Observability and Estimation Uncertainty Analysis for PMU Placement Alternatives

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***Abstract*—The synchronized phasor measurement unit (PMU), developed in the 1980s, is considered to be one of the most important devices in the future of power systems. While PMU measurements currently cover fewer than 1% of the nodes in the U.S. power grid, the power industry has gained the momentum to advance the technology and install more units. However, with limited resources, the installation must be selective.**

Previous PMU placement research has focused primarily on the network topology, with the goal of finding configurations that achieve full network observability with a minimum number of PMUs. Here we present a new approach that also includes stochastic models for the signals and measurements, to characterize the observability and corresponding uncertainty of any given configuration of PMUs, whether that configuration achieves full observability or not. We hope that this approach can provide planning engineers with a new tool to help choose between PMU placement alternatives.

***Index Terms*—Power systems, Power transmission, Power transmission planning, Power system planning, Power system simulation, Power system state estimation.**

I. Introduction

STATE estimation plays a crucial role in determining the health of a power system. System state can be estimated using the available measurements and network information. The phasor measurement unit (PMU), measures both the magnitude and phase angle of the electrical waves in a power grid. With the ability to impact future power system monitoring and operation methods, the PMU has been identified as one of the key enabling technologies for the future "smart grid."

Traditionally, power grid measurements are provided by remote terminal units (RTU) at the substations. RTU measurements include real/reactive power flows, power injections, and magnitudes of bus voltages and branch currents. The most commonly used state estimation measurement are:

- Line power flow measurements: the real and reactive power flow along the transmission lines or transformers.
- Bus power injection measurements: the real and reactive power injected at the buses.
- Voltage magnitude measurements: the voltage magnitudes of the buses.

Under certain circumstances such as state estimation of distribution systems, the line current magnitude measurements may be considered, which provide the current flow magnitudes along the transmission lines or transformers. The installation of PMUs makes two additional types of measurements available:

- Voltage phasor measurements: the phase angles and magnitudes of voltage phasors at system buses.
- Current phasor measurements: the phase angles and magnitudes of current phasors along transmission lines or transformers.

Recent developments of phasor measurement technologies provide high-speed sensor data (typically 30 samples/second) with precise time synchronization [1], [2]. Synchronized phasor measurements are commonly referred to as *synchrophasors*. This is in comparison with traditional Supervisory Control And Data Acquisition (SCADA) RTU measurements, which have cycle times of five seconds or longer and are not time synchronized.

Widely scattered in the power system network and synchronized from the common global positioning system (GPS) radio clock, PMUs can provide real-time synchrophasor data to the SCADA system to facilitate the time-critical applicatons such as dynamic state estimation and dynamic stability analysis [3]. With properly placed and sufficient numbers of PMUs, time stamped synchrophasors can be used to estimate the whole system status in real-time.

Significant previous work has been dedicated to the selection of the best locations to install new PMUs [4]–[7]. Several algorithms have been developed, primarily with the aim of utilizing a minimum number of PMUs to ensure full network observability. However, two issues caught our attention:

- First, the observability of the entire system is not an "all or nothing" problem. Due to various resource limitations, we do not have the luxury of installing a complete set of PMUs throughout the power grid. Yet we are not starting from scratch either. In practice PMUs are added to the grid incrementally, and it would be useful to have some guidance on where to install them.
- Second, the power system networks usually have complex topologies in practice, so that more than one solution for the same minimum number of PMUs will be obtained. In such cases, the planning engineers need to make a choice from among these solutions.

In this paper, we focus on these two issues. Instead of searching for the minimum set of PMUs to cover the entire power system, we present an approach to determine the effects of the existing PMUs on the estimation of the network state. Furthermore, to help the engineers make decisions regarding different candidate plans, we quantify the impact of the plans based on state space models and steady-state uncertainty estimates.

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II. Background and Related Work

The phasor measurement unit (PMU) was first developed and utilized in [1], [2]. Considering partially observable systems (with an inadequate number of PMUs) the authors in [8] presented an estimation algorithm based on singular value decomposition (SVD), which did not require the complete network system to be observable prior to estimation.

Optimal PMU placement for full observability was studied in [4]. An algorithm for finding the minimum number of PMUs required for power system state estimation was developed, in which simulated annealing optimization and graph theory were utilized in formulating and solving the problem.

In [7] the authors focused on the analysis of network observability and PMU placement when using a mixed measurement set. They developed an optimal placement algorithm for PMUs using integer programming. In [5], a strategic PMU placement algorithm was developed to improve the bad data processing capability of state estimation by taking advantage of the PMU technology. Furthermore, the PMU placement problem was re-studied and a generalized integer linear programming formulation is presented in [6].

Our work, however, is taking a different approach: we provide a method for evaluating any candidate PMU placement design, which can be especially useful when there are multiple placement candidates.

III. PMUs in the Power System State Estimation

With the advent of satellite clock synchronization via GPS, PMUs have achieved a level of precision that typically exceeds the conventional measurements, which makes phasor telemetry a valuable source of measurements. Nowadays, PMUs are becoming more and more attractive in various power system applications such as system monitoring, protection, control, and stability assessment.

The benefit of PMUs is especially evident when it comes to power system state estimation. Conventional power system state estimation uses real and reactive power as measurements, to estimate the bus phasor voltages, As a result, the relationship between measurements and states is non-linear. The solution is always obtained by linearizing the model and solving it in an iterative fashion.

The invention of PMUs alleviates this problem by providing the phasors of voltages and currents measured at a given substation. Hence, by measuring the bus phasor voltages and line phasor currents, the relationship between measurements and states becomes linear. When the measurement equation is linear, the estimation algorithm is direct and significantly faster than the non-linear ones.

IV. Observability Checking

For modeling power systems, we consider two types of measurements: PMU measurements and *zero injection* measurements.

A PMU installed at a specific bus is capable of measuring not only the bus voltage phasor, but also the current phasors along all the lines incident to the bus. Hence in addition to the phasor voltage at this bus, we are able to compute the phasor voltages of all of its neighboring buses.

If a bus has neither generators nor loads, then the sum of flows on all the associated branches to the bus is zero. This equality normally corresponds to a pseudo measurement called the *zero injection measurement*, which has proven to be useful in system state estimation [7]. According to Kirchhoff's current law, we know that at any bus in the power grid, the sum of currents flowing into that bus is equal to the sum of currents flowing out of that bus. Thus, for an injection-measured bus and its n neighbors ($(n+1)$ buses total), if the phasor voltages at any n out of the $(n+1)$ buses are known (observable), then the remainder can also be computed and hence become observable.

Based on these assumptions, we developed the observability checking algorithm shown in Algorithm 1.

Algorithm 1: Observability checking for a given set of PMU locations and zero injection buses.

Input: PMU buses location P_bus and zero injection buses location Z_bus
Output: Observable buses location O_bus
$O\_bus = \emptyset$
$O\_bus = P\_bus$ (the buses that can be measured directly by PMUs)
for $i \leftarrow 1$ **to** $|P\_bus|$ **do**
    $O\_bus = O\_bus \cup neighbors(P\_bus_i)$
(adding the buses that can be estimated through at least one adjacent PMU bus)
$flag = 1$
while *flag* **do**
    $flag = 0$
    for $j \leftarrow 1$ **to** $|Z\_bus|$ **do**
        if $(|Z\_bus_j \cup neighbors(Z\_bus_j)) \cap O\_bus| = |Z\_bus_j \cup neighbors(Z\_bus_j)| - 1$ **then**
            $O\_bus = O\_bus \cup Z\_bus_j \cup neighbors(Z\_bus_j)$
            $flag = 1$
            (adding the buses that can be estimated using the zero injection information. According to the Kirchhoff's law. For a zero injection bus and its neighborhood, if all but one is known, then the unknown one can be estimated)

V. PMU Placement Evaluation

Ideally, we want the PMU placement to be optimal in the sense that it *maximizes observability* while *minimizing cost*. This is especially true for large-scale complex systems. The authors in [6], [7] presented a numerical formulation of this problem, and developed an optimal placement algorithm for PMUs using integer linear programming. This formulation allows easy analysis of the network observability and concerns about the full coverage of the system. However, the integer linear programming approach may not be sufficient for determining the optimal locations of PMUs. The reason is that very often there will be multiple optimal solutions with the same minimum number of PMUs. In such cases, one would want a means for comparing the different solutions, as some will actually offer lower estimate uncertainty and latency.

Built on the previous work [9], [10], our approach is to use a stochastic estimate of the *asymptotic* or *steady-state* error covariance, as a quantitative metric for the performance evaluation. Our approach is generalized for any PMU placement layout, without requirements for achieving full observability.

To estimate the steady-state error covariance, we first consider the relevant state space models.

A. State Space Models

The state space models are the most basic yet extensively used mathematical models in power system state estimation. An assumed *linear* system can be modeled as a pair of linear stochastic process and measurement equations

$$x_k = Ax_{k-1} + w_{k-1} \quad (1)$$

$$z_k = Hx_k + v_k \quad (2)$$

where $x \in \mathcal{R}^n$ is the state vector, $z \in \mathcal{R}^m$ is the measurement vector, A is a $n \times n$ matrix that relates the state at the previous time step $k-1$ to the state at the current step k in the absence of either a driving function or process noise[1], and H is a $m \times n$ matrix that relates the state to the measurement z_k. The process noise w_k and measurement noise v_k are assumed to be mutually independent random variables, spectrally white, and with normal probability distributions

$$p(w) \sim N(0, Q) \quad (3)$$

$$p(v) \sim N(0, R), \quad (4)$$

where the process noise covariance Q and measurement noise covariance R matrices are often assumed to be constant.

For any state estimate $\hat{x}_k$ the estimate error can be defined as $e_k \equiv x_k - \hat{x}_k$ and estimate error covariance as $P_k \equiv E[e_k e_k^T]$. For the state estimate $\hat{x}_k$ at the time step k, the estimate error covariance P_k contains important information, reflecting the uncertainty of the estimation. Methods such as the Kalman Filtering techniques [11], [12], estimate the state by minimizing the *a posteriori* estimate error covariance, in a recursive prediction-correction manner.

B. Steady State Estimation

Although the *a posteriori* estimate error covariance P_k changes over time, the steady-state error covariance can be described as

$$P^{\infty} = \lim_{k \to \infty} E[e_k e_k^T], \quad (5)$$

where x and $\hat{x}$ represent the true and estimated states respectively, and E denotes statistical expectation. Note that we do not actually attempt to estimate x_k and $\hat{x}_k$. Instead we estimate P^{∞} directly from state-space models of the system using stochastic models for the various noise sources.

[1]In practice, the matrix A may change with each time step, but it is assumed to be constant here.

The Discrete Algebraic Riccati Equation (DARE) represents a closed-form solution to the steady-state covariance P^{∞} [13]. Assuming the process and measurement noise elements are uncorrelated, the DARE can be written in this form:

$$\begin{aligned} P^{\infty} =& AP^{\infty}A^T + Q \\ &- AP^{\infty}H^T(R + HP^{\infty}H^T)^{-1}HP^{\infty}A^T \end{aligned} \quad (6)$$

We use the MacFarlane-Potter-Fath "Eigenstructure Method" [13] to calculate the DARE solution P^{∞} as follows. Given the model parameters A, Q, H, and R from the last subsection, we first calculate the $2n \times 2n$ discrete-time Hamiltonian matrix

$$\Psi = \begin{bmatrix} A + QA^{-T}H^TR^{-1}H & QA^{-T} \\ A^{-T}H^TR^{-1}H & A^{-T} \end{bmatrix}. \quad (7)$$

We then form

$$\begin{bmatrix} B \\ C \end{bmatrix} = [e_1, e_2, ..., e_n] \quad (8)$$

from the n characteristic eigenvectors $[e_1, e_2, ..., e_n]$ of Ψ. Finally, using B and C we can compute the steady-state error covariance as

$$P^{\infty} = BC^{-1}. \quad (9)$$

P^{∞} indicates the expected state estimation uncertainty corresponding to a candidate design. Intuitively, given PMU placements leading to the same level of observability, the lower the uncertainty is, the more we prefer this design.

C. The Process Model

For modeling the state and covariance changes over time, there are several ways to identify the parameters A and Q, both *a priori* and on line [10], [13]. We consider the power system to be reasonably stable; hence following the methodology described in [14], a quasi-static model of a power system has been used in our approach. The oscillations in the state variables of this model are assumed to be small, thus the states of the power system at time step $(k+1)$ are the same as those at time step k, except for some zero mean, white Gaussian noise. Hence our process model reduces to

$$x_k = x_{k-1} + w_{k-1}. \quad (10)$$

D. The Measurement Model

Traditionally, real and reactive power measurements are used in the power system state estimation, resulting in nonlinear measurement models. An iterative algorithm must be employed in order to solve for the state of the system. Now with the help of PMUs, the measurement model becomes linear: we simply let the phasor bus voltages be the state, while phasor bus voltages and phasor currents be the measurements. In this way we are able to use non-iterative estimation algorithms, which makes the computation much faster. For a power system, the measurement model can be written in the general form

$$z = Hx + v, \quad (11)$$

where $x \in \mathcal{R}^n$ is the complex state vector, $z \in \mathcal{R}^m$ is the complex measurement vector, and H is a $m \times n$

complex matrix that relates the state to the measurement z_k. The measurement noise elements in v are assumed to be mutually independent random variables, white, and with normal probability distributions

$$p(v) \sim N(0, R). \tag{12}$$

For computational convenience, we convert the complex valued measurement model into a real valued measurement model as in [8]:

$$\begin{bmatrix} Re(z) \\ Im(z) \end{bmatrix} = \begin{bmatrix} Re(H) & -Im(H) \\ Im(H) & Re(H) \end{bmatrix} \begin{bmatrix} Re(x) \\ Im(x) \end{bmatrix} + \begin{bmatrix} Re(v) \\ Im(v) \end{bmatrix} \tag{13}$$

where $Re(\cdot)$ and $Im(\cdot)$ stand for the real and imaginary parts of the corresponding vector/matrix respectively.

After the procedure of observability checking, all the observable buses are divided into one or more levels, according to the "directness" of their observations: with Level 1 containing the most directly observable buses, and the buses with more "indirect" observations labeled with higher levels. Notice that if the power grid only uses PMU measurements, the observable buses belong to either Level 1 or Level 2; however, if both PMU measurements and injection measurements (which could be zero injection pseudo-measurements) are taken into account, there may be more than two levels of observable buses.

1) Level 1 buses: If the bus i is in Level 1 (meaning that there is a PMU placed on this bus), the measurement equation is

$$z_i = x_i + v_i, \tag{14}$$

where z_i is the measured complex voltage at bus i, x_i is the "true" complex voltage at bus i and v_i is the complex measurement noise of this PMU. Thus for all the buses in Level 1, we can write the measurement equation in the matrix form

$$z_V = I \cdot x_{L_1} + v_V, \tag{15}$$

where z_V is the complex voltage measurement subvector, I is the identity matrix, x_{L_1} is the complex state subvector ("true" complex voltages at all Level 1 buses) and v_V is the voltage measurement noise subvector.

2) Level 2 buses: If the bus i is in Level 2 (meaning that there is at least one PMU placed on an adjacent bus), then for each PMU placed at some adjacent bus j, the measurement equation will be

$$z_{ji} = \begin{bmatrix} Y_{ji} & -Y_{ji} \end{bmatrix} \begin{bmatrix} x_j \\ x_i \end{bmatrix} + v_j, \tag{16}$$

where z_{ji} is the measured complex current at bus j (towards bus i), Y_{ji} is the admittance of line (j, i), x_j and x_i are the "true" complex voltages at bus j and i respectively, and v_j is the complex measurement noise of this PMU. So for all the buses in Level 1 and Level 2, we have the measurement equation

$$z_C = \begin{bmatrix} Y_{CL_1} & Y_{CL_2} \end{bmatrix} \begin{bmatrix} x_{L_1} \\ x_{L_2} \end{bmatrix} + v_C, \tag{17}$$

where z_C is the complex current measurement subvector, Y_{CL_1} and Y_{CL_2} are the line admittance matrices that relates Level 1 and Level 2 bus voltages to z_C respectively, x_{L_1} and x_{L_2} are the complex state subvector ("true" complex voltages at all Level 1 and Level 2 buses), and v_C is the current measurement noise subvector. In this equation, the measurement matrix $\begin{bmatrix} Y_{CL_1} & Y_{CL_2} \end{bmatrix}$ has each row sum up to zero. If the number of PMUs installed is sufficient, or the power grid is well-connected, it is quite possible that a Level 2 bus has more than one PMU-installed neighbor buses. In such case, we allow the contributions from each adjacent PMU to be fused at this bus.

3) Level 3 or above buses: If the bus i is in Level 3 or above (meaning that the bus voltage can be calculated, but there is no PMU placed on itself or any adjacent bus), then the complex bus voltage must be obtained by using the information about the net injection current measurements, and applying Kirchhoff's Current Law. Generally there are two possibilities:

- an injection measurement is taken at bus i; or
- an injection measurement is taken at bus j, which is adjacent to bus i.

Overall, assuming there are totally l levels of observable buses, the measurement equation can be written as

$$z_I = \begin{bmatrix} Y_{IL_1} & Y_{IL_2} & Y_{IL_3} \cdots & Y_{IL_l} \end{bmatrix} \begin{bmatrix} x_{L_1} \\ x_{L_2} \\ \vdots \\ x_{L_l} \end{bmatrix} + v_I, \tag{18}$$

where z_I is the complex injection current measurement subvector (zero pseudo-measurements), Y_{IL_1} through Y_{IL_l} are the node admittance matrices of all observable buses that related to z_I, x_{L_1} through x_{L_l} represent the complex state subvector for all observable buses, and v_I is the current measurement noise subvector.

Combining the above three cases, the complete measurement model can be expressed as

$$\begin{bmatrix} z_V \\ z_C \\ z_I \end{bmatrix} = \begin{bmatrix} I & 0 & 0 \cdots & 0 \\ Y_{CL_1} & Y_{CL_2} & 0 \cdots & 0 \\ Y_{IL_1} & Y_{IL_2} & Y_{IL_3} \cdots & Y_{IL_l} \end{bmatrix} \begin{bmatrix} x_{L_1} \\ x_{L_2} \\ \vdots \\ x_{L_l} \end{bmatrix} + \begin{bmatrix} v_V \\ v_C \\ v_I \end{bmatrix}. \tag{19}$$

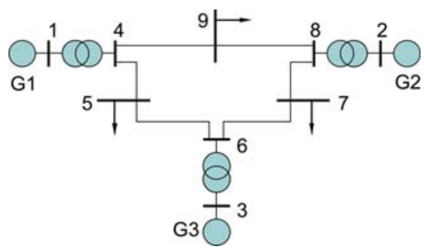
The measurement model can be converted into real valued model using the technique mentioned before.

VI. Simulation Results

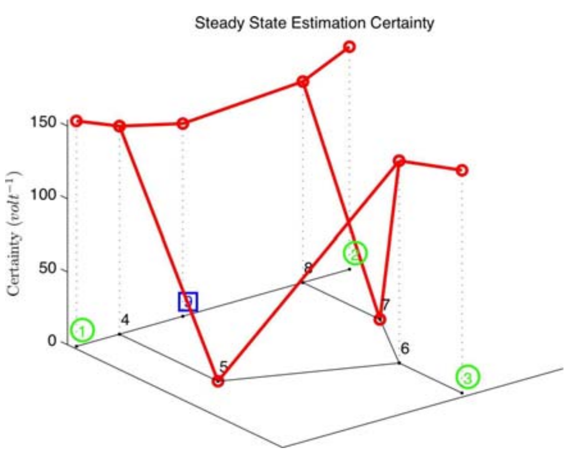
In this section, we first apply our observability checking and estimation uncertainty analysis to two multi-machine systems. We then apply our approach to the evaluation of four different optimal PMU placement alternatives, to find the best one.

A. Observablity Checking and Estimation Uncertainty

Given the sets of PMU buses and zero injection buses, our observability algorithm returns a list of observable buses classified at different levels. Then the steady state error covariance P^{∞} of these observable buses is computed, with each diagonal element P_{ii}^{∞} (i.e. the variance) indicating the estimation uncertainty of the corresponding bus. The estimation uncertainty of any unobservable bus is set to $P_{ii}^{\infty} = \infty$. To better illustrate the results, we define $1/\sqrt{P_{ii}^{\infty}}$ to be the estimation "certainty" (information) of the corresponding bus. In this way, the certainty of any unobservable bus is set to be zero; whereas for any observable bus, the higher certainty value it has, the better it can be estimated.



(a) The 3-machine 9-bus system. Figure reproduced from [3].



(b) The certainty analysis of test case 3.

Fig. 1. The small test system and the plotting result of case 3

We executed our algorithm on one small test system consisting of three machines in a looped network of nine buses as shown in Fig. 1(a). Table I presents the observability analysis of three test cases, and Fig. 1(b) illustrates one of the certainty plots. In all the following certainty plots, the green-colored numbers in cirles represent the PMU buses, while the blue-colored numbers in squares represent the zero-injection buses. Then similarly, our method was tested on a larger 16-machine 68-bus system as shown in Fig. 2(a), representing interconnected New England Test system (NETS) and New York Power System, with also three cases shown in Table II and the certainty analysis of the last one in Fig. 2(b).

B. Comparison Among Multiple Optimal Solutions

In this subsection, we used the small test system in Fig. 1(a) as an example and only considered PMU measurements. Using

TABLE I
OBSERVABILITY CHECKING FOR THE 3-MACHINE 9-BUS SYSTEM

Case	PMU buses	0-inj buses	Observable buses
1	1, 2	None	L1: 1, 2 L2: 4, 8
2	1, 2, 3	None	L1: 1, 2, 3 L2: 4, 6, 8
3	1, 2, 3	9	L1: 1, 2, 3 L2: 4, 6, 8 L3: 9

TABLE II
OBSERVABILITY CHECKING FOR THE 16-MACHINE 68-BUS SYSTEM

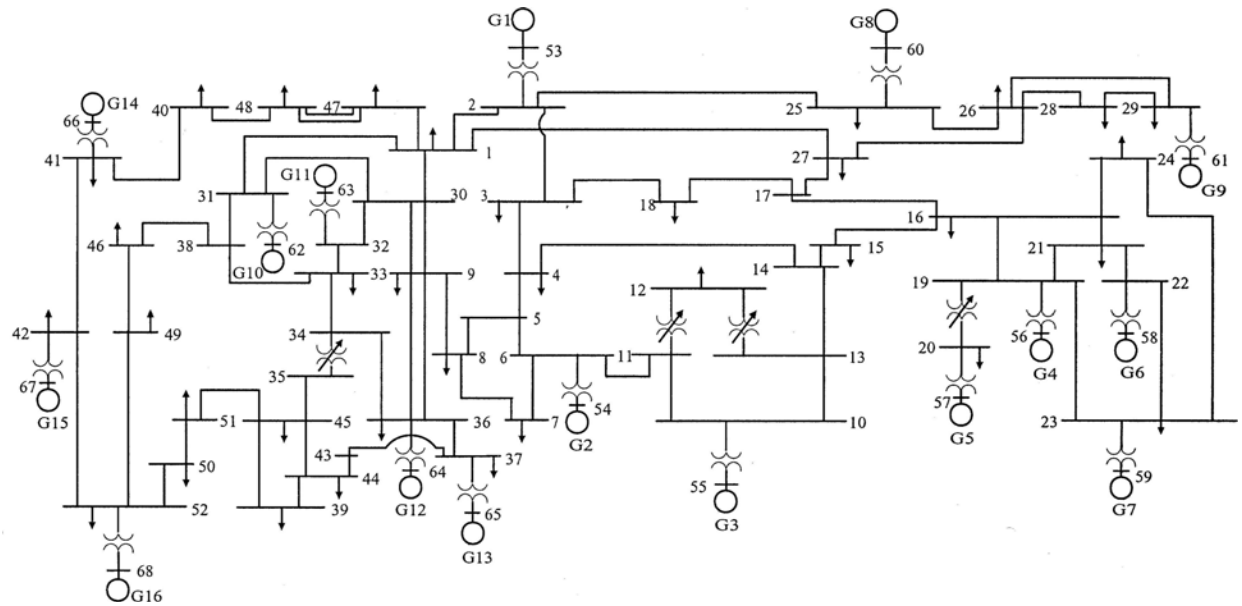
Case	PMU buses	0-inj buses	Observable buses
1	1, 11, 21,	None	L1: 1, 11, 21 L2: 2, 6, 10, 12, 16, 22, 27, 30, 31, 47
2	1, 11, 21, 31, 41, 51	None	L1: 1, 11, 21, 31, 41, 51 L2: 2, 6, 10, 12, 16, 22, 27, 30, 38, 40, 42, 45, 47, 50, 62, 66
3	1, 11, 21, 31, 41, 51	12, 38, 40, 42, 50	L1: 1, 11, 21, 31, 41, 51 L2: 2, 6, 10, 12, 16, 22, 27, 30, 38, 40, 42, 45, 47, 50, 62, 66 L3: 13 48 52 L4: 67

the linear integer programming method, we could in fact obtain four "optimal" solutions, with PMUs installed at buses $\{1, 6, 8\}$, $\{2, 4, 6\}$, $\{3, 4, 8\}$ and $\{4, 6, 8\}$ respectively. They can be called "optimal" in the sense that they all make the entire system observable by using a minimum number of PMUs. To determine which PMU placement strategy is the best, we demonstrate the certainty plots of them in Fig. 3-6. Furthermore, it is up to the users to decide what criterion they prefer using to make their specific choices. For instance, one can use the *maximum*, the *minimum*, the *mean* of these "certainties" (Fig. 7), or even some weighted function of them. In our example, we can immediately tell that by all means, the PMU placement at buses $\{4, 6, 8\}$ outperforms other strategies.

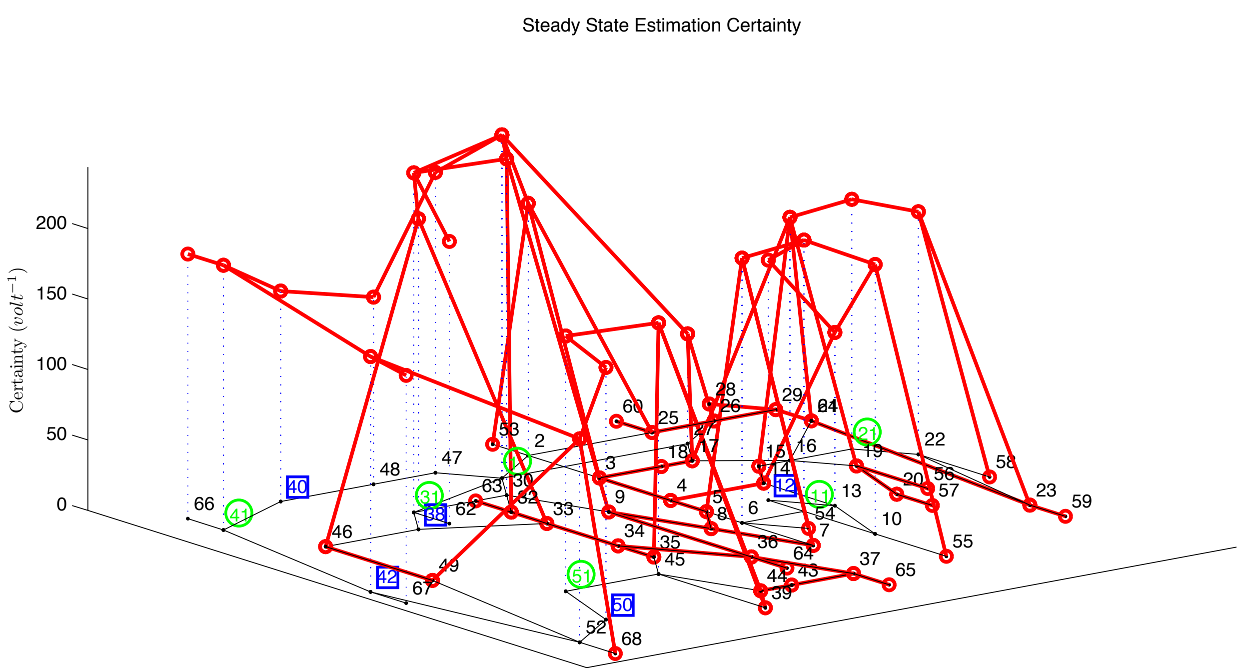
VII. CONCLUSIONS AND FUTURE WORK

We have presented a stochastic framework to estimate the steady-state performance of any candidate PMU placement design, and example visualizations of the results. The results can be readily computed without running the actual estimation procedure; and it is general enough to accommodate any PMU placement design, regardless of achieving full observability.

While we have initially chosen to work with quasi-static process models, we plan to incorporate more dynamic models. We are also beginning to investigate the incorporation of nonlinear models such as would be required to estimate the internal states of generators. We also plan to extend the measurement set to include conventional RTU measurements. Finally, we plan to employ our steady-state approach in a system-wide sensor placement optimization framework.



(a) The 16-machine 68-bus system. Figure reproduced from [15].



(b) The certainty analysis of test case 3

Fig. 2. The large test system and the plotting result of case 3

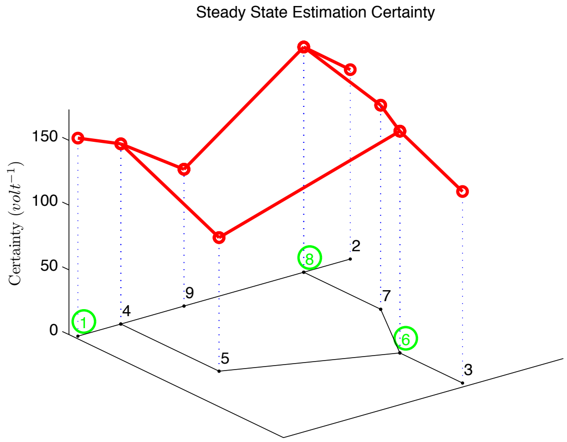


Fig. 3. PMUs installed at buses 1, 6 and 8

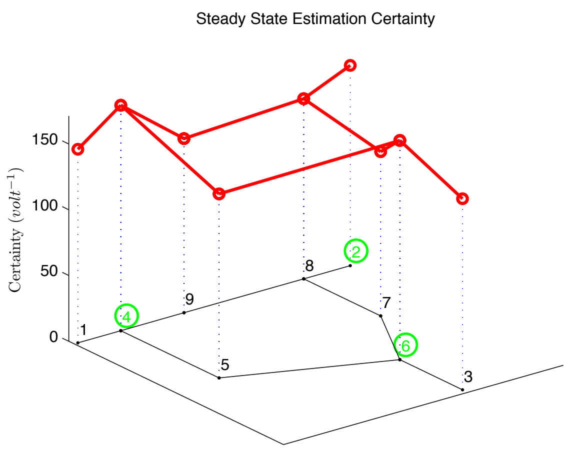


Fig. 4. PMUs installed at buses 2, 4 and 6

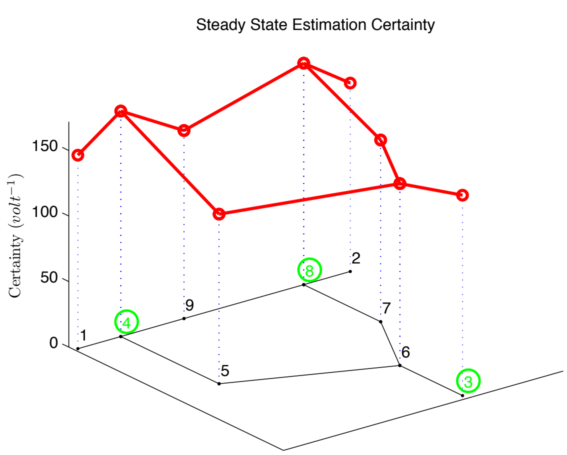


Fig. 5. PMUs installed at buses 3, 4 and 8

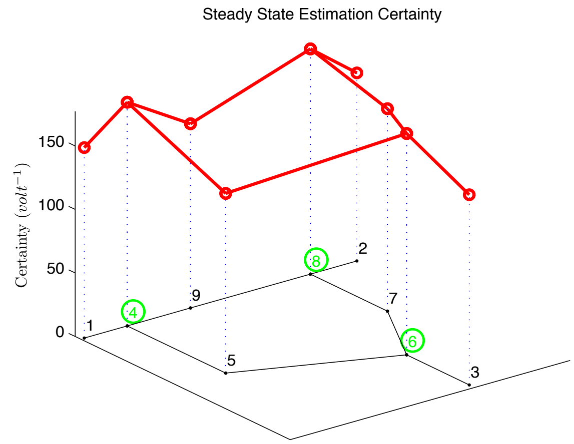


Fig. 6. PMUs installed at buses 4, 6 and 8

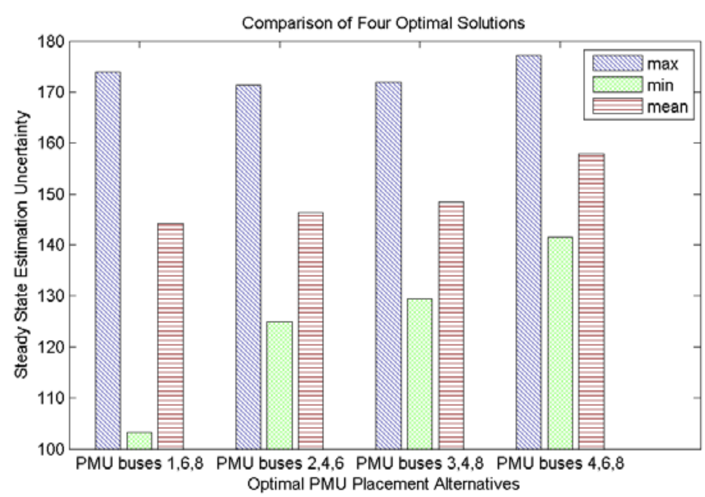


Fig. 7. The comparison of the four "optimal" solutions, based upon three criteria: *max*, *min* and *mean*

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