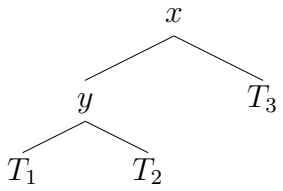
COMP 122
Algorithms and Analysis
Fall 2004
Second Mid Semester Exam
Tuesday, Nov. 2, 2004
Closed Book - Closed Notes
Don't forget to write your name or ID and pledge on the exam sheet.
This exam has three pages.

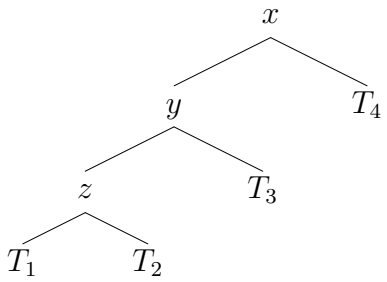
1. (12 points) (a) Give an upper bound on the height of a red black tree having n internal nodes. _____ (b) If a red black tree has black height 15, what is its maximum height? _____ (c) If a red black tree has black height 15, what is the minimum number of internal nodes in the tree? _____

2. (10 points) Give a sequence of seven elements in the range $\{1, 2, 3, \ldots, 7\}$ which, when inserted into an initially empty binary search tree, will produce a perfectly balanced tree. _____

3. (10 points) Suppose T is a height balanced binary tree. Suppose that T_1, T_2, T_3, and T_4 are height-balanced binary trees of height h. (a) Assume that the following tree is a binary search tree. Is this tree necessarily height balanced? Why or why not? _____



(b) Assume that the following tree is a binary search tree. Is it necessarily height balanced? Why or why not? _____



4. (10 points) Suppose a hash table has a load factor of 99/100. (a) If open addressing is being used, assuming simple uniform hashing, what is the expected number of probes in an unsuccessful search? _____ (b) If hashing with chaining is being used, assuming simple uniform hashing, what is the expected time for an unsuccessful search? _____

5. (10 points) (a) How fast can one sort n elements in the range $\{1, 2, \ldots, n\}$ using $O(n)$ storage? Give an asymptotic bound and a brief justification. _____ b) How fast can one sort $\sqrt{n}$ elements in the range $\{1, 2, \ldots, n\}$ using $O(\sqrt{n})$ storage? Give an asymptotic bound and a brief justification. _____

6. (10 points) Suppose there are n binary numbers, each m bits in length at most. Suppose one sorts these numbers using radix sort, treating each bit as a separate digit. What is the asymptotic time required to do this? _____

7. (10 points) Consider the following set of elements, where $a : b$ denotes an element with key a and priority b. Construct a treap from these elements. $1 : 5, 2 : 3, 3 : 6, 4 : 1, 5 : 4, 6 : 2$.

8. (12 points) (a) How many comparisons are needed to find the maximum of n elements? Give an asymptotic bound. _____ (b) How many comparisons are needed to find the maximum and second largest of n elements? Give an asymptotic bound. _____ (c) How many comparisons are needed to find the median of n elements? Give an asymptotic bound. _____

9. (10 points) For hashing by the multiplication method, which of the following values for A is best: (a) .125 (b) 0.01 (c) .618 (d) .618033988 _____

10. (10 points) For hashing by the division method, which of the following is the best value for the modulus m? Give a brief justification for your answer.

(a) 256 (b) 128 (c) 10 (d) 181 _____

11. (6 points) If the hash table size is 200 and there are 120 elements in the table, what is the load factor? _____

12. (10 points) Suppose one is using hashing by the multiplication method and A is .528 and the table size m is 20 and the key k is 15. Which bin will this key hash to? _____

13. (10 points) Suppose one is using hashing by the division method and the table size m is 53 and the key k is 225. Which bin will this key hash to? _____

14. (10 points) Suppose one is doing universal hashing and the table size m is 50 and there are 1000 hash functions in all in the set H. Let x and y be two keys. What is the maximum number of hash functions h in H such that h(x) = h(y), according to universal hashing? _____

EXTRA CREDIT: (10 points) Let n_i be a random variable for the number of elements that end up in the i^{th} bucket when doing bucket sort of n objects, assuming there are n buckets and each object has an equal chance of ending up in each bucket. What is $E[n_i]$? _____ What is $E[n_i^2]$? _____

EXTRA CREDIT: (10 points) Suppose n elements hash randomly to n slots. Let n_i be the number of elements that hash to slot i. What is the expected value $E[\Sigma_i n_i^2]$? _____

EXTRA CREDIT: (10 points) (a) How many comparisons are needed to find the maximum and minimum elements of a set of six elements? _____ (b) How many comparisons are needed to find the second largest element of a set of eight elements? _____ Justify your answers briefly.