Statistics on Diffeomorphisms in a Log-Euclidean Framework

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Abstract. In this article, we focus on the computation of statistics of invertible geometrical deformations (i.e., diffeomorphisms), based on the generalization to this type of data of the notion of *principal logarithm*. Remarkably, this logarithm is a simple 3D vector field, and can be used for diffeomorphisms close enough to the identity. This allows to perform *vectorial* statistics on diffeomorphisms, while preserving the invertibility constraint, contrary to Euclidean statistics on displacement fields.

Overview

In this article, which is an extended abstract of [1], we focus on the computation of statistics of general *diffeomorphisms*, i.e. of geometrical deformations (non-linear in general) which are both one-to-one and regular (as well as their inverse). To quantitatively compare non-linear registration algorithms, or in order to constrain them, computing statistics on *global* deformations would be very useful as was done in [6] with *local* statistics.

The computation of statistics is closely linked to the issue of the *parameterization* of diffeomorphisms. Many algorithms, as in [5], provide transformations are always diffeomorphic, and parameterize them via their displacement field. However, Euclidean means of displacement fields do not necessary yield invertible deformations, which makes Euclidean statistics on these parameters problematic for diffeomorphisms. In [7], it was proposed to parameterize arbitrary diffeomorphisms with Geodesic Interpolating Splines control points [4], and then to perform Euclidean operations on these low-dimensional parameters. However, although this guarantees the invertibility of the results, this may not be adequate for the whole variety of invertible transformations used in medical imaging.

To fully take into account the group structure of diffeomorphisms, it has been proposed to parameterize dense deformations with Hilbert spaces of *time-varying* speed vector fields, which yield geometrical deformations via the integration of an Ordinary Differential Equation (ODE) during one unit of time [8, 3]. In [9], it is suggested that the linear space of intial momenta of the geodesics of these spaces could provide an appropriate setting for statistics on diffeomorphisms.

However, this is illustrated in [9] only in the case of landmark matching. To our knowledge, this statistical framework has not been used yet in the *general case*, certainly because of the *iterative nature* of the computation of the mean in this setting, which requires very stable numerical algorithms to converge.

In this work, we introduce a novel parameterization of diffeomorphisms, based on the generalization of the *principal logarithm* to non-linear geometrical deformations. Interestingly, this corresponds to parameterizing diffeomorphisms with *stationnary* speed vectors fields. As for matrices, this logarithm can be used only for transformations close enough to the identity. However, our preliminary numerical experiments on 3D non-rigid registration suggest that this limitation affects only very large deformations, and may not be problematic for image registration results. This novel setting is the infinite-dimensional analogous of the Log-Euclidean framework proposed in [2] for tensors. In this framework, usual Euclidean statistics can be performed on diffeomorphisms via their logarithms, whith excellent mathematical properties like inversion-invariance.

In [1], our contributions are presented as follows. We first present the Log-Euclidean framework for diffeomorphisms, which is closely linked to the notion of *one-parameter subgroups*. Then, we present two efficient algorithms to compute the exponential of a vector field and the logarithm of a diffeomorphism, which are exemplified on synthetic data. Finally, we apply our framework to non-linear registration results to compute a Log-Euclidean mean deformation between a 3D atlas and a dataset of 9 T_1 MR images of human brains.

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