COMP 122
Algorithms and Analysis
Fall 2004
Final Exam
Thursday, Dec. 16, 2004
Closed Book - Closed Notes
Don't forget to write your name or ID and pledge on the exam sheet.
This exam has three pages.

1. (12 points) For each problem, write in the blank all elements F of the set $\{\Theta, O, o, \Omega, \omega\}$ such that the statement $f(x) = F(g(x))$ is a correct statement of the asymptotic relationship between f and g. Thus if $f(x) = \Omega(g(x))$ and $f(x) = \Theta(g(x))$ and $f(x) = O(g(x))$ are the only three valid asymptotic relationships between f and g, write Ω, Θ, O in the blank. Hint: $f(x) = \omega(g(x))$ means f grows faster than g, in the limit. $f(x) = o(g(x))$ means f grows slower than g, in the limit. Θ means they grow the same, roughly speaking.

a). $f(x) = x^2 + 1, g(x) = 2x^2 + x$. ____________________
b). $f(x) = 2^x + 4x^2, g(x) = 3^x + x + 1$. ____________________
c). $f(x) = x^3, g(x) = 3x + 2$. ____________________
d). $f(x) = 2x + 1, g(x) = 4\log^2 x - 1$. ____________________
e). $f(x) = 2\sqrt{x}, g(x) = 3\log x$. ____________________
f). $f(x) = 3\log_2 x, g(x) = 2\log_3(2x)$. ____________________

2. (10 points) Solve the recurrence $T(n) = 3T(n/3) + \Theta(n)$. Indicate which solution method you used. ____________________

3. (4 points) What is the asymptotic expected time for quicksort? ____________________
4. (4 points) What is the asymptotic worst case time bound for quicksort? ____________________
5. (4 points) What is the asymptotic worst case time bound for heapsort?____________________
6. (4 points) What is the asymptotic expected time for heapsort? ____________________

7. (12 points) (a) Give an upper bound on the height of a red black tree having n internal nodes. ____________________ (b) If a red black tree has black height 12, what is its maximum height? ____________________ (c) If a red black tree has black height 12, what is the minimum number of internal nodes in the tree? ____________________

8. (12 points) (a) How many comparisons are needed to find the maximum of n elements? Give an asymptotic bound. ____________________ (b) How many comparisons are needed to find the maximum and second largest of n elements? Give an asymptotic bound. ____________________ (c) How many comparisons are needed to find the median of n elements? Give an asymptotic bound. ____________________

9. (10 points) For hashing by the division method, which of the following is the best value for the modulus m? Give a brief justification for your answer. (a) 256 (b) 128 (c) 10 (d) 181 ____________________

10. (6 points) If a hash table has size 200 and there are 150 elements in the table, what is the load factor? ____________________

		b	a	a	b	c
b						
c						
a						
b						
a						

11. (10 points) Fill in the above table with the numbers $c[i, j]$ and the arrows used in the longest common subsequence algorithm. Show the longest common subsequence. ____________________

12. (10 points) Suppose the symbols a,b,c,d,e,f are used with the probabilities 1/21, 2/21, 3/21, 4/21, 5/21, and 6/21, respectively. Construct an optimal prefix code (a Huffman code) for these symbols.

13. (6 points) Suppose that a directed graph contains the following edges. List the strongly connected components. $\{(a,b),(b,c),(c,a),(d,e),(e,f),(f,g),$ $(g,d),(h,i),(i,j),(j,k),(k,h),(l,m),\ (m,n),\ (n,l),(f,i),\ (c,e),(j,b),(k,l)\}$ ____

14. (5 points) For each of the following problems, give an asymptotic time bound for the best known algorithms, in terms of the number V of vertices in the graph and the number E of edges:
(a) To determine if an undirected graph is connected.______________
(b) To find the strongly connected components of a directed graph. ______________
(c) To determine if a directed graph has a cycle. ______________
(d) Given a directed graph G and a vertex x of G, to find the lengths of the shortest paths from x to every other vertex in G. ______________
(e) To find the connected components of an undirected graph. ______________

15. (10 points) Indicate an (undirected) edge (x,y) having weight z by (x,y,z). Consider the graph G having the edges $\{(a,f,1),(f,c,2),(b,d,1),$ $(d,e,4),(d,f,3)\}$. Suppose the set of edges A contains just the edge $(b,d,1)$.
(a) Which edge would be added to A next in Kruskal's method? ______________
(b) Which edge would be added to A next in Prim's method? ______________

16. (6 points) For the following questions, give the best method to use to solve the single-source sortest paths problem on the kind of weighted, directed graph specified. Choices: A. Bellman-Ford B. Dijkstra C. An algorithm that is simpler and faster than Dijkstra's algorithm on the specified kind of graphs.
(a) For graphs with no negative weight edges. ______________
(b) For graphs with no negative weight cycles. ______________
(c) For directed acyclic graphs ______________

17. (10 points) EXTRA CREDIT: Compute $\sum_{j=0}^{\infty} \frac{j}{3^j}$. ______________

18. (10 points) EXTRA CREDIT: Solve the recurrence relation $T(n) = 2T(\sqrt{n}) + \Theta(\log n)$ ______________

19. (10 points) EXTRA CREDIT: (10 points) (a) How many comparisons are needed to find the maximum and minimum elements of a set of six elements? ______________ (b) How many comparisons are needed to find the second largest element of a set of eight elements? ______________ Justify your answers briefly.