Medical Image Synthesis via Monte Carlo Simulation

An Application of Statistics in Geometry
&
Building a Geometric Model with Correspondence

James Z. Chen, Stephen M. Pizer, Edward L. Chaney, Sarang Joshi, Joshua Stough

Presented by: Joshua Stough
Medical Image Display & Analysis Group, UNC
midag.cs.unc.edu
Population Simulation Requires Statistical Profiling of Shape

**Goal:** Develop a methodology for generating realistic synthetic medical images *AND* the attendant “ground truth” segmentations for objects of interest.

**Why:** Segmentation method evaluation.

**How:** Build and sample probability distribution of shape.
**Basic Idea**

- **New images via deformation of template geometry and image.**

- **Characteristics**
  - Legal images represent statistical variation of shape over a training set.
  - Image quality as in a clinical setting.
The Process

Template: $I_0, S_0$

Training: $\{I_t\}$

Registration

$I_t \cong H_t(I_0)$

Generate FPM

Analyze $\{H_t\}$ via PCA on $\{\text{FPM displacements}\}$ to yield $\text{Pr(\text{FPM})}$

Template transformation

$H_{s,t} = H_s \cdot H_t^{-1}$

Generate random diffeomorphic

$H_s$: Sample $\text{Pr(\text{FPM})}$ and interpolate

Synthetic images from template $I_t$:

$I'_s = H_{s,t}(I_t)$

$S'_s = S_s = H_s(S_0)$

Synthetic images from template $I_0$:

$I_s = H_s(I_0)$

$S_s = H_s(S_0)$

James Chen
Registration

- Registration – Composition of Two Transformations
  - Linear – MIRIT, Frederik Maes
    - Affine transformation, 12 dof
  - Non-linear – Deformation Diffeomorphism, Joshi

- For all $I_t$, $I_t \cong H_t(I_0)$ and $S_t \cong H_t(S_0)$

Image Warp by Fluid Deformation
Consequence of an Erroneous $H_t$

Template: $I_0, S_0$
Training: $\{I_t\}$

Registration
$I_t \approx H_t(I_0)$

Generate FPM

Analyze $\{H_t\}$ via PCA on $\{\text{FPM displacements}\}$ to yield $\Pr(\text{FPM})$

Template transformation
$H_{s,t} = H_s H_t^{-1}$

Synthetic images from template $I_t$:
$I_s' = H_{s,t}(I_t)$
$S_s' = S_s = H_s(S_0)$

Generate random diffeomorphic
$H_s$: Sample $\Pr(\text{FPM})$ and interpolate

Synthetic images from template $I_0$:
$I_s = H_s(I_0)$
$S_s = H_s(S_0)$

James Chen
Generating the Statistics of $H_t$

Template: $I_0, S_0$
Training: $\{I_t\}$

Registration
$I_t \approx H_t(I_0)$

Generate FPM

Analyze $\{H_t\}$ via PCA on $\{\text{FPM displacements}\}$ to yield $\text{Pr(}\text{FPM)}$

Template transformation
$H_{s,t} = H_s \cdot H_{t}^{-1}$

Synthetic images from template $I_t$
$I_s' = H_{s,t}(I_t)$
$S_s' = S_s = H_s(S_0)$

Generate random diffeomorphic $H_s$: Sample $\text{Pr(}\text{FPM)}$ and interpolate

Synthetic images from template $I_0$
$I_s = H_s(I_0)$
$S_s = H_s(S_0)$

James Chen
Fiducial Point Model

- $H_t$ is *locally correlated*

- Fiducial point choice via greedy iterative algorithm

- $H_t'$ determined by Joshi Landmark Deformation Diffeomorphism

- The Idea: Decrease

$$\frac{1}{T \cdot N_0} \sum_{t=1}^{T} \sum_{x \in S_0} |H_t(x) - H_t'(x)|$$
FPM Generation Algorithm

1. Initialize \( \{F_m\} \) with a few geometrically salient points on \( S_0 \);
2. Apply the training warp function \( H_t \) on \( \{F_m\} \) to get the warped fiducial points: \( F_{m,t} = H_t(F_m) \);
3. Reconstruct the diffeomorphic warp field \( H'_t \) for the entire image volume based on the displacements \( \{F_{m,t} - F_m\} \);
4. For each training case \( t \), locate the point \( p_t \) on the surface of \( S_0 \) that yields the largest discrepancy between \( H_t \) and \( H'_t \);
5. Find most discrepant point \( p \) over the point set \( \{p_t\} \) established from all training cases. Add \( p \) to the fiducial point set;
6. Return to step 2 until a pre-defined optimization criterion has been reached.
A locally accurate warp via FPM landmarks

Volume overlap optimization criterion tracks mean warp discrepancy

Under 100 fiducial points, of thousands on surface

Warped Image and Warped Segmentation

ATLAS WARP TRAINING
Human Kidney Example

- 36 clinical CT images in the training set
- Monotonic Optimization
- 88 fiducial points sufficiently mimick inter-human rater results (94% volume overlap)
Fiducial Point Model Is an Object Representation with Positional Correspondence

- Positional correspondence is via the $H'$ interpolated from the displacements at the fiducial points.
- The correspondence makes this representation suitable for statistical analysis.
Statistical Analysis of the Geometry Representation

Template: $I_0, S_0$

Training: $\{I_t\}$

Registration

$I_t \approx H_t(I_0)$

Generate FPM

Analyze $\{H_t\}$ via PCA on $\{\text{FPM displacements}\}$ to yield $\text{Pr}(\text{FPM})$

Template transformation

$H_{s,t} = H_s \cdot H_t^{-1}$

Generate random diffeomorphic $H_s$: Sample $\text{Pr}(\text{FPM})$ and interpolate

Synthetic images from template $I_t$:

$I'_s = H_{s,t}(I_t)$

$S'_s = S_s = H_s(S_0)$

Synthetic images from template $I_0$:

$I'_s = H_s(I_0)$

$S'_s = H_s(S_0)$

James Chen
Principal Components Analysis of the FPM Displacements

\[
\begin{align*}
\vec{f}_0 &= ((x_{f_1}, y_{f_1}, z_{f_1}), (x_{f_2}, x_{f_2}, x_{f_2}), \ldots, (x_{f_M}, y_{f_M}, z_{f_M})) \\
\vec{f}_i &= H_i(\vec{f}_0) \\
\vec{f}_s &= \langle \vec{f} \rangle + \sum_{k=1}^{n} b_k \cdot \vec{p}_k \quad -3\sqrt{\lambda_k} \leq b_k \leq 3\sqrt{\lambda_k}
\end{align*}
\]

- Points in 3M-d space
- Analyze deviation from mean
- Example: first 7 modes of FPM cover 88% of the total variation.
Modes of Variation – Human Kidney

<table>
<thead>
<tr>
<th>Mode</th>
<th>ATLAS</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td><img src="atlas_2.png" alt="Image" /></td>
<td><img src="mean_2.png" alt="Image" /></td>
</tr>
<tr>
<td>-1</td>
<td><img src="atlas_1.png" alt="Image" /></td>
<td><img src="mean_1.png" alt="Image" /></td>
</tr>
<tr>
<td>+1</td>
<td><img src="atlas_1.png" alt="Image" /></td>
<td><img src="mean_1.png" alt="Image" /></td>
</tr>
<tr>
<td>+2</td>
<td><img src="atlas_2.png" alt="Image" /></td>
<td><img src="mean_2.png" alt="Image" /></td>
</tr>
</tbody>
</table>
Generating Samples of Image Intensity Patterns

Template: $I_0, S_0$
Training: $\{I_t\}$

Registration
$I_t \cong H_t(I_0)$

Generate FPM

Analyze $\{H_t\}$ via PCA on $\{FPM$ displacements$\}$ to yield $Pr(FPM)$

Template transformation
$H_{s,t} = H_s \cdot H^{-1}_t$

Synthetic images from template $I_t$:
$I'_s = H_{s,t}(I_t)$
$S'_s = S_s = H_s(S_0)$

Synthetic images from template $I_0$:
$I_s = H_s(I_0)$
$S_s = H_s(S_0)$

James Chen
## Results

**Synthesis of CT images of kidney region**

<table>
<thead>
<tr>
<th>Template</th>
<th>Synthesized from $H_s(I_0)$</th>
<th>Synthesized from $H_{s,t}(I_t)$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Axial Template" /></td>
<td><img src="image2" alt="Axial Synthesized" /></td>
<td><img src="image3" alt="Axial Synthesized" /></td>
</tr>
<tr>
<td><img src="image4" alt="Sagittal Template" /></td>
<td><img src="image5" alt="Sagittal Synthesized" /></td>
<td><img src="image6" alt="Sagittal Synthesized" /></td>
</tr>
<tr>
<td><img src="image7" alt="Coronal Template" /></td>
<td><img src="image8" alt="Coronal Synthesized" /></td>
<td><img src="image9" alt="Coronal Synthesized" /></td>
</tr>
</tbody>
</table>

Axial | Sagittal | Coronal | James Chen
Results
National Cancer Institute Grant P01 CA47982

References


