A Hierarchical Situation Calculus

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Abstract

A situation calculus is presented that provides a solution to the frame problem for hierarchical situations, that is, situations that have a modular structure in which parts of the situation behave in a relatively independent manner. This situation calculus is given in a relational, functional, and modal logic form. Each form permits both a single level hierarchy or a multiple level hierarchy, giving six versions of the formalism in all, and a number of sub-versions of these. For multiple level hierarchies, it is possible to give equations between parts of the situation to impose additional structure on the problem. This approach is compared to others in the literature.

Keywords

Situation calculus, frame problem, modal logic

1 Introduction

The situation calculus formalism permits reasoning about properties of situations that result from a given situation by sequences of actions [MH69]. A problem with this formalism is the necessity to include a large number of *frame axioms* that express the fact that actions do not influence many properties of a situation (for example, if you walk in a room, the color of the wall typically does not change). Non-monotonic logic is one approach for handling such frame axioms, by assuming that properties stay the same unless they can be proved to change, but this logic has additional complexity compared to classical logic. Reiter [Rei91] proposed an approach to the frame problem in first-order logic that avoids the need to specify all of the frame axioms. In this paper another

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approach to the frame problem based on *aspects* is presented. An aspect is essentially a relationship between a situation and one of its component parts. Each action and fluent (property of a situation) has an aspect, indicating which component of the situation the aspect or fluent refers to, and if the aspects satisfy a certain specified relation, it follows that the action does not influence the fluent. For example, if an action and fluent have different aspects, this can indicate that the action influences a different component of the situation than the fluent refers to. Using aspects to specify frame axioms can reduce their complexity, because many actions and fluents can have the same aspect. In addition to the use of aspects for specifying frame axioms, a number of approaches for deriving aspects of actions and fluents from more fundamental properties of a situation are also given.

2 Aspects

All methods for formalizing aspects considered here have the following general features: Actions modify situations, which are denoted by the variables s, t, and u. A situation is assumed to be composed of relatively independent parts and each part is called an *aspect* of the situation. Thus if a building has four rooms, each room could be considered an aspect of the building, because many actions will only influence one room of the building. In the traditional situation calculus, a situation s represents the entire world. In the aspect formalism, a situation s can represent just a portion of the world, for example, a room of a building. If α is an aspect, then $R_\alpha(s,t)$ indicates that situation t is an α aspect of situation s. If t is unique, it may be denoted by $f_\alpha(s)$. The symbol a denotes an action, and $a(s)$ denotes the situation that results when action a is performed in situation s. This is an abbreviation for $do(a,s)$, for example, $do(drop(x),s)$. Also, a may have preconditions, but it is not necessary to consider them here. A *fluent* is a relation or predicate that can be true or false in a situation. The symbols p and q denote fluents. The notation $p(s)$ indicates that fluent p is true in situation s. This is an abbreviation for $holds(p,s)$, for example, $holds(at(x,y),s)$. In the aspect formalism, some fluents may be properties of a portion of the world and others may not. If fluent p is not a property of the situation s, because p refers to something that is outside of the portion of the world modeled by s, then $p(s)$ is undefined. (For our purposes, $p(s)$ can have an arbitrary truth value in this case.) Similary, if an action a does not refer to the portion of the world modeled by s, then $a(s)$ is undefined. Actions and fluents can have aspects, indicating which parts of the situation they influence. For example, if an action a has as aspect a room of a building, this can indicate that the action can only influence properties of that room. If a fluent has as aspect a room of a building, this can indicate that the fluent is only a property of that room. Thus if action a and fluent p have different aspects, then performing a will not affect p. The symbols α_i and β_i denote aspects of fluents and actions, respectively, and are drawn from universes $\mathcal{A}_1$ and $\mathcal{A}_2$ of objects, respectively. Thus $\alpha_i \in \mathcal{A}_1$ and $\beta_i \in \mathcal{A}_2$. Often $\mathcal{A}_1 = \mathcal{A}_2$. For a fluent p, $p : \alpha$ indicates that p has aspect α. For an action a, $a : \beta$ indicates that action a has aspect β. Theoretically, a fluent or action could have more than one aspect, but the following discussion assumes that aspects of fluents and actions are unique. However, there can be many different actions and fluents that have the same aspect. Therefore the number of aspects can be much smaller than the number of fluents and actions.

For each problem domain and each formalism, it is necessary to give axioms specifying the aspects of fluents and actions. Let $F[a,p]$ be the *frame axiom*

$$\forall s(p(s) \equiv p(a(s))) \tag{1}$$

indicating that action a does not affect fluent p. Let $d(\alpha,\beta)$ be a *non-interference* property, where d is a predicate symbol; this property will be specified differently for each problem domain. Intuitively, $d(\alpha,\beta)$ means that actions of aspect β do not influence fluents of aspect α. A typical non-interference specification would be $(\alpha \neq \beta) \supset d(\alpha,\beta)$. For example, an action a and a fluent p that correspond to distinct rooms of a building would not interact. It is also possible to let aspects of fluents be elements of some set $\mathcal{F}$ and let aspects of actions be sets of elements from $\mathcal{F}$ Thus $\mathcal{A}_1 = \mathcal{F}$ and $\mathcal{A}_2 = 2^{\mathcal{F}}$. Then the non-interference specification could be $\alpha \notin \beta \supset d(\alpha,\beta)$. For example, an action a might influence two rooms r_1 and r_2 of a building; then its aspect might be the set $\{r_1, r_2\}$. A fluent p that is only a property of room r_3 might have aspect r_3. Because $r_3 \notin \{r_1, r_2\}$, performing action a would not influence fluent p. Another example is a telephone call between locations x and y that can only influence fluents having aspect x or y. Because the number of aspects can be much smaller than the number of fluents and actions, the axiomatization of d can be small relative to the axiomatization of the entire domain. For each formalism it is necessary to show the *non-interference axiom*

$$p : \alpha \wedge a : \beta \wedge d(\alpha,\beta) \supset F[a,p] \tag{2}$$

for all p, a, α, β. If $p : \alpha$ and $a : \beta$ and $d(\alpha,\beta)$, then p and a are said to be *disjoint*, otherwise they *intersect*. For each version of the formalism, it is only necessary to give frame axioms for a and p that intersect. This can considerably reduce the effort necessary to give frame axioms.

2.1 Sequences of Aspects

In formalisms based on sequences, actions and fluents have *sequences* of aspects instead of single aspects. The notation $\overline{\alpha}$ denotes a sequence of aspects, that is, $\alpha_1, \ldots, \alpha_n$. The sequences of aspects may have different lengths for different actions and fluents. For this formalism, one has $d(\overline{\alpha}, \overline{\beta})$ for the non-interference property. The non-interference axiom becomes

$$p : \overline{\alpha} \wedge a : \overline{\beta} \wedge d(\overline{\alpha}, \overline{\beta}) \supset F[a,p] \tag{3}$$

for all $p, a, \overline{\alpha}, \overline{\beta}$. If $p : \overline{\alpha}$ and $a : \overline{\beta}$ and $d(\overline{\alpha}, \overline{\beta})$, then p and a are said to be *disjoint*, otherwise p and a *intersect*, as before. A typical axiom would be $\exists i(\alpha_i \neq \beta_i) \supset d(\alpha,\beta)$. Intuitively, sequences of aspects represent more than one level hierarchy. For example, an aspect of a country could be a state and an aspect of a state could be a city in the state. Some actions might influence a whole country, some might influence just one state, and some might influence just one city in one state. These would correspond to different length sequences of aspects. Another example is binary trees; a sequence of 0 and 1 specifies a subtree of a binary tree, with 0 signifying left subtree and 1 signifying right subtree, for example. So an empty sequence refers to the whole tree, the sequence (1,1) refers to the right subtree of the right subtree, et cetera. If an action a has aspect (1,1) and a fluent p has aspect (0,1,0) then the action can only influence the right subtree of the right subtree, while the fluent is a property of a different subtree. Thus one would have $F[a,p]$. If the action has aspect (1,1) and the fluent has aspect (1,1,0) then the action can influence the fluent. Longer sequences indicate smaller influences of an action and fewer dependencies of a fluent.

2.2 Assuming the Non-Interference Axiom

The preceding discussion, both for aspects and sequences of aspects, is concerned with justifying the non-interference axiom from more basic properties. A simpler approach would be to *assert* the non-interference axiom for each problem domain; then it is only necessary to assign aspects to fluents and actions and give axioms specifying d. Even this approach can greatly simplify the frame axioms. Suppose there are m fluents $p_1, \ldots, p_m$ having aspect α and n actions $a_1, \ldots, a_n$ having aspect β. Then from the non-interference axiom, the specification of d, and the assertions $p_i : \alpha$, $1 \leq i \leq m$ and $a_j : \beta$, $1 \leq j \leq n$ it is possible to deduce the mn frame axioms

$$F[a_j, p_i], 1 \leq i \leq m, 1 \leq j \leq n \tag{4}$$

This approach requires only $m + n + 2$ small axioms, assuming that the specification of d requires only one small axiom, whereas directly specifying the mn frame axioms would require mn axioms. Of course, there could be other actions and fluents having other aspects β' and α'. For each pair (α, β) of aspects such that $d(\alpha, \beta)$, one obtains mn frame axioms, where there are m fluents of aspect α and n actions of aspect β. This approach essentially classifies actions and fluents and uses this classification to formalize general frame axioms having many specific frame axioms as consequences. This method alone gives an economical solution to the frame problem in some cases.

However, the question remains as to how the non-interference axiom is justified. It is profitable to consider various semantics and axiomatizations from which the non-interference axiom is derivable. This helps to gain insight into this axiom, as well as providing guidance concerning when the non-interference property should be used and how it can be formalized.

A number of approaches to axiomatizing the non-interference property are given. In many cases, more than one formalism is applicable. The choice between sequential and non-sequential formalisms can be dictated largely by the problem structure, whether the domain has a single level or multi-level hierarchy. The choice between existential and universal versions of the relational formalisms is less clear. The functional formalisms appear simpler than the relational ones, and probably are preferable when they apply. They also may permit more equational reasoning. It would also be possible to have mixed universal and existential relational formalisms. The modal formalisms are equivalent to the relational ones, and the choice between them may be a matter of taste.

2.3 Examples

An example, the blocks world example, will illustrate some of the features of the aspect formalism, even though this example is not particularly well suited to aspects. There are two fluents, $on(x, y)$ (block x is on block y) and $clear(x)$ (nothing is on x). The domain consists of blocks and the floor. The action is $move(x, y)$, move block x on block y, with preconditions that both x and y be clear. Let the aspect axioms be $on(x, y) : y$, $clear(x) : x$, $on(x, z) \supset move(x, y) : \{y, z\}$, and $\neg(\exists z) on(x, z) \supset move(x, y) : \{y\}$. For simplicity, assume every block is on something. In general, the aspect of a block is the block it rests on (or the floor), and moving a block x has as aspect both its old and new locations. Let the disjointness predicate d be axiomatized by $\alpha \notin \beta \supset d(\alpha, \beta)$.

The general non-interference axiom, axiom 2, has the following instance:

$$clear(w) : \alpha \wedge move(x, y) : \beta \wedge \alpha \notin \beta \wedge holds(clear(w), s) \supset holds(clear(w), do(move(x, y), s)) \tag{5}$$

From this instance and the axioms $clear(w) : w$, $on(x, z) \supset move(x, y) : \{z, y\}$, and $w \neq y \wedge w \neq z \supset w \notin \{z, y\}$ the following frame axiom follows.

$$w \neq y \wedge w \neq z \wedge on(x, z) \wedge holds(clear(w), s) \supset holds(clear(w), do(move(x, y), s)) \tag{6}$$

Note that the condition $w \neq z$ of the frame axiom is not necessary, so this frame axiom is weaker than necessary. For problem domains with more of a hierarchical structure, frame axioms derived from aspects should generally be as strong as possible. Another instance of the non-interference axiom is the following:

$$on(v,w) : \alpha \wedge move(x,y) : \beta \wedge \alpha \notin \beta \wedge holds(on(v,w),s) \supset holds(on(v,w), do(move(x,y),s)) \quad (7)$$

By similar reasoning, the following frame axiom follows from this instance:

$$w \neq y \wedge w \neq z \wedge on(x,z) \wedge holds(on(v,w),s) \supset holds(on(v,w), do(move(x,y),s)) \quad (8)$$

The condition $w \neq y$ is not necessary for this frame axiom.

The blocks world example can be made more hierarchical by assuming that there are a number of rooms with blocks in each room. An action that only involves the blocks in one room, and does not move them to another room, will not influence any other rooms. An action that moves a block from one room to another can influence both rooms. For the multi-room example, the aspect of a block can be a sequence (α_1, α_2) where α_1 gives the room the block is in and α_2 gives the block it rests on. The aspects of fluents and actions are obtained from the blocks they refer to. Then d can be defined so that an action and fluent referring to different rooms, are disjoint.

As another example illustrating sequences of aspects on a hierarchical domain, consider actions that change some of the pixels on a display and fluents that describe properties of a subset of the pixels. Let $\mathcal{A}_1$ and $\mathcal{A}_2$ be the set of subsets of the pixels. For each action a_j suppose that $a_j : \beta$ where β is the set of pixels modified by the action, and for each fluent p_i suppose that $p_i : \alpha$ where p_j is a property of the set α of pixels. Then from the non-interference axiom one obtains the following:

$$p_i : \alpha \wedge a_j : \beta \wedge \alpha \cap \beta = \phi \supset F[a_j, p_i]. \quad (9)$$

Now suppose that in addition there are actions that modify the memory of a computer but not the display. These can be given a "memory" aspect and the display actions and fluents can be given a "display" aspect to indicate that they do not interfere. If there are other objects in the room (such as a window and door) that are independent of the computer then actions and fluents that refer to them can be given "window" and "door" aspects, and all the computer actions and fluents can be given a "computer" aspect, to indicate that they do not interact. Some actions may influence the whole room. In this way one obtains a sequence of aspects; for display actions and fluents the sequence is of the form $(computer, display, S)$ where S is a set of pixels. For memory actions and fluents the sequence is $(computer, memory, T)$ where T is the set of memory cells affected by the action. Actions that influence the window and door would have sequences of the form $(window)$ and $(door)$, respectively. Actions that influence the whole room, such as a meteorite hitting, would have an empty sequence. Let the disjointness predicate d be axiomatized by $d((\alpha_1, \ldots, \alpha_m), (\beta_1, \ldots, \beta_n))$ if for some i, $\alpha_i \neq \beta_i$. Then if $\alpha_1, \ldots, \alpha_m$ is the sequence of aspects for a fluent p and $\beta_1, \ldots, \beta_n$ is the sequence of aspects for an action a, the frame axiom $F[a,p]$ follows from the non-interference axiom if for some i, $\alpha_i \neq \beta_i$. Thus actions on the display do not influence the memory or the door or the window, actions on the door or window do not influence the display or the memory, but actions on the whole room may influence anything.

2.4 Comparison to Other Approaches

The most relevant comparable approach to the frame problem is that of Reiter [Rei91], based on the work of Haas [Haa87], Pednault [Ped89], Schubert [Sch90] and Davis [Dav90]. Reiter essentially gives an axiom for each fluent p that says, For all actions a, if a is possible in situation s, then $p(a(s))$ holds if either (1) a is an action that makes p true in situation s, or (2) $p(s)$ is already true and a is not an action that can make p false in situation s. One such axiom is needed for each fluent, so the number of such axioms is equal to the number of fluents. This approach avoids the frame problem by focusing on actions that *change* fluents rather than actions that do not change fluents. Thus it is not necessary to give all the frame axioms $F[a,p]$.

In particular, Reiter gives the following *successor state axiom* for each fluent R:

$$Poss(a,s) \supset [R(do(a,s)) \equiv \gamma_R^+(a,s) \vee R(s) \wedge \neg\gamma_R^-(a,s)] \tag{10}$$

where $Poss(a,s)$ expresses that action a is possible in situation s, $\gamma_R^+(a,s)$ is true if action a can make R become true and $\gamma_R^-(a,s)$ is true if action a can make R become false, informally speaking. Thus $R(do(a,s))$ is true if a caused R to become true or if $R(s)$ was already true and a did not cause R to become false.

A problem with Reiter's approach is that the successor state axiom for p can be very complicated, if many actions influence p; of course, this complexity appears to be unavoidable in any approach. Another problem is that it is necessary to consider *all* actions. Perhaps there are many actions that can influence p but only a small subset of them is necessary for a planning problem. In fact, knowledge concerning all actions that influence a fluent may not even be available. Reiter's approach requires that all of the actions be mentioned, whether they will be used or not. The traditional formalization of the frame axioms avoids this problem. The aspect approach also avoids this problem. But it is straightforward to modify Reiter's approach to handle unknown actions; it is only necessary to quantify the successor state axiom over all a in some set S of actions that is of interest, assuming that S is sufficient for planning or reasoning purposes. It may also be true that for some action a of interest one does not know its effect on a fluent R. In this case, the corresponding predicate $\gamma_R^+(a,s)$ or $\gamma_R^-(a,s)$ need not be axiomatized, if the remaining axioms are sufficient for planning or reasoning purposes.

In Reiter's approach, to show that $R(a(s))$, assuming $R(s)$, it is necessary to demonstrate $\neg\gamma_R^-(a,s)$. This involves testing a against other actions, one by one, and verifying that a is distinct from all actions that can make R false. In the traditional approach, the frame axiom $R(s) \supset R(a(s))$ serves the purpose much more simply. Proofs in the aspect based system are also simple and intuitive. Therefore proofs are more complex in Reiter's system than in the traditional system and in the aspect based system. Such complex proofs may be more difficult for an automatic reasoning system to find. The successor state axiom in Reiter's system is compact, but unintuitive. One would presumably justify it by appealing to the traditional frame axioms $F[a,p]$. Therefore, for the sake of correctness, it may be preferable to use the traditional frame axioms or the aspect formalism. Similarly, a proof from the traditional frame axioms $F[a,p]$ or from axioms for aspects may be more convincing to a user than a proof from Reiter's successor state axiom. Of course, it is possible to obtain a proof using the successor state axiom and then modify it to obtain a proof from the traditional frame axioms.

Reiter's approach has an extra cost, namely, the necessity to reason about equality between actions. The aspect formalism also has a cost, namely, the need to reason about aspects. Each approach may complicate the reasoning process in a different way. Reiter's definition of equality between actions also means that equality on actions is defined in a non-extensional way.

Another problem with Reiter's approach, noted in Scherl and Levesque [SL93], is that it can be difficult to incorporate *constraints* between fluents, for example, when one fluent implies another.

The successor state axiom essentially implies that the only way a fluent can become true is if an action makes it true. Lin and Reiter [LR94] discuss how to overcome this by modifying the successor state axiom. For the aspect approach, constraints are not a problem.

An advantage of Reiter's formalism is its conciseness. The aspect based formalism can also be concise, if for all but a small number of action-fluent pairs (a, p), a and p are disjoint, that is, $p : \alpha \wedge a : \beta \wedge d(\alpha, \beta)$. In such domains, the aspect formalism may be preferable. However, in other domains, the conciseness of Reiter's formalism may be preferable. In fact, the aspect based system is concise if the number of pairs (a, p) such that a and p intersect is not much larger than the number of pairs (a, p) such that $F[a, p]$ is false, because the latter will contribute to the complexity of either formalism, as well as to that of the traditional formalism.

One can combine Reiter's approach with aspects by using the successor state axiom *and* the non-interference axiom. When the non-interference axiom can be used to demonstrate $F[a, p]$, it may provide simpler proofs than the successor state axiom. When $F[a, p]$ either is false or cannot be shown using the non-interference axiom, the successor state axiom may be used. For this combined formalism it is also necessary to axiomatize d, include the non-interference axiom (or other assertions from which it can be derived), and specify the aspects of actions and fluents.

A prior aspect formalism for the situation calculus was presented in [PZ97]. This formalism also treated hierarchical situations and defined aspects of situations. However, the axiomatization and semantics were different. The relation between a predicate p of situation s and a predicate q on an aspect of s was not considered. Also, there was no consideration of a situation variable as representing only one component of a situation.

There have been many other approaches to the frame problem, but none directly related to the aspect formalism. Turner [Tur97] shows how to represent commonsense knowledge using default logic in the situation calculus. He is concerned not only with the truth of fluents but on causality relations between them. Pirri and Reiter [PR99] formalize situations as sequences of actions and discuss soundness and completeness issues for planning with deterministic actions. Miller and Shanahan [MS94] discuss narratives in the situation calculus (sequences of actions about which one has incomplete information). Reiter [Rei93] discusses methods to prove that certain properties are true in all states accessible from a given state by a sequence of actions. He discusses formalizing databases using the situation calculus and reasoning about their properties. Finzi, Pirri, and Reiter [FPR00] discuss open world planning and the associated theorem proving problem, using essentially the formalism of Reiter [Rei91]. They also convert the planning problem to a propositional problem. Scherl and Levesque [SL93] consider the frame problem in the context of actions that gain knowledge, such as looking up a telephone number. They also use Reiter's formalism. It is interesting that they use Moore's [Moo85] "possible world" formalism for knowledge, which is closely related to the modal aspect formalisms given below. Störr and Thielscher [ST00] give an equational formulation of the situation calculus, which is based on the work of Hölldobler and Thielscher [HT95]; this is a different formalism from that of Reiter, and also distinct from the aspect formalism. In their formalism, the *fluent calculus*, situations are represented essentially as logical formulae, that is, conjunctions of fluents that are true in the situation.

Next a variety of formalisms for justifying the non-interference axiom for aspects are presented. These formalisms will aid in understanding this axiom, as well as providing guidance about when and how this axiom may be used.

3 Relational Formalisms

3.1 The Relational Formalism (Existential Version)

In this formalism, there are *relations* between situations, denoted by R_α where α is an aspect. The intuitive meaning of $R_\alpha(s,t)$ is that the situation t is an α-aspect of situation s. Thus if s represents a building, t can represent one room of the building. This formalism has the following axioms:

$$\forall a\alpha\beta(d(\alpha,\beta) \wedge a : \beta \supset (R_\alpha(s,t) \equiv R_\alpha(a(s),t))) \tag{11}$$

$$\forall p\alpha(p : \alpha \supset \exists q\forall s(p(s) \equiv \exists t(q(t) \wedge R_\alpha(s,t)))) \tag{12}$$

In words, axiom 11 says that α and β are non-interfering if the α-aspect of situation s is the same as the α-aspect of situation $a(s)$ for an action a of aspect β. For example, painting one room of a building does not affect the other rooms of the building. Axiom 12 states that a fluent p has aspect α if $p(s)$ only depends on the α aspect of situation s. The q in axiom 12 is denoted by p_α. For example, let $p(s)$ be the property that a building is heated in situation s. If the heater is in room r_4, and $q(t)$ specifies that the heater is on, and α is r_4, then $p(s) \equiv \exists t(q(t) \wedge R_\alpha(s,t))$. Intuitively, the building is heated if the heater is on, so that an action that does not influence room r_4 will not affect fluent p. To be more precise, because this version is existential, there can be several heaters; $R_\alpha(s,t)$ means that t is *one* of the heaters of the building, and the building is heated if any one of the heaters is on. Also, if the circulation of blood through the body is a property of the heart, then an action that does not influence the heart will not affect the circulation. In these examples, the predicate q has a natural interpretation in the problem, but the formalism can be used even when q does not have a natural interpretation.

Theorem 1 *The non-interference axiom, axiom 2, follows from axioms 11 and 12.*

Proof: Suppose $p : \alpha$ and $a : \beta$ and $d(\alpha,\beta)$. From $p : \alpha$ and formula 12,

$$\exists q\forall s(p(s) \equiv \exists t(q(t) \wedge R_\alpha(s,t))). \tag{13}$$

From $a : \beta$, $d(\alpha,\beta)$, and formula 11,

$$R_\alpha(s,t) \equiv R_\alpha(a(s),t)) \tag{14}$$

Therefore $p(s) \equiv \exists t(q(t) \wedge R_\alpha(s,t))) \equiv \exists t(q(t) \wedge R_\alpha(a(s),t))) \equiv p(a(s))$.

3.2 The Relational Formalism (Universal Version)

This is the same as the existential version of the relational formalism except that axiom 12 has a universal quantifier instead of an existential quantifier:

$$\forall a\alpha\beta(d(\alpha,\beta) \wedge a : \beta \supset (R_\alpha(s,t) \equiv R_\alpha(a(s),t))) \tag{15}$$

$$\forall p\alpha(p : \alpha \supset \exists q\forall s(p(s) \equiv \forall t(R_\alpha(s,t) \supset q(t)))) \tag{16}$$

As an illustration, because this version is universal, there can be several heaters; $R_\alpha(s,t)$ means that t is *one* of the heaters of the building, but the building is only heated ($p(s)$) if all of the heaters are on ($\forall t(R_\alpha(s,t) \supset q(t))$, where $q(t)$ means that heater t is on).

Theorem 2 *Axiom 2 follows from axioms 15 and 16.*

Proof: Suppose $p : \alpha$ and $a : \beta$ and $d(\alpha, \beta)$. From $p : \alpha$ and formula 16,

$$\exists q \forall s(p(s) \equiv \forall t(R_\alpha(s,t) \supset q(t))). \tag{17}$$

From $a : \beta$, $d(\alpha, \beta)$, and formula 15,

$$R_\alpha(s,t) \equiv R_\alpha(a(s),t) \tag{18}$$

Therefore $p(s) \equiv \forall t(R_\alpha(s,t) \supset q(t)) \equiv \forall t(R_\alpha(a(s),t) \supset q(t)) \equiv p(a(s))$.

3.3 The Sequential Relational Formalism (Existential Version)

For this formalism, $R_{\alpha_1,\ldots,\alpha_n}(s,t)$ is defined as $\exists u(R_{\alpha_1}(s,u) \wedge R_{\alpha_2,\ldots,\alpha_n}(u,t)$ for $n > 1$. This formalism has the following axioms:

$$\forall a \overline{\alpha} \overline{\beta}(d(\overline{\alpha}, \overline{\beta}) \wedge a : \overline{\beta} \supset (R_{\overline{\alpha}}(s,t) \equiv R_{\overline{\alpha}}(a(s),t))) \tag{19}$$

$$\forall p \overline{\alpha}(p : \overline{\alpha} \supset \exists q \forall s(p(s) \equiv \exists t(q(t) \wedge R_{\overline{\alpha}}(s,t)))) \tag{20}$$

The q in axiom 20 is denoted by $p_{\overline{\alpha}}$.

Theorem 3 *Axiom 3 follows from axioms 19 and 20.*

Proof: Similar to the proof of theorem 1

This formalism has some additional properties relating sequences of aspects:

$$\forall p(p : \alpha_1 \ldots \alpha_n \supset p : \alpha_1 \ldots \alpha_{n-1}) \tag{21}$$

Intuitively, a fluent is a property of a state if it is a property of a city in the state.

$$0 \leq k < m \wedge d(\alpha_1 \ldots \alpha_k, \beta_1 \ldots \beta_n) \supset d(\alpha_1 \ldots \alpha_m, \beta_1 \ldots \beta_n) \tag{22}$$

Intuitively, an action that does not influence a state will not influence any city in the state. Also, if $d(\overline{\alpha}, \overline{\beta})$ is defined by $\exists i(\alpha_i \neq \beta_i)$ then in addition

$$0 \leq k < n \wedge d(\alpha_1 \ldots \alpha_m, \beta_1 \ldots \beta_k) \supset d(\alpha_1 \ldots \alpha_m, \beta_1 \ldots \beta_n) \tag{23}$$

Intuitively, if an action that only influences a state does not influence city X, then an action that only influences a city in the state, does not influence city X. The proofs of these properties (21, 22, 23) are straightforward.

3.4 The Sequential Relational Formalism (Universal Version)

This is the same as the existential version of the sequential relational formalism except that axiom 20 has a universal quantifier instead of an existential quantifier:

$$\forall a\overline{\alpha}\overline{\beta}(d(\overline{\alpha},\overline{\beta}) \wedge a : \overline{\beta} \supset (R_{\overline{\alpha}}(s,t) \equiv R_{\overline{\alpha}}(a(s),t))) \tag{24}$$

$$\forall p\overline{\alpha}(p : \overline{\alpha} \supset \exists q \forall s(p(s) \equiv \forall t(R_{\overline{\alpha}}(s,t) \supset q(t)))) \tag{25}$$

Also, $R_{\alpha_1,\ldots,\alpha_n}(s,t)$ is defined as above.

Theorem 4 *Axiom 3 follows from axioms 24 and 25.*

Proof: Similar to the proof of theorem 2.

The properties 21 and 22 hold for this formalism as well, as does 23 under the appropriate definition of d.

4 Functional Formalisms

4.1 Simple Functional Formalism

If in the relational formalism, all R_α are in fact functions, that is, for all s there is exactly one t such that $R_\alpha(s,t)$, then a functional formalism can be used. In this case, t is a function of s, denoted $f_\alpha(s)$. Intuitively, if the four rooms of a building correspond to four distinct aspects, then each aspect defines a function of the building. If an aspect of a class is an arbitrary student in the class, then this is not a function, so one cannot use a functional formalism. In a straightforward manner, one obtains the following axioms for the functional formalism.

$$\forall a\alpha\beta(d(\alpha,\beta) \wedge a : \beta \supset f_\alpha(s) = f_\alpha(a(s))) \tag{26}$$

$$\forall p\alpha(p : \alpha \supset \exists q \forall s(p(s) \equiv q(f_\alpha(s)))) \tag{27}$$

As an illustration, because this version is functional, there is just one heater; $f_\alpha(s)$ is the heater of building s, and the building is heated ($p(s)$) if the heater is on ($\forall s(p(s) \equiv q(f_\alpha(s)))$ where $q(t)$ means that heater t is on).

Theorem 5 *Axiom 2 follows from axioms 26 and 27.*

Proof: Suppose $p : \alpha$ and $a : \beta$ and $d(\alpha,\beta)$. From $p : \alpha$ and formula 27,

$$\exists q \forall s(p(s) \equiv q(f_\alpha(s))). \tag{28}$$

From $a : \beta$, $d(\alpha,\beta)$, and formula 26,

$$f_\alpha(s) = f_\alpha(a(s)). \tag{29}$$

Therefore $p(s) \equiv q(f_\alpha(s)) \equiv q(f_\alpha(a(s))) \equiv p(a(s))$.

4.2 Sequential Functional Formalism

This corresponds to the sequential relational formalism when each relation R_α is a function, as above. One then has

$$f_{\alpha_1 \ldots \alpha_n} = f_{\alpha_n} f_{\alpha_{n-1}} \cdots f_{\alpha_2} f_{\alpha_1} \tag{30}$$

$$\forall a \overline{\alpha} \overline{\beta} (d(\overline{\alpha}, \overline{\beta}) \wedge a : \overline{\beta} \supset f_{\overline{\alpha}}(s) = f_{\overline{\alpha}}(a(s))) \tag{31}$$

$$\forall p \overline{\alpha} (p : \overline{\alpha} \supset \exists q \forall s (p(s) \equiv q(f_{\overline{\alpha}}(s)))) \tag{32}$$

Theorem 6 *Axiom 3 follows from axioms 31 and 32.*

Proof: Similar to the proof of theorem 5

The sequential functional formalism also has properties 21 and 22, and 23 holds if d is defined as stated.

5 Collective Formalisms

The preceding formalisms help to justify the non-interference axiom. However, they do not give insight into the definition of d. It would be helpful to derive the axiomatization of d as well from more basic properties, to help to understand this predicate and to provide guidance about how to axiomatize it. When $d(\alpha, \beta)$ is defined as $\alpha \neq \beta$, the preceding discussion is sufficient, because it seems reasonable that actions and fluents that influence different aspects of a situation will not interact. However, if $d(\alpha, \beta)$ is defined by $\alpha \notin \beta$ or $\alpha \cap \beta = \phi$, then more justification is appropriate. What is the meaning of the underlying elements of these sets, in a general context, and what is the meaning of this particular definition of d? The following formalisms help to answer these questions.

5.1 The Collective Relational Formalism (Existential Version)

In this formalism, as before, there are relations between situations. Aspects of actions and fluents are assumed to be sets of elements of some underlying set $\mathcal{A}$ (for example, the students in a class, or the cities in a state). For *each* $x \in \mathcal{A}$, there is a relation R_x on situations. The intuitive meaning of $R_x(s,t)$ is that the situation t is the x-aspect of situation s. Thus if s represents a class, x can be an aspect corresponding to a student in the class, and t is a situation representing the properties of that student. Also, $d(\alpha, \beta)$ iff $\alpha \cap \beta = \phi$. This formalism has the following axioms:

$$\forall a x \beta (x \notin \beta \wedge a : \beta \supset (R_x(s,t) \equiv R_x(a(s),t))) \tag{33}$$

$$\forall p \alpha (p : \alpha \supset \exists q \forall s (p(s) \equiv (\forall x \in \alpha) \exists t (q_x(t) \wedge R_x(s,t)))) \tag{34}$$

In words, axiom 33 says that action a of aspect β can only influence the x portions of situation s for $x \in \beta$. Thus if $x \notin \beta$ then the x portion of situation s is the same as the x portion of situation $a(s)$. For example, if an action has aspect $\{r_1, r_2\}$ where r_i are rooms of a building, then the action can only influence these rooms, so it does not influence room r_3 because $r_3 \notin \{r_1, r_2\}$. Axiom 34 states that a fluent p has aspect α if $p(s)$ only depends on the x aspects of situation s for $x \in \alpha$. That is, there is a predicate q such that $p(s)$ iff for all x in α, there is some situation t that is an x aspect of s such that $q_x(t)$. As an example, consider a university s to be superior $(p(s))$if every faculty member x of s is above average in performance $(q_x(t))$. Then $p : \alpha$ where α is the set of faculty members of the university. Let a be the action of increasing the student enrollment. Then $a : \beta$ where β is the set of students of the university. Axiom 33 states that action a does not influence any of the faculty members of the university. Axiom 34 states that the fluent p only depends on the faculty members; $R_x(s,t)$ if t is the situation representing faculty member x and $q_x(t)$ if t is a superior faculty member. From these axioms it follows that the university will still be superior if student enrollment increases.

Theorem 7 *The non-interference axiom, axiom 2, with d defined by $\alpha \cap \beta \neq \phi \supset d(\alpha, \beta)$, follows from axioms 33 and 34.*

Proof: Suppose $p : \alpha$ and $a : \beta$ and $\alpha \cap \beta \neq \phi$. From $p : \alpha$ and formula 34,

$$\exists q \forall s (p(s) \equiv (\forall x \in \alpha) \exists t (q_x(t) \wedge R_x(s,t))) \tag{35}$$

From $a : \beta$ and formula 33,

$$\forall x (x \notin \beta \supset (R_x(s,t) \equiv R_x(a(s),t))) \tag{36}$$

Therefore $p(s) \equiv (\forall x \in \alpha)\exists t(q_x(t) \wedge R_x(s,t)) \equiv (\forall x \in \alpha)\exists t(q_x(t) \wedge R_x(a(s),t)) \equiv p(a(s))$.

5.2 The Collective Relational Formalism (Universal Version)

This is the same as the existential version of the relational formalism except that axiom 34 has a universal quantifier instead of an existential quantifier:

$$\forall a x \beta (x \notin \beta \wedge a : \beta \supset (R_x(s,t) \equiv R_x(a(s),t))) \tag{37}$$

$$\forall p \alpha (p : \alpha \supset \exists q \forall s (p(s) \equiv (\forall x \in \alpha) \forall t (R_x(s,t) \supset q_x(t)))) \tag{38}$$

This has the same intuition as the existential version, except that each student and faculty member corresponds to a set of situtations, all of which must have the property q_x.

Theorem 8 *The non-interference axiom, axiom 2, with d defined by $\alpha \cap \beta \neq \phi \supset d(\alpha, \beta)$, follows from axioms 37 and 38.*

Proof: Suppose $p : \alpha$ and $a : \beta$ and $\alpha \cap \beta \neq \phi$. From $p : \alpha$ and formula 38,

$$\exists q \forall s (p(s) \equiv (\forall x \in \alpha) \forall t (R_x(s,t) \supset q_x(t))) \tag{39}$$

From $a : \beta$ and formula 37,

$$\forall x (x \notin \beta \supset (R_x(s,t) \equiv R_x(a(s),t))) \tag{40}$$

Therefore $p(s) \equiv (\forall x \in \alpha)\forall t(R_x(s,t) \supset q_x(t)) \equiv (\forall x \in \alpha)\exists t(R_x(a(s),t) \supset q_x(t)) \equiv p(a(s))$.

5.3 The Collective Functional Formalism

This is analogous to the functional formalism, with collective aspects:

$$\forall ax\beta(x \notin \beta \wedge a : \beta \supset f_x(s) = f_x(a(s))) \tag{41}$$

$$\forall p\alpha(p : \alpha \supset \exists q\forall s(p(s) \equiv (\forall x \in \alpha)q_x(f_x(s)))) \tag{42}$$

The intuition is as above, but more natural; for every aspect x there is a unique situation $f_x(s)$ representing x.

Theorem 9 *Axiom 2 with d defined by* $\alpha \cap \beta \neq \phi \supset d(\alpha, \beta)$*, follows from axioms 41 and 42.*

Proof: Suppose $p : \alpha$ and $a : \beta$ and $\alpha \cap \beta \neq \phi$. From $p : \alpha$ and formula 42,

$$\exists q\forall s(p(s) \equiv (\forall x \in \alpha)q_x(f_x(s))). \tag{43}$$

From $a : \beta$ and formula 41,

$$x \notin \beta \supset f_x(s) = f_x(a(s)) \tag{44}$$

Therefore $p(s) \equiv (\forall x \in \alpha)q_x(f_x(s)) \equiv (\forall x \in \alpha)q_x(f_x(a(s))) \equiv p(a(s))$.

It would be possible to extend the collective formalisms to sequential collective formalisms, as well.

6 Modal Formalisms

There are four more aspect-based situation calculus formalisms based on modal logic [Eme90]. These formalisms make use of the modal operators [ ] and <>. In modal logic, [ ]A means "necessarily A" and <> A means "possibly A." The modal situation calculus formalisms introduce a number of modal operators, one for each action and fluent. Thus if a is an action, $[a]A$ means that after a executes, assertion A is necessarily true. If α is an aspect, then $[\alpha]A$ means that in all worlds obtained from the current world by relation R_α, assertion A is true. Also, $\langle\alpha\rangle A$ means that in some world obtained from the current world by relation R_α, assertion A is true. The frame axiom $F(a, p)$ is expressed in the modal formalisms as the following assertion:

$$p \equiv [a]p \tag{45}$$

Intuitively, this means that p is true in the current world if and only if it is true in all worlds reachable by action a. Modal logic has the same rules as first-order logic plus additional ones; the only rules we need are the following:

$$(X \supset Y) \supset ([a]X \supset [a]Y). \tag{46}$$

$$(X \supset Y) \supset ([\alpha]X \supset [\alpha]Y). \tag{47}$$

$$(X \supset Y) \supset (\langle\alpha\rangle X \supset \langle\alpha\rangle Y). \tag{48}$$

We also need the axioms

$$p : \alpha \supset [a](p : \alpha) \tag{49}$$

and

$$a' : \beta \supset [a](a' : \beta) \tag{50}$$

which say that aspects are preserved under actions. Also, if a formula X is provable (without assumptions), then one can deduce the formulas $[a]X$,$[\alpha]X$, and $\langle\alpha\rangle X$ as well.

6.1 Simple Modal Formalism, [ ] Version

This formalism is analogous to the universal version of the relational formaliism. This formalism has the following axioms:

$$\forall a\alpha\beta(d(\alpha,\beta) \wedge a : \beta \supset ([\alpha]X \equiv [a][\alpha]X)) \tag{51}$$

$$\forall p\alpha(p : \alpha \supset \exists q(p \equiv [\alpha]q)) \tag{52}$$

Theorem 10 *The non-interference axiom 2 follows from formulas 51 and 52.*

Proof: Suppose $p : \alpha$ and $a : \beta$ and $d(\alpha, \beta)$. From $p : \alpha$ and formula 52,

$$\exists q(p \equiv [\alpha]q). \tag{53}$$

From $a : \beta$, $d(\alpha, \beta)$, and formula 51,

$$[\alpha]X \equiv [a][\alpha]X \tag{54}$$

Therefore $p \equiv [\alpha]q \equiv [a][\alpha]q \equiv [a]p$. The last step $[a][\alpha]q \equiv [a]p$ makes use of $[a](p : \alpha)$ from which it is derivable. The assertion $[a](p : \alpha)$ is derivable from the assumption $p : \alpha$ and axiom 49.

6.2 Simple Modal Formalism, <> Version

This formalism is analogous to the existential version of the relational formaliism. This formalism has the following axioms:

$$\forall a\alpha\beta(d(\alpha,\beta) \wedge a : \beta \supset (\langle\alpha\rangle X \equiv [a]\langle\alpha\rangle X)) \tag{55}$$

$$\forall p\alpha(p : \alpha \supset \exists q(p \equiv \langle\alpha\rangle q)) \tag{56}$$

Theorem 11 *The non-interference axiom 2 follows from formulas 55 and 56.*

Proof: Suppose $p : \alpha$ and $a : \beta$ and $d(\alpha, \beta)$. From $p : \alpha$ and formula 56,

$$\exists q(p \equiv \langle\alpha\rangle q). \tag{57}$$

From $a : \beta$, $d(\alpha, \beta)$, and formula 55,

$$\langle\alpha\rangle X \equiv [a]\langle\alpha\rangle X \tag{58}$$

Therefore $p \equiv \langle\alpha\rangle q \equiv [a]\langle\alpha\rangle q \equiv [a]p$. The last step makes use of the assumption $p : \alpha$ and axiom 49, as before.

6.3 Sequential Modal Formalism, [ ] Version

This formalism is analogous to the sequential universal relational formalism. Sequences of aspects correspond to sequences of modal operators. This formalism has the following axioms:

$$[\alpha_1\alpha_2\ldots\alpha_n]X \equiv [\alpha_1][\alpha_2]\ldots[\alpha_n]X \tag{59}$$

$$\forall a\overline{\alpha}\overline{\beta}(d(\overline{\alpha},\overline{\beta}) \wedge a:\overline{\beta} \supset ([\overline{\alpha}]X \equiv [a][\overline{\alpha}]X)) \tag{60}$$

$$\forall p\overline{\alpha}(p:\overline{\alpha} \supset \exists q(p \equiv [\overline{\alpha}]q)) \tag{61}$$

Theorem 12 *The non-interference axiom 3 follows from formulas 60 and 61.*

Proof: Similar to the proof of theorem 10.

6.4 Sequential Modal Formalism, <> Version

This formalism is analogous to the sequential existential relational formalism. This formalism has the following axioms:

$$\langle\alpha_1\alpha_2\ldots\alpha_n\rangle X \equiv \langle\alpha_1\rangle\langle\alpha_2\rangle\ldots\langle\alpha_n\rangle X \tag{62}$$

$$\forall a\overline{\alpha}\overline{\beta}(d(\overline{\alpha},\overline{\beta}) \wedge a:\overline{\beta} \supset (\langle\overline{\alpha}\rangle X \equiv [a]\langle\overline{\alpha}\rangle X)) \tag{63}$$

$$\forall p\overline{\alpha}(p:\overline{\alpha} \supset \exists q(p \equiv \langle\overline{\alpha}\rangle q)) \tag{64}$$

Theorem 13 *The non-interference axiom 3 follows from formulas 63 and 64.*

Proof: Similar to the proof of theorem 11.

The sequential modal formalisms also have properties 21 and 22, and 23 holds as well if d is defined as stated.

Collective modal formalisms could also be defined, but it seems less natural to quantify over modalities.

7 Constrained Formalisms

The sequential formalisms are natural for domains that are tree structured, but some domains have a different structure. It is possible to represent these as well using *constraints* on aspects. For example, in the existential sequential relational formalism, the *commutative* axiom

$$\forall st\alpha_1\alpha_2(R_{\alpha_1,\alpha_2}(s,t) \equiv R_{\alpha_2,\alpha_1}(s,t)) \tag{65}$$

expresses a constraint on the structure of situations. If the set of aspects is $\{0,1\}$, then situations without the commutative axiom have a tree structure, but with the commutative axiom, the structure becomes a mesh. A mesh structure might be more appropriate for navigating in a two-dimensional space, or finding locations on city streets. Of course, there are many other constraints corresponding to different problem structures.

Such constraints require special care when specifying d. Suppose one specifies d by $\exists i(\alpha_i \neq \beta_i) \supset d(\alpha,\beta)$. Suppose $a:\beta$ and $p:\alpha$ where $\alpha = \beta = (0,1)$. Because there is no i such that $\alpha_i \neq \beta_i$, it appears that $F[a,p]$ is not derivable. In fact, it is possible to derive $F[a,p]$ as follows: From assertion 19 with $\overline{\alpha} = (1,0)$ and $\overline{\beta} = (0,1)$, one obtains

$$R_{(1,0)}(s,t) \equiv R_{(1,0)}(a(s),t). \tag{66}$$

From $R_{(0,1)}(x,y) \equiv R_{(1,0)}(x,y)$ it follows that

$$R_{(0,1)}(s,t) \equiv R_{(0,1)}(a(s),t). \tag{67}$$

From assertion 20 and $p:(0,1)$ it follows that

$$p(s) \equiv \exists t(q(t) \wedge R_{(0,1)}(s,t)). \tag{68}$$

From this and assertion 67 it follows that $p(s) \equiv p(a(s))$, which is $F[a,p]$. Thus one has $F[a,p]$ for p having any of the aspects $(0,0),(0,1),(1,0),(1,1)$, which means that action a has no effects at all, under a reasonable interpretation. To avoid this problem, it is necessary to specify d differently so that $d(\alpha,\beta)$ *unless* $\alpha = \beta$ is derivable using commutativity. Thus in this example, $d((0,1),(1,0))$ would not be asserted.

It is also possible to obtain the effect of the commutative axiom in other sequential formalisms. In the universal sequential relational formalism, it is possible to use the same commutativity axiom. In the sequential functional formalism, instead of the commutative axiom one has the following:

$$\forall st\alpha_1\alpha_2 f_{\alpha_1,\alpha_2} = f_{\alpha_2,\alpha_1}. \tag{69}$$

In the sequential modal formalism, the [ ] version, the corresponding commutative axiom is

$$[\alpha_1\alpha_2]X \equiv [\alpha_2\alpha_1]X \tag{70}$$

and in the sequential modal formalism, the $\langle\rangle$ version, the corresponding commutative axiom is

$$\langle\alpha_1\alpha_2\rangle X \equiv \langle\alpha_2\alpha_1\rangle X. \tag{71}$$

Of course, for a two-dimensional space, another possibility is to have aspects of the form (x,y) where x and y are real numbers giving x and y co-ordinates. Then it would be appropriate to specify the d relation using geometrical constraints of some kind.

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