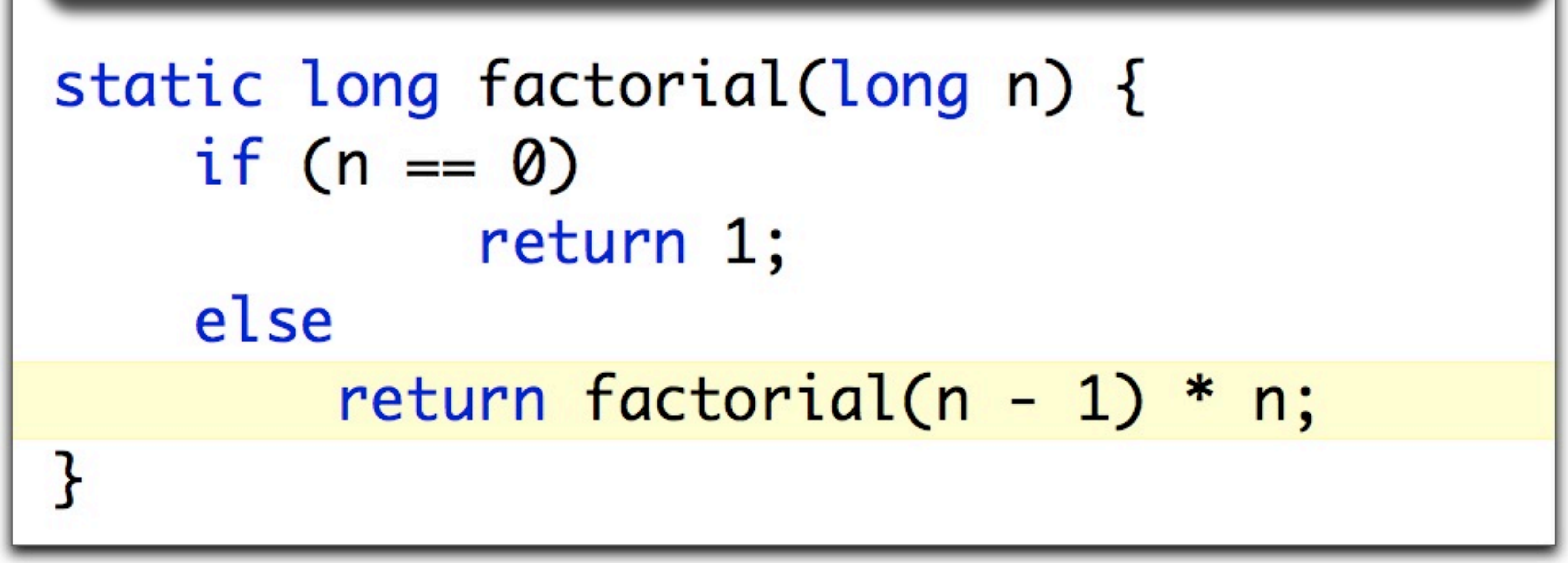
  (repeated several thousand times)
```

Stack Overflow Example

So how can we implement **arbitrary loops** with recursion if we have only finite memory?



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```

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  (repeated several thousand times)
```

Tail Recursion

If a **recursive call is the last statement/expression** of a subroutine to be evaluated, then the **already-allocated stack frame of the caller is reused**.

Stack frame = local execution context.

- ➡ If nothing remains to be executed, then stack frame contents are no longer required.
- ➡ Conceptually, instead of allocating a new stack frame, **the compiler simply generates a jump** to the beginning of the subroutines code.
  - ‣ A bit more complicated with indirect recursion…
- ➡ **Elegant recursion compiled to efficient loop**.

Tail Recursion Example

Prolog supports proper tail recursion.

```
naive_fact(1, 0) :- !.
naive_fact(X, N) :-
  Prev is N - 1,
  naive_fact(Y, Prev),
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```

```
fact(X, N) :- fact(X, N, 1, 0).
fact(X, N, Accumulator, Index) :-
  Index = N, !,
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  Next is Index + 1,
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  fact(X, N, Fact, Next).
```

Output:

```
?- naive_fact(X, 1000).
ERROR: Out of local stack
?- fact(X, 1000).
X =
402387260077093773543702433923003985719374864210714632543799910429938512398629020592044208486969404800479988610197196058631666872994808558901323829669944590997424504087073759918
823627727188732519779505950995276120874975462497043601418278094646496291056393887437886487337119181045825783647849977012476632889835955735432513185323958463075557409114262417474
349347553428646576611667797396668820291207379143853719588249808126867838374559731746136085379534524221586593201928090878297308431392844403281231558611036976801357304216168747609
675871348312025478589320767169132448426236131412508780208000261683151027341827977704784635868170164365024153691398281264810213092761244896359928705114964975419909342221566832572
080821333186116811553615836546984046708975602900950537616475847728421889679646244945160765353408198901385442487984959953319101723355556602139450399736280750137837615307127761926
849034352625200015888535147331611702103968175921510907788019393178114194545257223865541461062892187960223838971476088506276862967146674697562911234082439208160153780889893964518
263243671616762179168909779911903754031274622289988005195444414282012187361745992642956581746628302955570299024324153181617210465832036786906117260158783520751516284225540265170
483304226143974286933061690897968482590125458327168226458066526769958652682272807075781391858178889652208164348344825993266043367660176999612831860788386150279465955131156552036
093988180612138558600301435694527224206344631797460594682573103790084024432438465657245014402821885252470935190620929023136493273497565513958720559654228749774011413346962715422
845862377387538230483865688976461927383814900140767310446640259899490222221765904339901886018566526485061799702356193897017860040811889729918311021171229845901641921068884387121
855646124960798722908519296819372388642614839657382291123125024186649353143970137428531926649875337218940694281434118520158014123344828015051399694290153483077644569099073152433
278288269864602789864321139083506217095002597389863554277196742822248757586765752344220207573630569498825087968928162753848863396909959826280956121450994871701244516461260379029
309120889086942028510640182154399457156805941872748998094254742173582401063677404595741785160829230135358081840096996372524230560855903700624271243416909004153690105933983835777
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Recursive sub-goal is **not last** to be evaluated!

Output:

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263243671616762179168909779911903754031274622289988005195444414282012187361745992642956581746628302955570299024324153181617210465832036786906117260158783520751516284225540265170
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093988180612138558600301435694527224206344631797460594682573103790084024432438465657245014402821885252470935190620929023136493273497565513958720559654228749774011413346962715422
845862377387538230483865688976461927383814900140767310446640259899490222221765904339901886018566526485061799702356193897017860040811889729918311021171229845901641921068884387121
855646124960798722908519296819372388642614839657382291123125024186649353143970137428531926649875337218940694281434118520158014123344828015051399694290153483077644569099073152433
278288269864602789864321139083506217095002597389863554277196742822248757586765752344220207573630569498825087968928162753848863396909959826280956121450994871701244516461260379029
309120889086942028510640182154399457156805941872748998094254742173582401063677404595741785160829230135358081840096996372524230560855903700624271243416909004153690105933983835777
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8236277271887325197795059509952761208749754624970436014182780946464962910563938874378864873371191810458257836  849977012476632889835955735432513185323958463075557409114262417474
3493475534286465766116677973966688202912073791438537195882498                                                                                                       57304216168747609
6758713483120254785893207671691324484262361314125087802080002                                                                                                       09342221566832572
0808213331861168115536158365469840467089756029009505376164758                                                                                                       37615307127761926
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2632436716167621791689097799119037540312746222899880051954444                                                                                                       16284225540265170
4833042261439742869330616908979684825901254583271682264580665                                                                                                       65955131156552036
0939881806121385586003014356945272242063446317974605946825731                                                                                                       11413346962715422
8458623773875382304838656889764619273838149001407673104466402                                                                                                       41921068884387121
8556461249607987229085192968193723886426148396573822911231250                                                                                                       44569099073152433
2782882698646027898643211390835062170950025973898635542771967                                                                                                       44516461260379029
3091208890869420285106401821543994571568059418727489980942547                                                                                                       90105933983835777
9394109700277534720000000000000000000000000000000000000000000                                                                                                       00000000000000000
0000000000000000000000000000000000000000000000000000000000000
```

Proper tail recursion: recursive sub-goal occurs last.

Inline Expansion

Subroutine granularity.

- ➡ **Using many, very short subroutines is good** software engineering practice.
  - ‣ Easier to understand and debug.
- ➡ However, **subroutine calls incur overhead**.

Inline subroutines.

- ➡ Semantically, like a normal subroutine.
  - ‣ **Type checking**, etc.
- ➡ However, instead of generating a call, compiler "**copy&pastes**" **subroutine code into caller**.
  - ‣ Like macro expansion.
  - ‣ **Increases code size**, but **call overhead is avoided**.

Inline Expansion Example

```
#include <stdio.h>

int normal_function(void)
{
    return 1;
}


inline int inline_function(void)
{
    return 2;
}

int main(int argc, char** argv)
{
    printf("result = %d\n",
       normal_function() + inline_function());
    return 0;
}
```

C99 Example.

Inline Expansion Example

```
#include <stdio.h>

int normal_function(void)
{
    return 1;
}


inline int inline_function(void)
{
    return 2;
}

int main(int argc, char** argv)
{
    printf("result = %d\n",
        normal_function() + inline_function());
    return 0;
}
```

Inline keyword is a **hint** to the compiler to include body instead of generating a call.

C99 Example.

Inline Expansion Example

```
080483a4 <normal_function>:
 80483a4:       55                      push   %ebp
 80483a5:       89 e5                   mov    %esp,%ebp
 80483a7:       b8 01 00 00 00          mov    $0x1,%eax
 80483ac:       5d                      pop    %ebp
 80483ad:       c3                      ret

080483b8 <main>:
 80483b8:       55                      push   %ebp
 80483b9:       89 e5                   mov    %esp,%ebp
 80483bb:       83 e4 f0                and    $0xfffffff0,%esp
 80483be:       83 ec 10                sub    $0x10,%esp
 80483c1:       e8 de ff ff ff          call   80483a4 <normal_function>
 80483c6:       83 c0 02                add    $0x2,%eax
 80483c9:       89 44 24 04             mov    %eax,0x4(%esp)
 80483cd:       c7 04 24 a0 84 04 08    movl   $0x80484a0,(%esp)
 80483d4:       e8 ff fe ff ff          call   80482d8 <printf@plt>
 80483d9:       b8 00 00 00 00          mov    $0x0,%eax
 80483de:       c9                      leave
 80483df:       c3                      ret
```

Generated machine code.

Inline Expansion

Call generated for normal function.

```
080483a4 <normal_function>:
 80483a4:       55                      push   %ebp
 80483a5:       89 e5                   mov    %esp,%ebp
 80483a7:       b8 01 00 00 00          mov    $0x1,%eax
 80483ac:       5d                      pop    %ebp
 80483ad:       c3                      ret

080483b8 <main>:
 80483b8:       55                      push   %ebp
 80483b9:       89 e5                   mov    %esp,%ebp
 80483bb:       83 e4 f0                and    $0xfffffff0,%esp
 80483be:       83 ec 10                sub    $0x10,%esp
 80483c1:       e8 de ff ff ff          call   80483a4 <normal_function>
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```

Generated machine code.

Inline Expansion Example

```
080483a4 <normal_function>:
                                        push   %ebp
                                        mov    %esp,%ebp
                                        mov    $0x1,%eax
                                        pop    %ebp
                                        ret

080483b8 <main>:
 80483b8:       55                      push   %ebp
 80483b9:       89 e5                   mov    %esp,%ebp
 80483bb:       83 e4 f0                and    $0xfffffff0,%esp
 80483be:       83 ec 10                sub    $0x10,%esp
 80483c1:       e8 de ff ff ff          call   80483a4 <normal_function>
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 80483d9:       b8 00 00 00 00          mov    $0x0,%eax
 80483de:       c9                      leave
 80483df:       c3                      ret
```

The “`return 2`” was inlined; no call to inline_function generated.

Generated machine code.

Exception Handling

How to report errors?

With error or return codes.

- ➡ Commonly done in C.
- ➡ Tedious and error-prone.
  - ‣ Hard to read, complex control flow.
  - ‣ Easy to forget.

With (unstructured) jumps.

- ➡ Also error prone.

Exceptions: structured error handling.

- ➡ **Checked** exceptions: **anticipated failures** that can occur in correct program.
  - ‣ e.g., `IOException`: user could have specified incorrect file.
- ➡ **Unchecked** exceptions: errors that indicate **programmer error** or catastrophic **system failure**.
  - ‣ e.g., `IllegalArgumentException`: misuse of API.
  - ‣ e.g., `OutOfMemoryError`: program can't do anything about it.

Exception Handling

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- ➡ Also error pro

In many languages (e.g., C++, Python,…), all exceptions are unchecked.

(<u>checked</u>: compiler raises error if possible exception is not handled or propagated)

Exceptions: structured error handling.

- ➡ **Checked** exceptions: **anticipated failures** that can occur in correct program.
  - ‣ e.g., `IOException`: user could have specified incorrect file.
- ➡ **Unchecked** exceptions: errors that indicate **programmer error** or catastrophic **system failure**.
  - ‣ e.g., `IllegalArgumentException`: misuse of API.
  - ‣ e.g., `OutOfMemoryError`: program can’t do anything about it.

Expression Evaluation

Statement vs. Expression

- ➡Imperative languages often differentiate between “statements” and “expressions”.
- ➡Functional languages usually focus on expressions.

Expressions

- ➡Can be evaluated to yield a **value**.
- ➡E.g., in Java, “`1 + 2`”, “`Math.sqrt(2)`”.

Statements

- ➡Give imperative languages **sequential nature**.
- ➡E.g., in Java, “`if`” is a statement; it cannot occur in expressions.

Expression Evaluation

Expressions usually consist of **operators**, **operands** (literals, variables, and subexpressions), and subroutine calls.

➡Functional languages usually focus on expressions.

Expressions

➡Can be evaluated to yield a **value**.

➡E.g., in Java, "`1 + 2`", "`Math.sqrt(2)`".

Statements

➡Give imperative languages **sequential nature**.

➡E.g., in Java, "`if`" is a statement; it cannot occur in expressions.

Unary, Binary, and Ternary Operators

Unary: Operator has single operand.

Example: logical negation

Binary: Operator has two operands.

Examples: logical and, addition

Ternary: Operator has three operands.

Example: `? :` (conditional expression) in C-like languages

Prefix, Infix, and Postfix Operators

Prefix: Operator before Operand

Examples: ++, !

Infix: Operator between Operands

Examples: &&, ||, +=, ==

Postfix: Operator after Operand

Examples: ++

Operators as Function Applications

Operators are not inherently special.

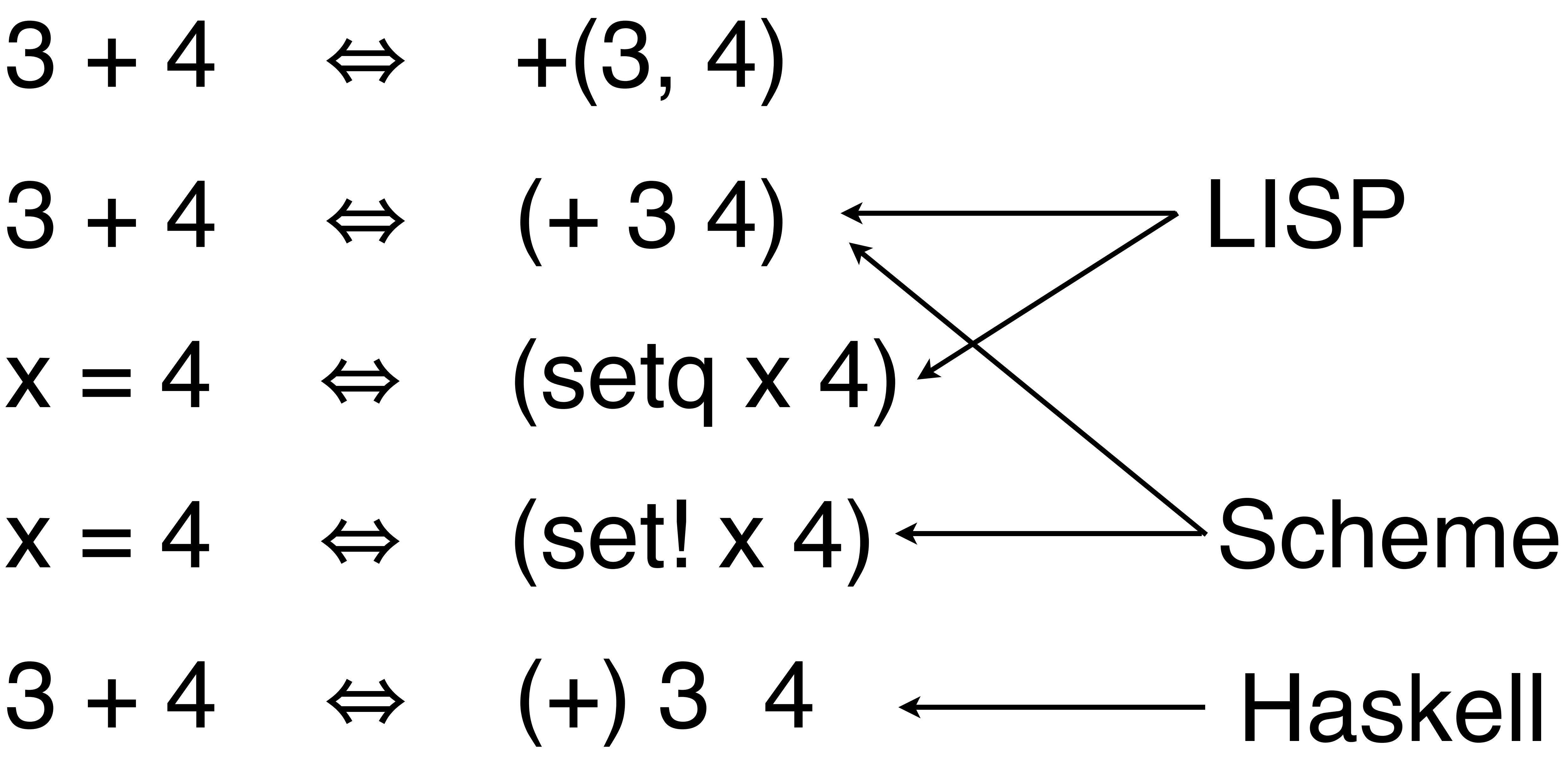
➡ An operator is a **function/subroutine with human-friendly syntax**.

`<operand1> <op> <operand2>`

is the same as

`<op>(<operand1>, <operand2>)`

Examples:



3 + 4 ⇔ +(3, 4)

3 + 4 ⇔ (+ 3 4)

x = 4 ⇔ (setq x 4)

x = 4 ⇔ (set! x 4)

3 + 4 ⇔ (+) 3 4

LISP

Scheme

Haskell

Operators as Function Applications

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`<operand1> <op> <operand2>`

is the same as

`<op>(<operand1>, <operand2>)`

Examples: 3 + 4 ⇔ +(3, 4)

This is a **purely syntactic transformation** that **can be done by the parser**.

The semantic analysis, optimization, and code generation phases only need to implement one concept: subroutine calls.

3 + 4 ⇔ (+) 3 4 ⟵ Haskell

Operators as Function Applications

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```

is the same as

```
<op>(<operand1>, <operand2>)
```

Compilation of operators.

➡ Some operators **correspond directly to machine instructions**.

- e.g., integer addition
- These are called ***built-in*** or ***primitive*** functions.

➡ Which operations are primitive is entirely machine-dependent.

- e.g., some machines require **software floating point** emulation.

➡ **Avoiding a subroutine call** in the case of primitive functions is a compile-time **optimization similar to inlining**.

Operators as Function Applications

Operators are not inherently special.

➡ An operator is a **function/subroutine with human-friendly syntax**.

<operand1> <op> <operand2>

is the same as

<op>(*<operand1>, <operand2>*)

However, classic **imperative language design** treats operators as a concept that is different from a regular subroutine abstraction. This is a **serious design limitation**.

- e.g., **in Pascal, C, and Java**, operators are unrelated to functions/procedures (even if they are implemented in software) and are **syntactically different**.
- e.g., **in C++**, the user can override select operators with custom methods, but the user **cannot define new operators**.

Operator Precedence

```
<operand1> <op1> <operand2> <op2> <operand3>
```

Automatically transformed by parser into subroutine calls…

Operator Precedence

```
<operand1> <op1> <operand2> <op2> <operand3>
```

Problem: how to match operators to operands?

```
<op1>(<operand1>, <op2>(<operand2>, <operand3>))
```

or

```
<op2>(<op1>(<operand1>, <operand2>), <operand3>))
```

Operator Precedence

```
<operand1> <op1> <operand2> <op2> <operand3>
```

Tie breaking rules.

- ➡ Each operator is assigned a **numeric precedence value**.
- ➡ Operators are evaluated in order of decreasing precedence.
  - ‣ Conceptually, **implicit parentheses** are inserted to disambiguate expression.
- ➡ e.g., multiplication usually has higher precedence than addition.

Operator Precedence

```
<operand1> <op1> <operand2> <op2> <operand3>
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  - ‣ Conceptually, **implicit parentheses** are inserted to disambiguate expression.
- ➡ e.g., multiplication usually has higher precedence than addition.

If `<op1>` has higher precedence than `<op2>`, then:

```
<op2>(<op1>(<operand1>, <operand2>), <operand3>))
```

Operator Associativity

What if both operators have the same precedence?

```
<operand1> <op1> <operand2> <op2> <operand3>
```

Consistent placement of implicit parentheses.

- Either start on left or start on right right.
  - Called **left-associative** and **right-associative**.
- Determines result if operator is not commutative.
  - **Note**: addition/multiplication not necessarily commutative on a computer due to **overflow** / **underflow** / **loss of precision**.

Operator Associativity

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Java:

```
a = b = c;   ⇔   a = (b = c);
a / b / c;   ⇔   (a / b) / c;
```

Short-Circuit Operators

Value of expressions does not always depend on all operands.

- Logical **and**: if first operand is false.
- Logical **or**: if first operand is true.
- **Short-circuit**: only evaluate second operand if result is required.
  - i.e., use **lazy evaluation**!

Uses.

- This is an optimization: put the computationally cheap tests first.
- Short-circuit operators are often used to guard potentially erroneous sub-expressions.

Java:

```
HashMap dict = null;
// ...
// possibly initialized by other code
if (dict != null && dict.contains("key"))
  // do something;
```

Short-Circuit Operators

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  erro

Potential null dereference (dict) **guarded by short-circuit operator**: **equivalent to call-by-name** invocation of subroutine named &&.

Java:

```
HashMap dict = null;
// ...
// possibly initialized by other code
if (dict != null && dict.contains("key"))
  // do something;
```

Nested Subroutines

Subroutines definitions within subroutines.

- ➡Subroutine definition creates a local, **nested scope**.
- ➡**Orthogonality**: it should be possible to define new, nested subroutines in a subroutine's local scope.
- ➡Allows **decomposing large subroutines into smaller parts** without "leaking" names into surrounding namespace.

History.

- ➡Introduced in **Algol 60**; adopted by many modern languages (e.g., Pascal, Python, Scheme, etc.).
- ➡**Ignored by C** and most descendants.
  - ‣In C, originally probably for **ease of implementation**.
  - ‣However, **gcc supports nested functions** as an extension.
  - ‣In Java, there isn't really a good reason not to include it…

Nested Subroutines: Example

```
def long_running_operation(list_of_work_items):
    def progress(i):
        print "finished %d of %d" % (i, len(list_of_work_items))
    while not done:
        # ... complicated logic ...
        progress(current_index)
        # ... more complicated logic...
```

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        # ... complicated logic ...
        progress(current_index)
        # ... more complicated logic...
```

Nested subroutine: remove UI clutter from main logic.
(especially useful if GUI code is involved)

Nested Subroutines: Example

```
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    def progress(i):
        print "finished %d of %d" % (i, len(list_of_work_items))
    while not done:
        # ... complicated logic ...
        progress(current_index)
        # ... more complicated logic...
```

Nesting of scopes: bindings from enclosing scope(s) visible.

Higher-Order Functions

Subroutines as arguments and return values.

- ➡ A function (i.e., subroutine) that either accepts (a reference to) another function as an argument or yields a subroutine as its return value is called a **higher-order function**.
- ➡ This allows users to write very flexible functions.
- ➡ Caller can “customize” implemented algorithm.

Python:

```
def update_elements(update, array):
  for i in range(len(array)):
    array[i] = update(array[i])


def scale_by_ten(x):
  return x * 10

a = [1, 2, 3, 4, 5]

update_elements(scale_by_ten, a)

print a # prints [10, 20, 30, 40, 50]
```

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update_elements(scale_by_ten, a)

print a # prints [10, 20, 30, 40, 50]
```

A higher-order function: caller can customize the update that is applied to each element.

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  return x * 10

a = [1, 2, 3, 4, 5]

update_elements(scale_by_ten  a)

print a # prints [10, 20, 30, 40, 50]
```

Separation of Concerns:

The loop “knows” nothing about scaling, and the scaling operation “knows” nothing about arrays.

DRY: write the loop once and **reuse** it with different update functions.

Higher-Order Functions: Subroutines as Arguments

Example: customized sort order.

```
def last_char(x):
  return x[-1]

strings = ["just", "some", "number", "of", "character", "sequences"]

print 'by length', sorted(strings, key=len)
print 'by last', sorted(strings, key=last_char)

# Output:
# by length ['of', 'just', 'some', 'number', 'character', 'sequences']
# by last ['some', 'of', 'number', 'character', 'sequences', 'just']
```

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# Output:
# by length ['of', 'just', 'some', 'number', 'character', 'sequences']
# by last ['some', 'of', 'number', 'character', 'sequences', 'just']
```

Python:
Negative indices count from the end of the list, thus, **x[-1] is the last element**.

Higher-Order Functions: Subroutines as Arguments

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# Output:
# by length ['of', 'just', 'some', 'number', 'character', 'sequences']
# by last ['some', 'of', 'number', 'character', 'sequences', 'just']
```

Algorithmic Customization:
Python allows items in a list be sorted based upon an arbitrary **key function**.

Higher-Order Functions: Subroutines as Arguments

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# Output:
# by length ['of', 'just', 'some', 'number', 'character', 'sequences']
# by last ['some', 'of', 'number', 'character', 'sequences', 'just']
```

Java does not support higher-order functions:
The same effect is achieved by `Collections.sort()` by accepting a reference to a **`Comparator`** instance.

(Which is significantly less elegant and natural.)

Anonymous Functions

```
def last_char(x):
  return x[-1]

strings = ["just", "some", "number", "of", "character", "sequences"]

print 'by length', sorted(strings, key=len)
print 'by last', sorted(strings, key=last_char)

# Output:
# by length ['of', 'just', 'some', 'number', 'character', 'sequences']
# by last ['some', 'of', 'number', 'character', 'sequences', 'just']
```

Definition of short "use once" functions.

- Defining a function and coming up with a **good name** for each "customization" can be tedious.
- Thus, it may be convenient to use **unnamed functions**.
- I.e., instead of defining a function and then referring to it, anonymous functions allow us to simply write **a function literal**.
- Due to their theoretical roots, these are often called **lambda expressions**.

Anonymous Functions

Unnecessarily verbose: the definition "boilerplate code" is much longer than the actual logic, the function is only used once.

```
def last_char(x):
  return x[-1]

strings = ["just", "some", "number", "of", "character", "sequences"]

print 'by length', sorted(strings, key=len)
print 'by last', sorted(strings, key=last_char)

# Output:
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- ➡ Due to their theoretical roots, these are often called **lambda expressions**.

Anonymous Functions

```
strings = ["just", "some", "number", "of", "character", "sequences"]

print 'by length', sorted(strings, key=len)
print 'by last', sorted(strings, key=lambda x: x[-1])

# Output:
# by length ['of', 'just', 'some', 'umber', 'character', 'sequences']
# by last ['some', 'of', 'number', character', 'sequences', 'just']
```

Better alternative: use an anonymous function (indicated by lambda keyword) to achieve same effect.

Definiti

- Defin each “custo
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Closures

Nested subroutines that “capture” their referencing environment.

Free variables.

- In a subroutine ***F***, a variable that is **neither a formal parameter** of ***F*** **nor a local variable** is called **a free variable**.
- What happens if ***F*** is a nested subroutine and **returned** by the subroutine in which it was defined?
  - Or if it is otherwise passed to code that may call it after the subroutine call in which was defined terminated.

```
int foo(int x)
{
    int y = 0;
    return x + y + z;
}
```

Closures

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  - Or if it is otherwise passed to code that may call it after the subroutine call in which was defined t

z is a free variable: neither local nor a parameter.

```
int foo(int x)
{
    int y = 0;
    return x + y + z;
}
```

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  - Or if it is otherwise passed to code that may call it after the subroutine call in which was defined terminated.

Closure.

- A subroutine that is "closed over" its **free variables**.
- Meaning: the **free variables stay bound** to whatever they became bound at definition time (see <u>lexical scoping</u>) and **remain valid**.
- This requires all entities that are referenced by closures **to be allocated on the heap**, since they may have to "outlive" the call in which the closure was created.
- Hence, closures are usually found **in garbage-collected languages**.
- **<u>Note</u>**: closures and anonymous functions are not the same concept!
  - Closures do not have to be anonymous.
  - Anonymous functions do not necessarily have free variables.

Closure Example: A "Hidden" Stack

Python:

```
def make_stack():
    mystack = []
    def _push(x):
        mystack.append(x)
    def _pop():
        val = mystack[-1]
        del mystack[-1]
        return val
    return (_push, _pop)

(a, b) = make_stack()
(c, d) = make_stack()

a(1); a(2); a(3)
c(9); c(8); c(7)

print 'b:', b(), b(), b()
print 'd:', d(), d(), d()
```

Output:

```
b: 3 2 1
d: 7 8 9
```

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```

Output:

```
b: 3 2 1
d: 7 8 9
```

Two **nested** subroutine definitions; the defined subroutines are returned as the return value.

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print 'd:', d(), d(), d()
```

Output:

```
b: 3 2 1
d: 7 8 9
```

Functions **a** and **b** share a common stack;
functions **c** and **d** share a different stack!

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a(1); a(2); a(3)
c(9); c(8); c(7)

print 'b:', b(), b(), b()
print 'd:', d(), d(), d()
```

Output:

```
b: 3 2 1
d: 7 8 9
```

The name `mystack` is a free variable; thus `_push()` and `_pop()` are closures.

Closure Example: A "Hidden" Stack

Python:

```
def make_stack():
    mystack = []
    def _push(x):
        mystack.append(x)
    def _pop():
        val = mystack[-1]
        del mystack[-1]
        return val
    return (_push, _pop)

(a, b) = make_stack()
(c, d) = make_stack()

a(1); a(2); a(3)
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print 'b:', b(), b(), b()
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Creates **hidden state** that is neither global nor local nor class-based.

Can be used to implement object systems.

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Java does not support closures: Again, it uses (inelegant) class-based workarounds, known as "**object-closures**".

This design choice is limiting. For example, the Swing GUI API would be a lot easier to use if Java had anonymous functions and closures; the need for "Listener" interfaces would be greatly reduced.

Partial Application

Specializing functions.

What happens if you supply "too few" actual parameters to a function?

- Normally, this is an error.
- Partial application allows the programmer to **specialize functions** by "fixing" some of the parameters.
- This is similar to a closure in that some parameters become "hidden."

Python:

```
from functools import partial

def scale_by(factor, x):
  return factor * x

scale_by_ten = partial(scale_by, 10)
scale_by_20  = partial(scale_by, 20)

print scale_by_ten(1), scale_by_20(1)
# prints 10 20
```

Haskell:

```
plus :: Int -> Int -> Int
plus a b = a + b

main = do
    let f = plus 10
    print (f 20)
    -- prints 30
```

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scale_by requires to parameters…

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…by **partially applying** one parameter, a new function is created that only requires a single parameter.

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This allows the creation of specialized versions **without duplicating the implemented logic.**

(DRY, good for maintenance)

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A function that maps **two** integers to an integer.

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Given only one parameter, Haskell automatically creates a function that maps **one** integer to an integer.

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> In the context of mathematics and **functional programming**, partial application is commonly called ***currying*** in honor of the logician **Haskell Curry** (1900–1982).

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Continuations

Simplified: snapshot of execution state.

- ➡ Stack + registers (incl. instruction pointer).
- ➡ Execution can be **resumed** (continue) form snapshot at later point in time.
- ➡ Very powerful abstraction.
  - ‣ e.g., can be used to implement exception handling.

Adoption.

- ➡ **Not widespread**.
- ➡ **Scheme** is the most-prominent example.
  - ‣ Well worth studying over the summer…
- ➡ Challenging to implement without extensive runtime system.

Co-Routines

Concurrent execution of subroutines.
- ➡ **Execution** of several subroutines is **interleaved**.
- ➡ Not by OS (e.g., processes), but by compiler / runtime system.

Uses.
- ➡ Emulate concurrency on a uniprocessor.
  - ‣ **Less overhead than actual multithreading**.
- ➡ Process simulation (SIMULA 67).
- ➡ Discrete-event simulation.

Adoption.
- ➡ Not supported by most main-stream programming languages.
  - ‣ Can be emulated in C with libraries (and inline assembly code).
- ➡ Relevance likely reduced on multicore systems.