Generalized Penetration Depth Computation and Applications to Motion Planning

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Background

Penetration depth (PD) is a distance measure to quantify the extent of inter-penetration between two overlapping, closed, geometric objects. PD computation is important in a number of applications, including physically-based modeling, robot motion planning, virtual reality, haptic rendering and computer games.

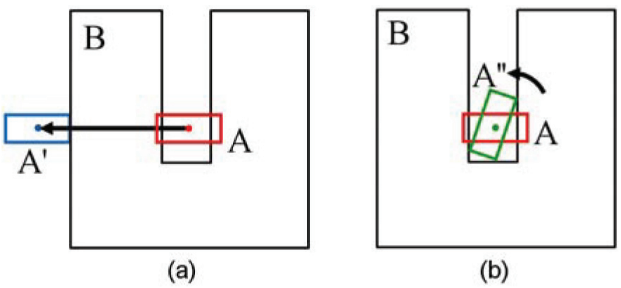

Fig. 1: (a) shows the translational PD between two overlapping objects **A** and **B**. (b) shows that when both translational and rotational transformation are allowed, the amount of the motion to separate **A** from **B** is much smaller than when only translation is allowed in (a).

The Challenge

Most of the prior work on PD computation has been restricted to *translational* PD or PD^t. The PD^t between two overlapping objects is defined as the minimum translational distance needed to separate the two overlapping objects. Many good algorithms to estimate the PD^t between convex and non-convex polyhedra are known. However, because PD^t does not take into account the rotational motion, it is not sufficient for many applications in practice (**Fig. 1**).

The Approach

We take into account the translational and rotational motion to describe the extent of two intersecting objects and refer to that extent of inter-penetration as the *generalized penetration depth* or PD^g. When an object undergoes rigid transformation, some point on the object traces the longest trajectory. The generalized PD between two overlapping objects is defined as the minimum of the longest trajectories of one object under all possible rigid transformations to separate the overlapping objects.

We prove a result that for convex polytopes, their PD^g is same as PD^t. As a result, the well known algorithms to compute the PD^t between the convex polytopes are directly applicable to PD^g.

For non-convex objects, we use the above result to compute a lower bound of PD^g by first computing the convex decomposition for each input models. Next, we take the maximum value of PD^t between all pairwise combinations of convex pieces as our lower bound on PD^g.

To compute an upper bound on PD^g for non-convex polytopes, we reduce it as a variant of a 3D convex containment problem, which can be optimized using linear programming.

In [6], we propose a gradient descent based PD^g optimization algorithm, and incrementally refine the solution on the contact space (**Fig. 3**). Moreover, DISP distance metric is employed since it can be efficiently computed by our *C-DIST* algorithm [5].

Highlights

- **A novel definition for PD^g;**
- **For convex models, we prove that their PD^g is same as PD^t;**
- **Lower and upper bound algorithms on PD^g for non-convex models;**
- **An efficient gradient descent based local PD^g algorithm;**
- **Application of PD^g to *C-obstacle query*;**
- **Application to complete motion planning.**

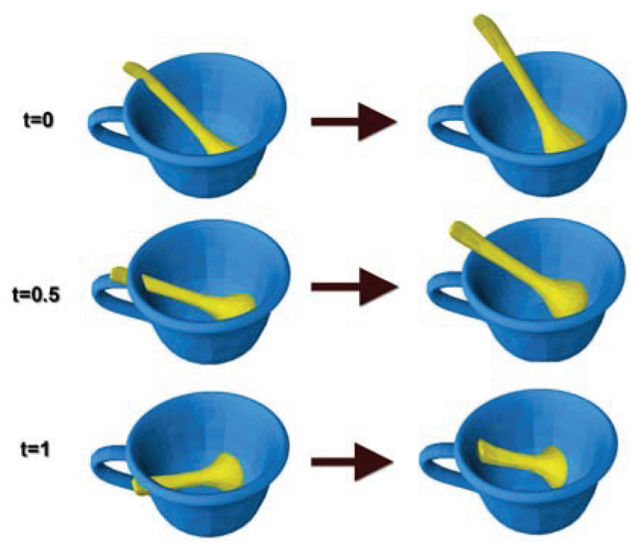

Fig. 2: In the left column, the 'spoon' collides with the 'cup' at *t=0*, *0.5*, and *1*. The right column shows for every *t* the corresponding collision-free configuration, which yields an upper bound on PD^g.

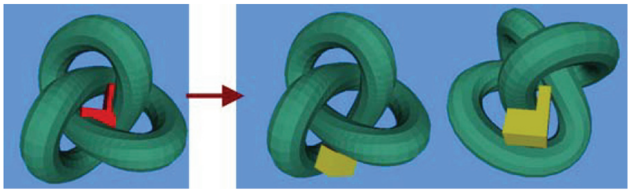

Fig. 3: PD^g computation by incremental refinement. Our algorithm can handle models with strong non-convexity [6].

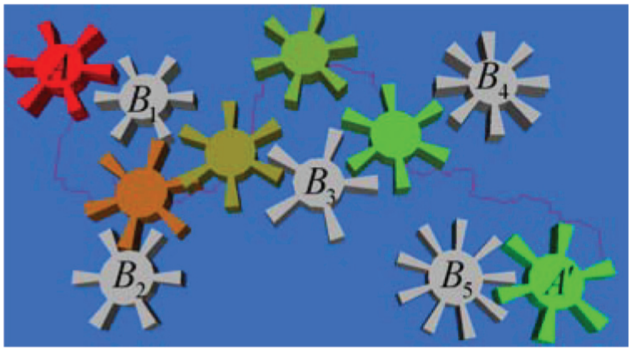

Fig. 4: An application of our C-obstacle query algorithm for a complete motion planner – the star-shaped roadmap method. The robot *Gear* needs to move from initial configuration *A* to goal configuration *A'* by translating and rotating within the shaded rectangular 2D region. We show the robot's intermediate configurations for the found path. The improved star-shaped roadmap method took about 110s for this example and achieved about 2.4 times speedup.

Application to Complete Motion planning

Our PD^g computation algorithm has been applied for motion planning to perform the *C-obstacle query*. Given a primitive in C-space, the C-obstacle query is formally defined as checking whether this primitive lies in C-obstacle space. Usually, the underlying query primitive is a cell.

The C-obstacle query is useful for cell decomposition based algorithms and other complete motion planning approaches, such as star-shaped roadmap and hybrid motion planning (**Figs. 4, 6**).

Another benefit of the *C-obstacle query* is to determine non-existence of any collision-free path for motion planning (**Fig. 5**).

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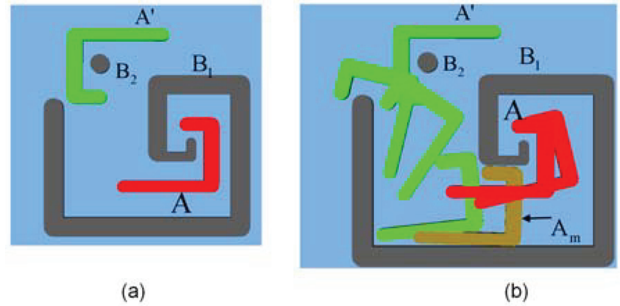

Fig. 5: '2D puzzle' example. (a) our algorithm can report the path non-existence between *A* and *A'* in *1.855s*. (b) is a modified version of (a) with the obstacle B_1 enlarged. We can find a collision-free path through a narrow passage among the obstacle [3].

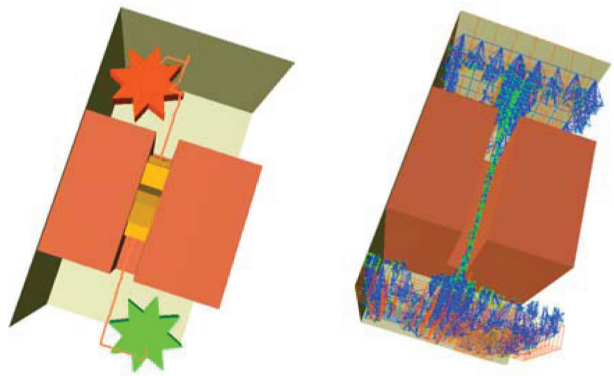

Fig. 6: By combing approximate cell decomposition (ACD) with probabilistic roadmaps (PRM), our hybrid algorithm is resolution-complete, and can efficiently find a collision free path or determine path non-existence for this 4-DOF problem [4].

Selected Publications

L-J. Zhang, Y.J. Kim, G. Varadhan, and D. Manocha. "Generalized Penetration Depth Computation," *ACM Solid and Physical Modeling Symposium (SPM06)*, 173-184, 2006.

L-J. Zhang, Y.J. Kim, G. Varadhan, and D. Manocha. "Fast C-obstacle Query Computation for Motion Planning," *IEEE International Conference on Robotics and Automation (ICRA)*, 3035-3040, 2006.

L-J. Zhang, Y. J. Kim, and D. Manocha. "A Simple Path Non-Existence Algorithm for Low DOF Robots," *International Workshop on the Algorithmic Foundations of Robotics (WAFR)*, 2006.

L-J. Zhang, Y. J. Kim, and D. Manocha. "A Hybrid Approach for Complete Motion Planning," UNC-CS Technical Report 06-022, 2006.

L-J. Zhang, Y. J. Kim, and D. Manocha. "C-DIST: Efficient Distance Computation for Rigid and Articulated Models in Configuration Space," *ACM Solid and Physical Modeling Symposium (SPM07)*, 2007, to appear.

L-J. Zhang, Y. J. Kim, and D. Manocha. "A Simple and Fast Algorithm for Generalized Penetration Depth Computation," *Robotics: Science and Systems Conference*, 2007, to appear.

For More Information

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