



Multi-Projector Techniques for Real-Time Visualizations in



Outline

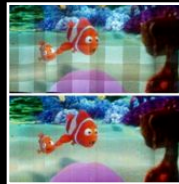
these slides:
www.uni-weimar.de/medien/AR



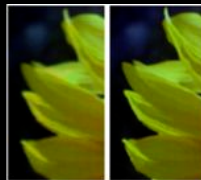
Introduction
Motivations and Applications



Geometric Correction
Planar, Non-Trivial, Complex Surfaces



Radiometric Compensation
Local and Global Light Effects



Advanced Techniques
View-Dependence, Multi-Focal Projection, Light Transport



Outlook
Limitations and Future Work



Introduction

Evolving Evolution

50s

60s

70s

80s

90s

2k

VR



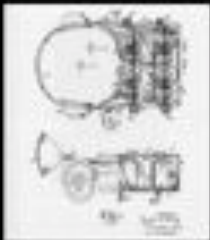
Heilig



Sutherland



Carnegie Mellon Univ.



Heilig / Corneau and Bryan



UNC



Courtesy to all who cannot be mentioned here



Matusik and Pfister



Gardín, et al



Courtesy to all who cannot be mentioned here



University



Manning et al



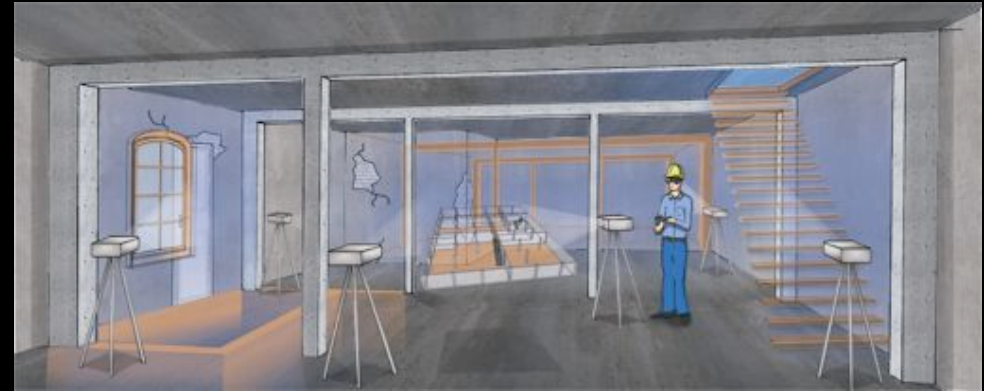
Franc Telecom: Henrysson, et al

AR

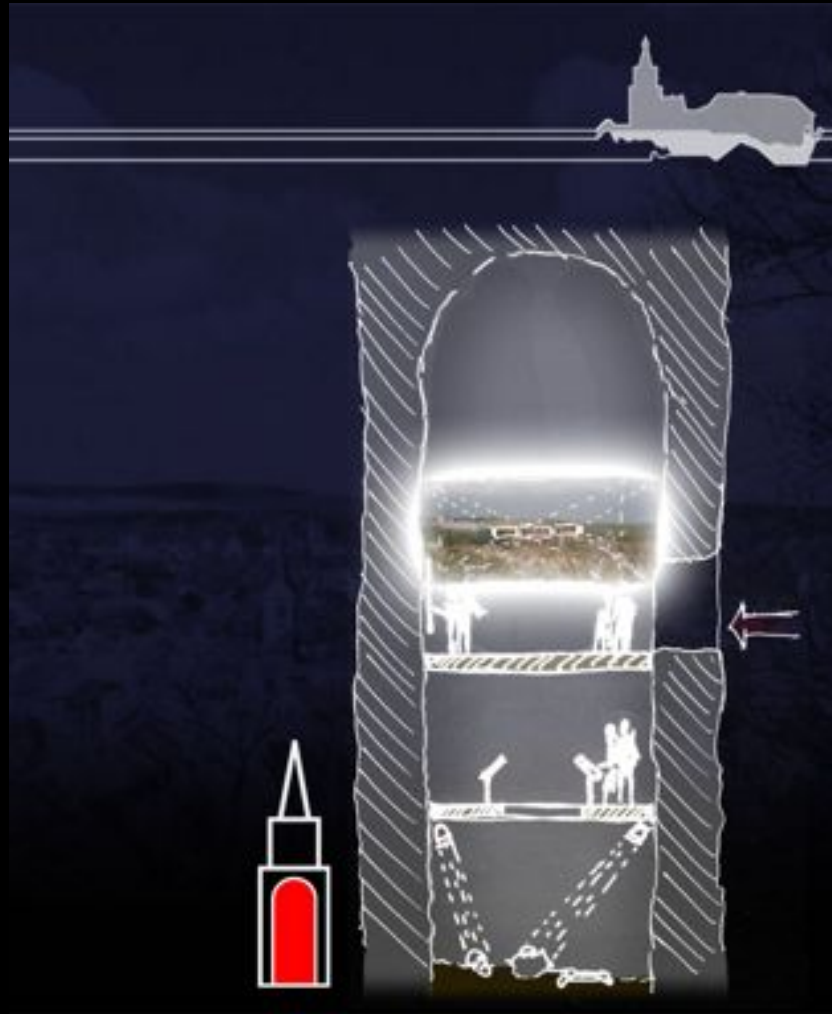
Spatial

Mobile

Motivation: Projection

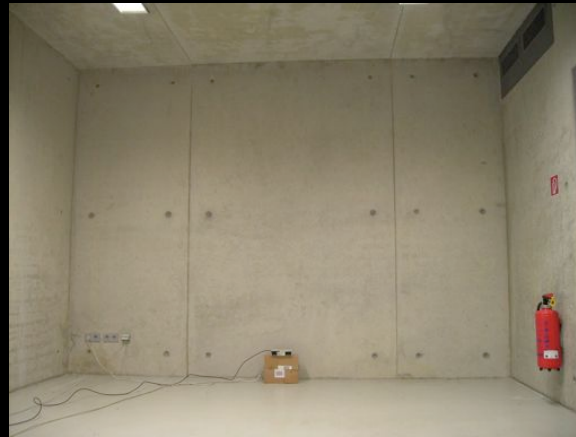
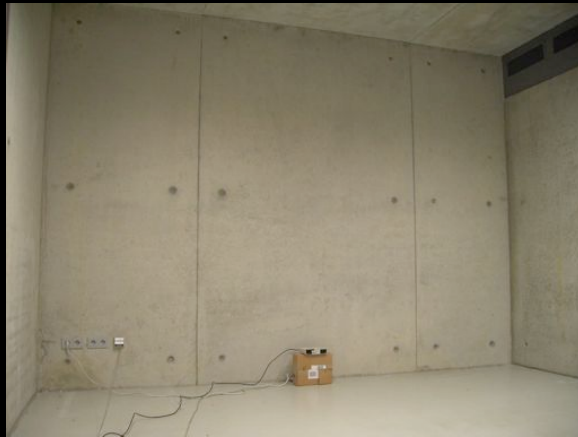


Application: Historic Sites and Museums



360° Surround
Projection in Castel
Tower
(Running project in
coop. with Bennert
Group)

Application: Architectural Visualization



Bimber et al, IEEE/ACM
ISMAR 2005

On-Site Architectural Visualizations
(Running project in coop. with Architecture Faculty, BUW)

Application: Pocket Projectors



Courtesy: InFocus

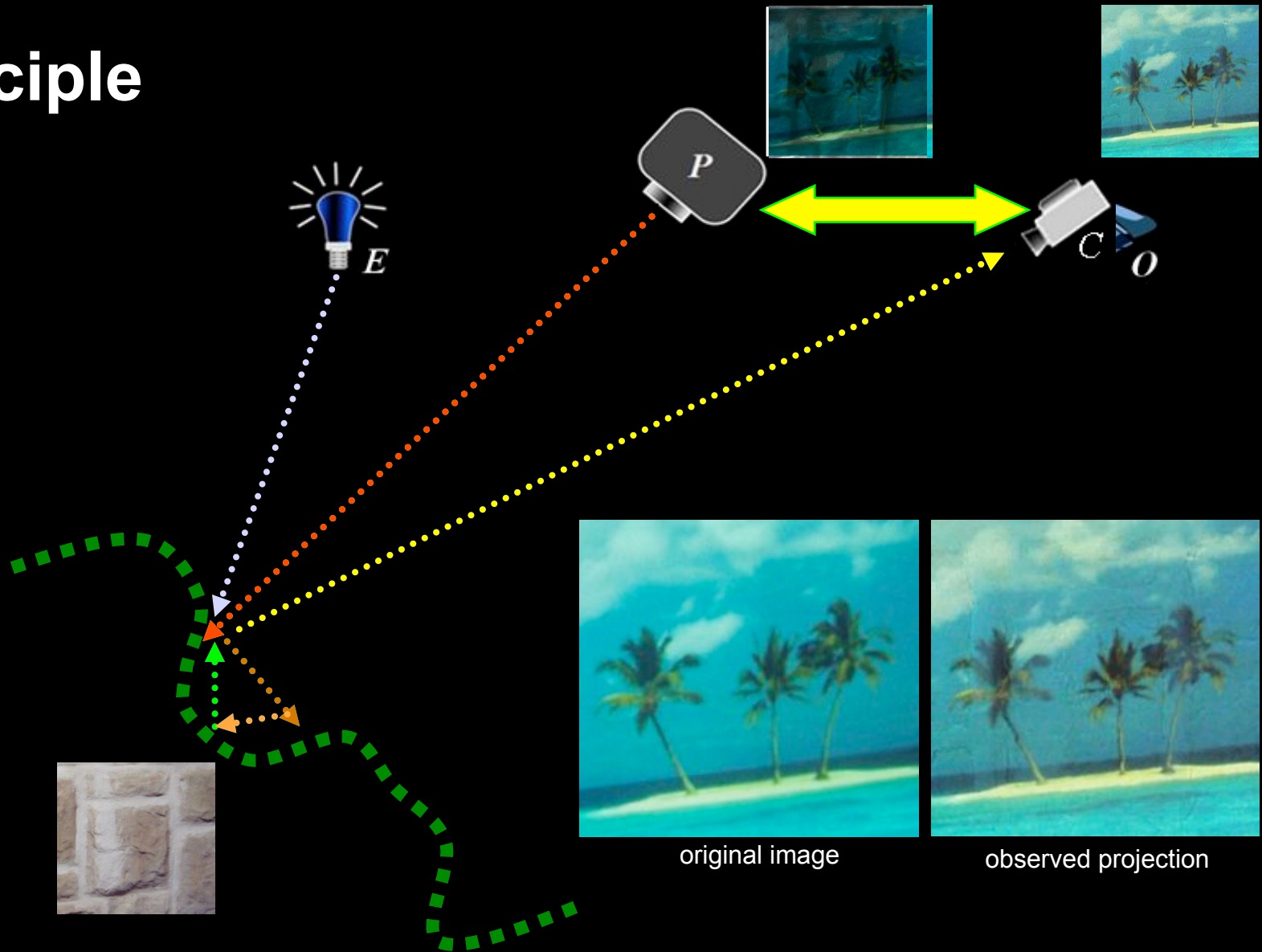


Courtesy: Siemens





Principle



Some Challenges



color blending



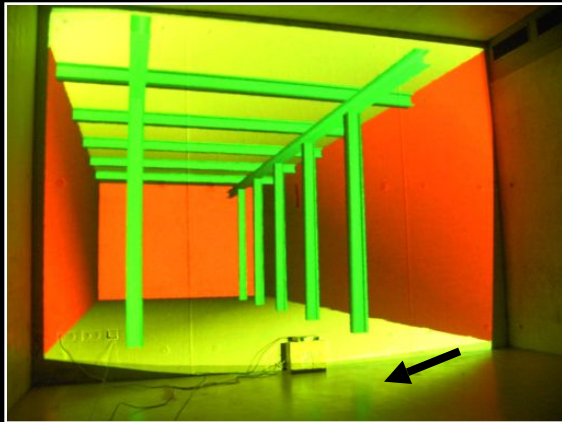
geometric warping



misregistration



regional defocus

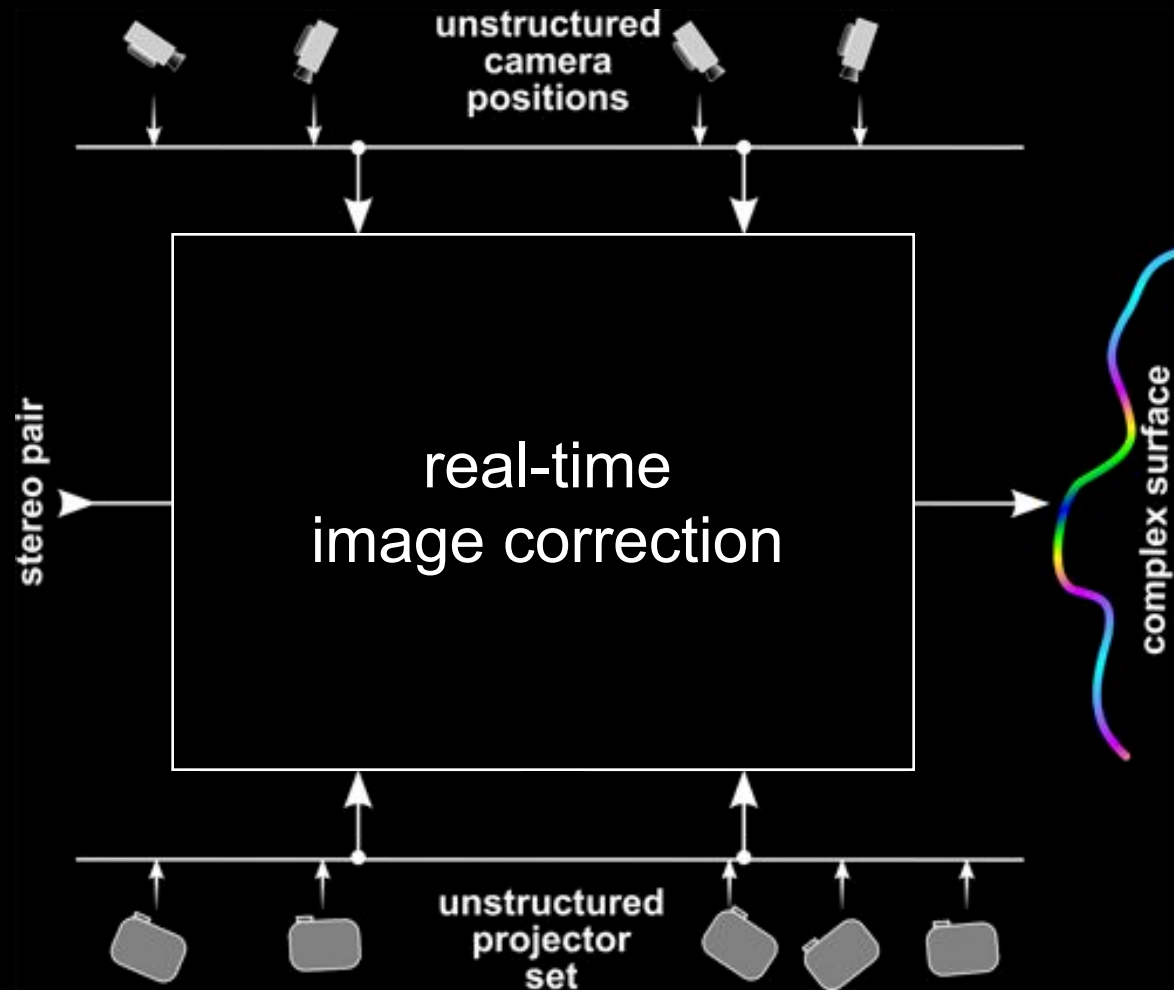


scattering



specular reflection, refraction, sub-surface scattering, inter-reflections, dispersion, diffraction, etc.

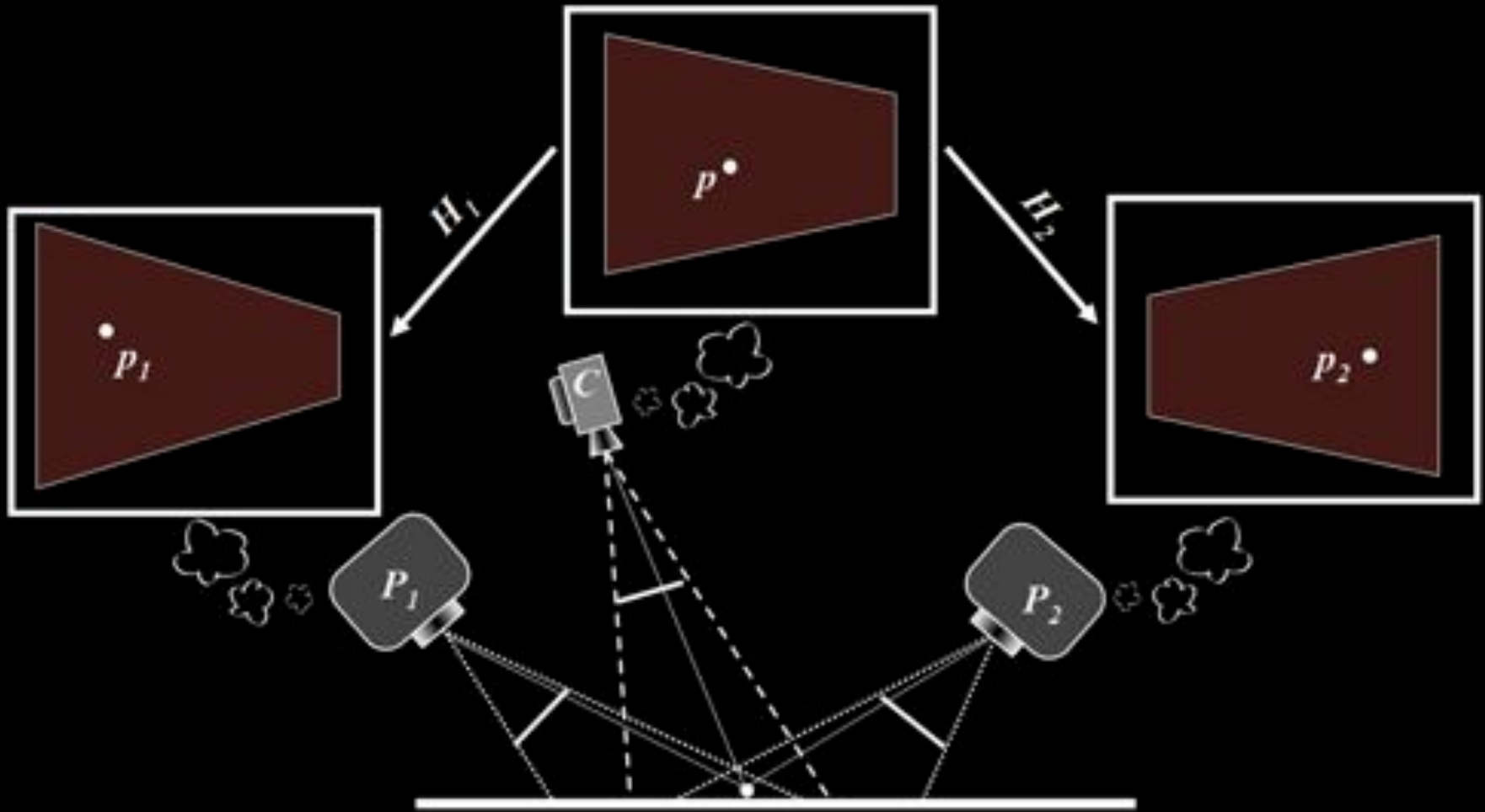
A Multi-Projector-Camera Approach





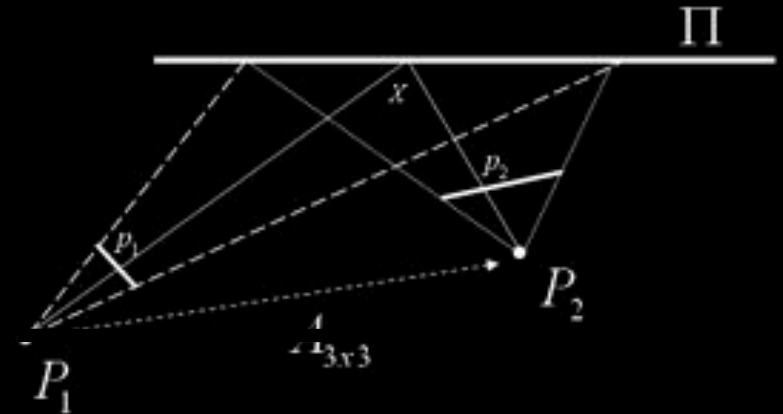
Geometric Correction

Planar Surfaces



Homography

- homography is a mapping between two projections over a plane
- can map pixel coordinates from one perspective to another
- equation system has to be solved to determine 8 parameters of matrix A
- can be used directly in transformation pipeline by multiplying the following matrix after projection (without perspective division):



$$p_2 \cong A_{3 \times 3} p_1$$

$$\begin{bmatrix} p_{2x} \\ p_{2y} \\ 1 \end{bmatrix} \cong \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 1 \end{bmatrix} \begin{bmatrix} p_{1x} \\ p_{1y} \\ 1 \end{bmatrix}$$

$$A_{4 \times 4} = \begin{bmatrix} a_{11} & a_{12} & 0 & a_{13} \\ a_{21} & a_{22} & 0 & a_{23} \\ 0 & 0 & 1 & 0 \\ a_{31} & a_{32} & 0 & 1 \end{bmatrix}$$

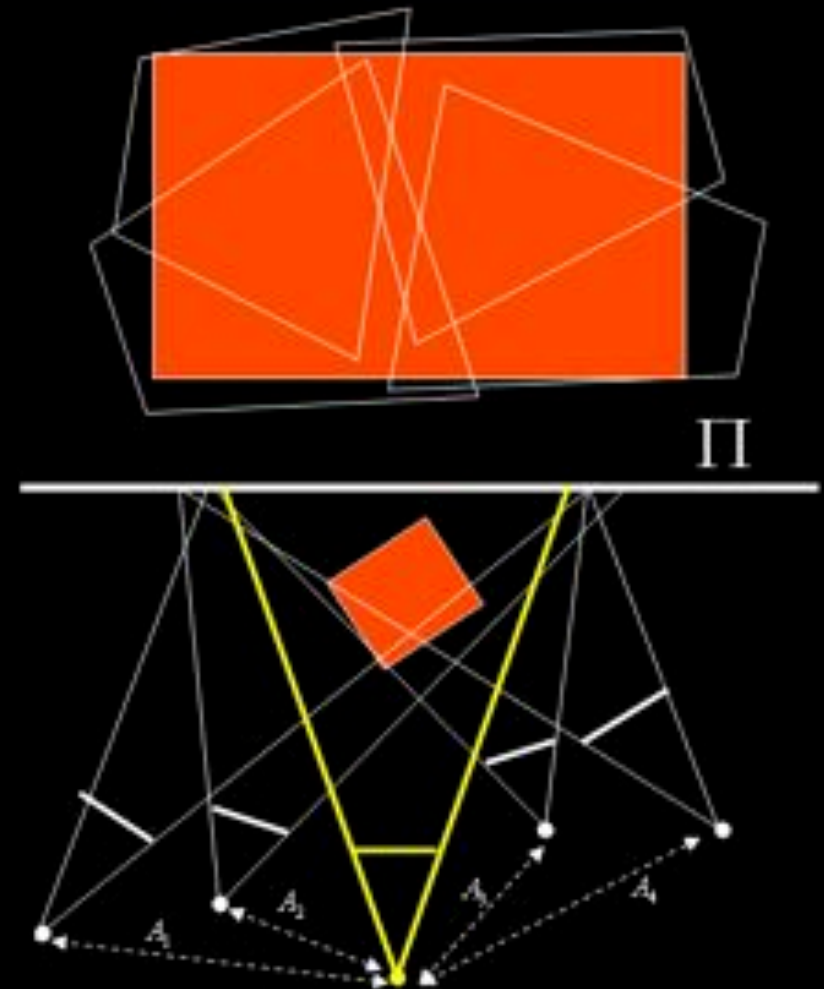
- ensure intact depth values with (approximately)

$$p_{2z} = \frac{p_{1z}}{(a_{31}p_{1x} + a_{32}p_{1y} + p_{1z})} \in [-1, 1]$$

$$A_{4 \times 4} = \begin{bmatrix} a_{11} & a_{12} & 0 & a_{13} \\ a_{21} & a_{22} & 0 & a_{23} \\ 0 & 0 & 1 - |a_{31}| - |a_{32}| & 0 \\ a_{31} & a_{32} & 0 & 1 \end{bmatrix}$$

Multi-Projector Registration

- registering multiple projectors onto a common planar surface
- map all perspective into a single target perspective via homographies
- target perspective can be camera perspective
 - automatic determination of matrix parameters via structured light
- rendering
 - render image for target perspective (if target perspective is **orthogonal** to plane, then it can be done with an off-axis projection of an observer!)
 - map pixels into individual projector views (i.e., multiply 4x4 version of homography matrix onto matrix stack [after projection] and ensure that depth values remain intact!)





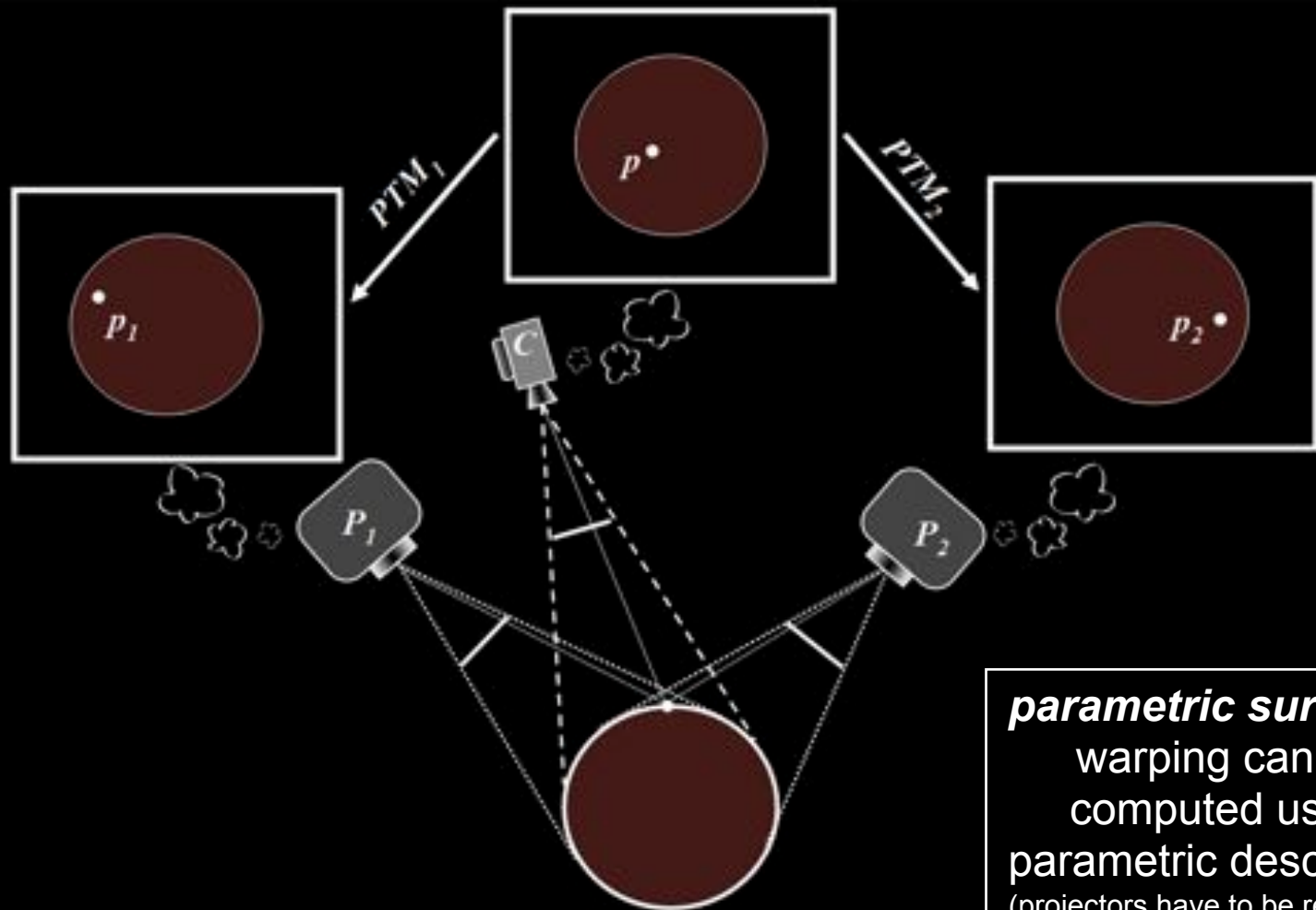
Example: Tiled Projection Screens



Courtesy: Brown, et al.,
IEEE TVCG, 2005

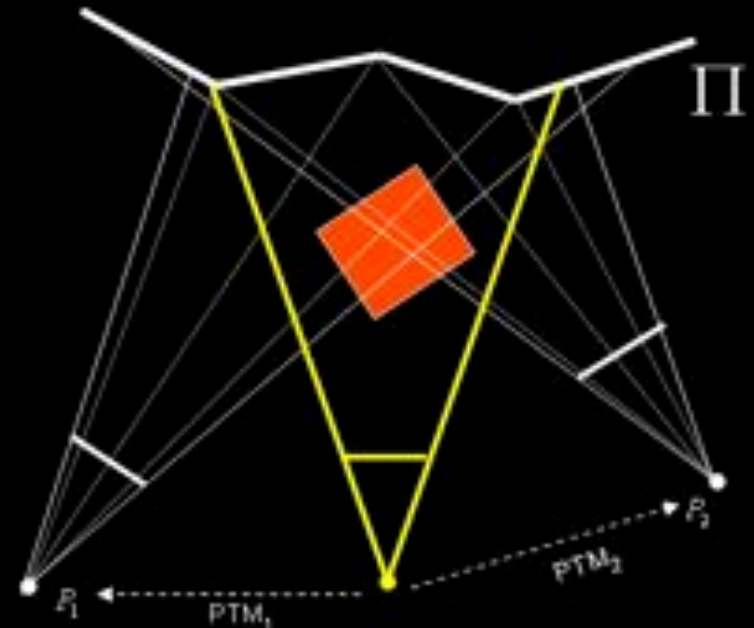


Non-Trivial Surfaces



Projective Texture Mapping

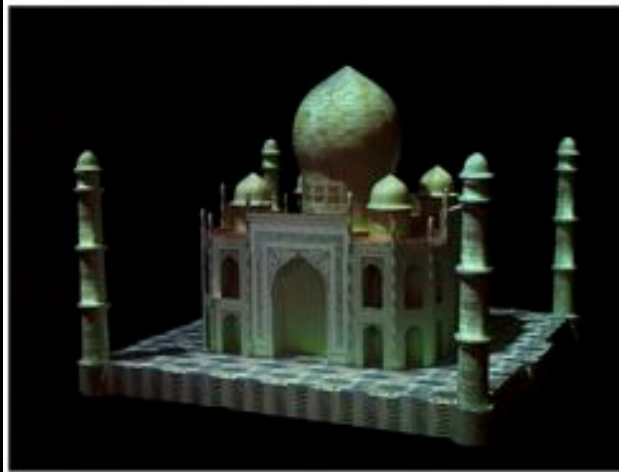
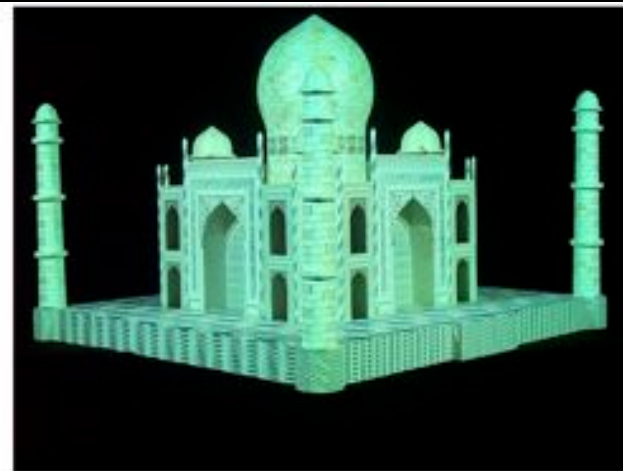
- given a geometric definition of the surface
 - scan or model
- determine intrinsic and extrinsic of projectors with respect to surface
 - measure projections of known 3D surface points on image plane of projector and solve equation system to determine parameters of matrix
- define virtual camera with same parameter for each projector
- render 3D model of surface, textured with images, from perspective of projectors/virtual cameras
- texture coordinates can be automatically generated from target perspective via projective texture mapping



$$\begin{pmatrix} wx \\ wy \\ wz \\ w \end{pmatrix} = \begin{bmatrix} f & \cdot & \cdot & \cdot \\ \cdot & f & \cdot & \cdot \\ \cdot & \cdot & \cdot & 1 \\ \cdot & \cdot & 1 & \cdot \end{bmatrix} \begin{bmatrix} R_{11} & R_{12} & R_{13} & t_x \\ R_{21} & R_{22} & R_{23} & t_y \\ R_{31} & R_{32} & R_{33} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{pmatrix} X \\ Y \\ Z \\ 1 \end{pmatrix}$$

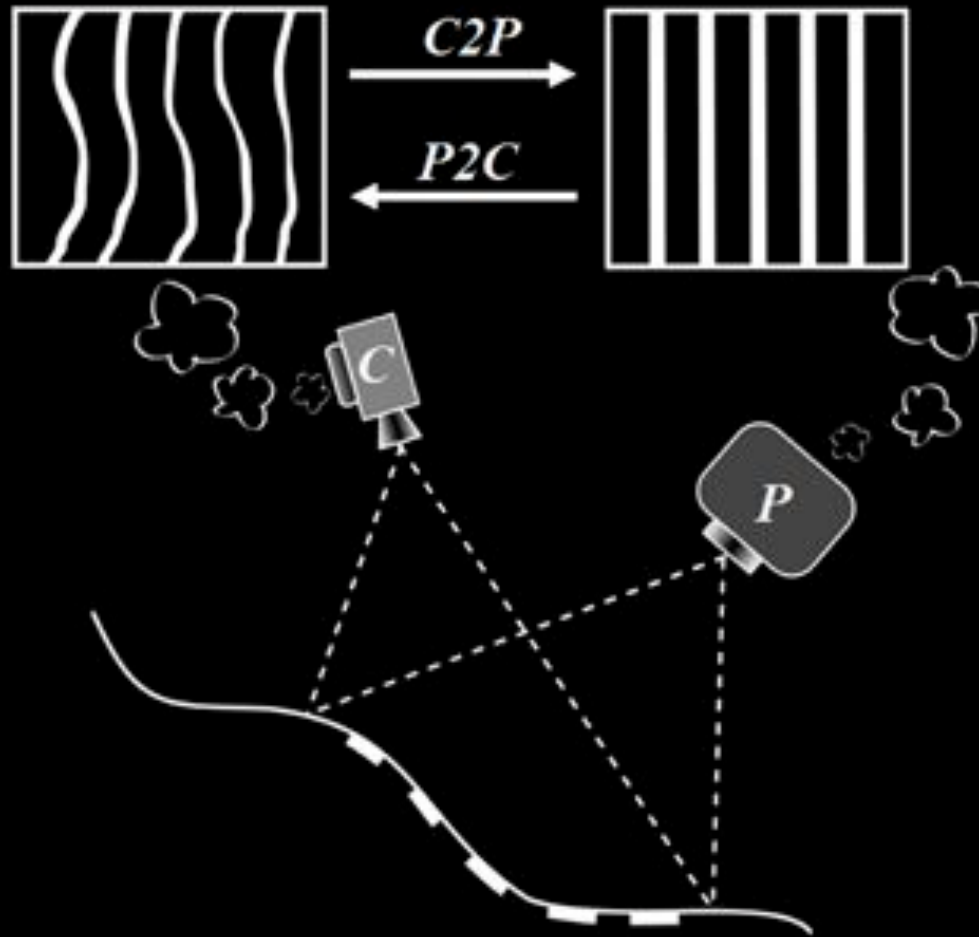


Example: Shader Lamps



Courtesy: Raskar, et al., EGRW 2001

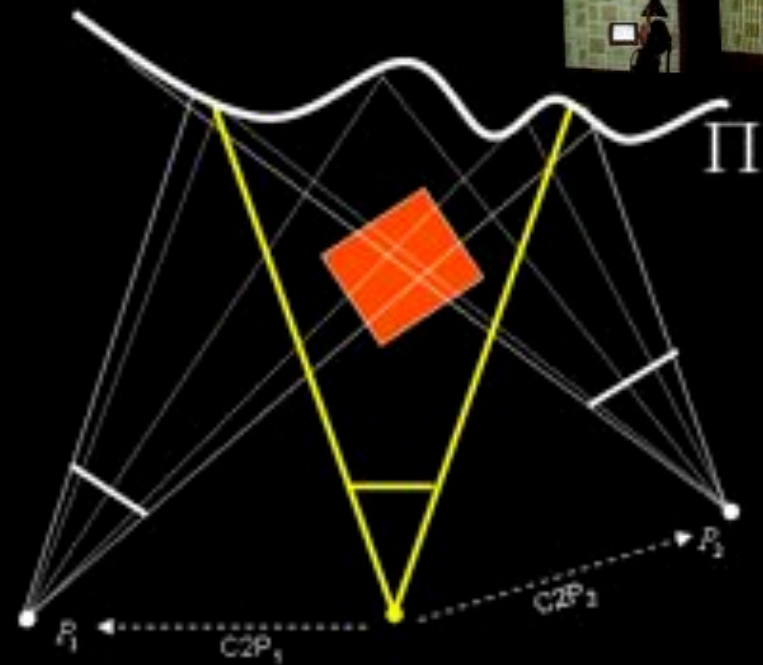
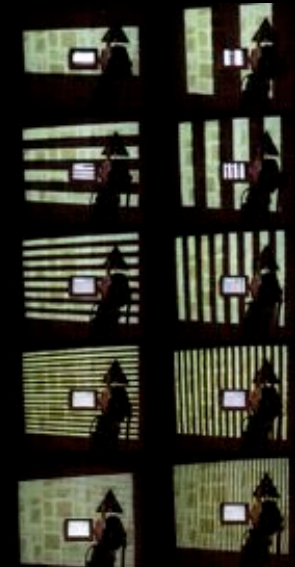
Complex Surfaces



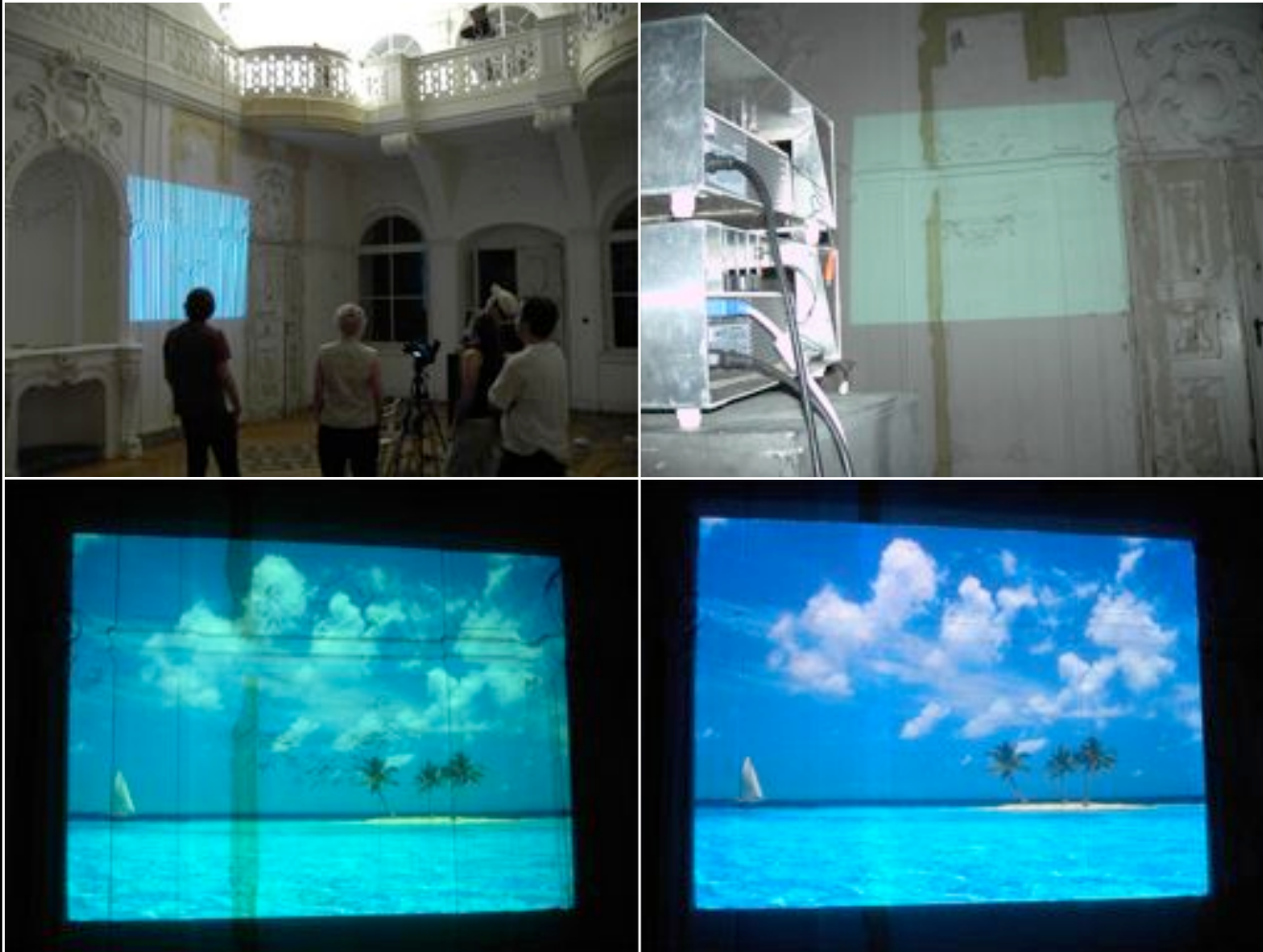
Pixel Displacement Mapping

- registering projections to such a surface by determining their intrinsic and extrinsic is too imprecise
 - non-linear lens distortion
 - errors in measuring fiducials
- rendering of 3D surface representation from perspective of projector might be too slow
 - high geometric complexity of model
 - many triangles to render
 - project, raster, texture
- measure per-pixel mapping between projector perspectives and target perspective (e.g., camera)
- render image from target perspective and map it (look-up) into perspective of projectors (e.g., pixel-shading)

problem:
works only for static target perspective!
(but image-based rendering approaches exist)



Example: Stucco Wall



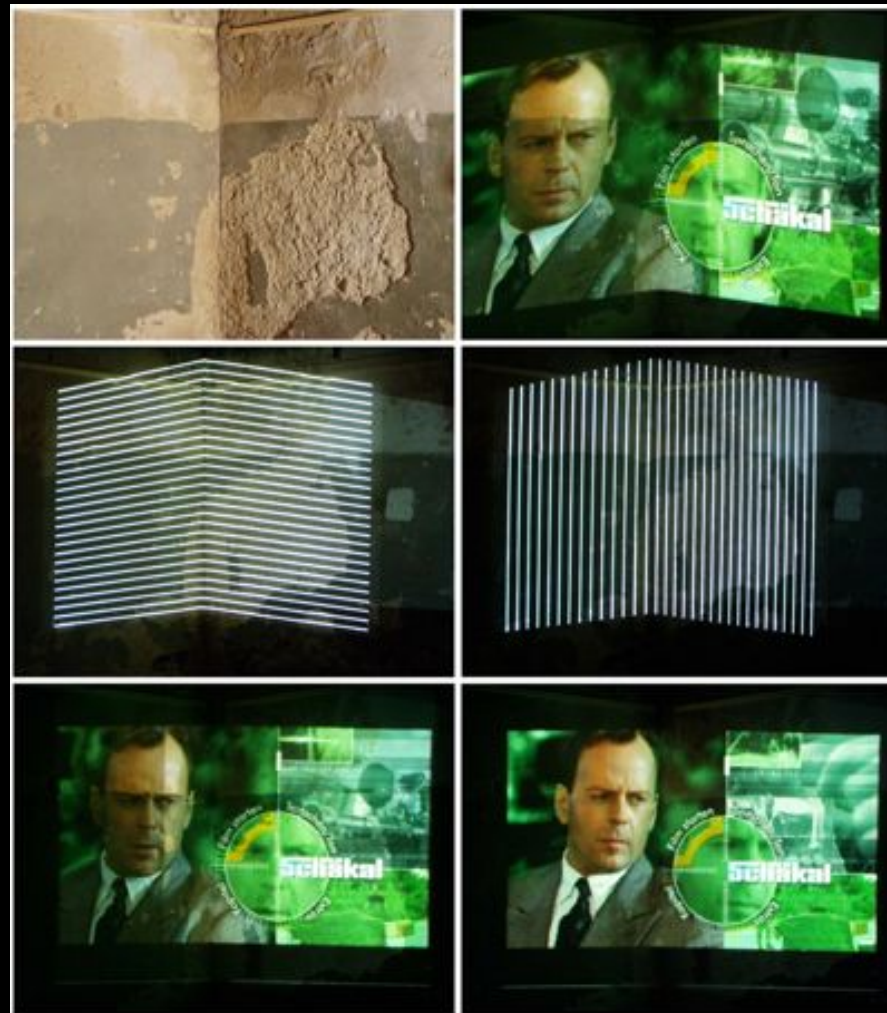
In coop. with
castle Etters-
burg

Example: Fossil Cast



In coop. with Senckenberg Museum

Example: Scruffy Room Corner



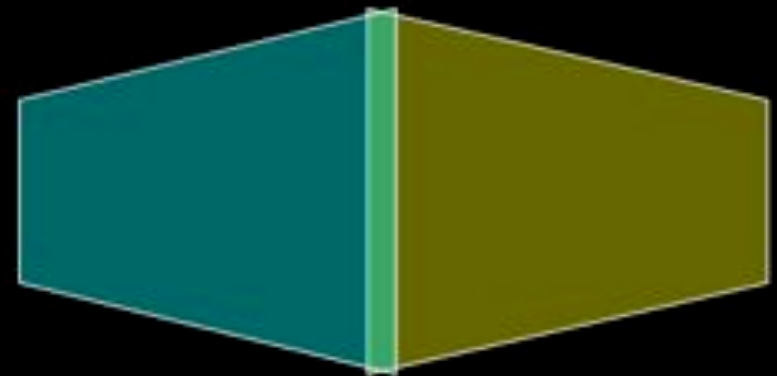
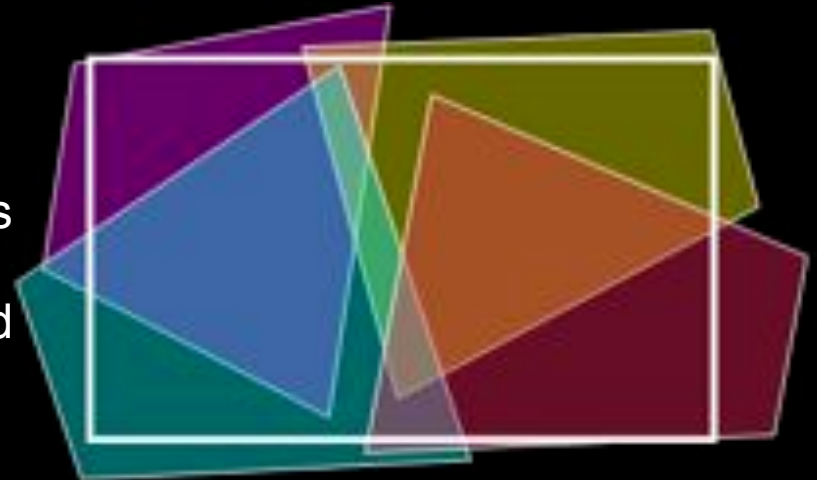
Bimber et al.,
IEEE Computer
2005



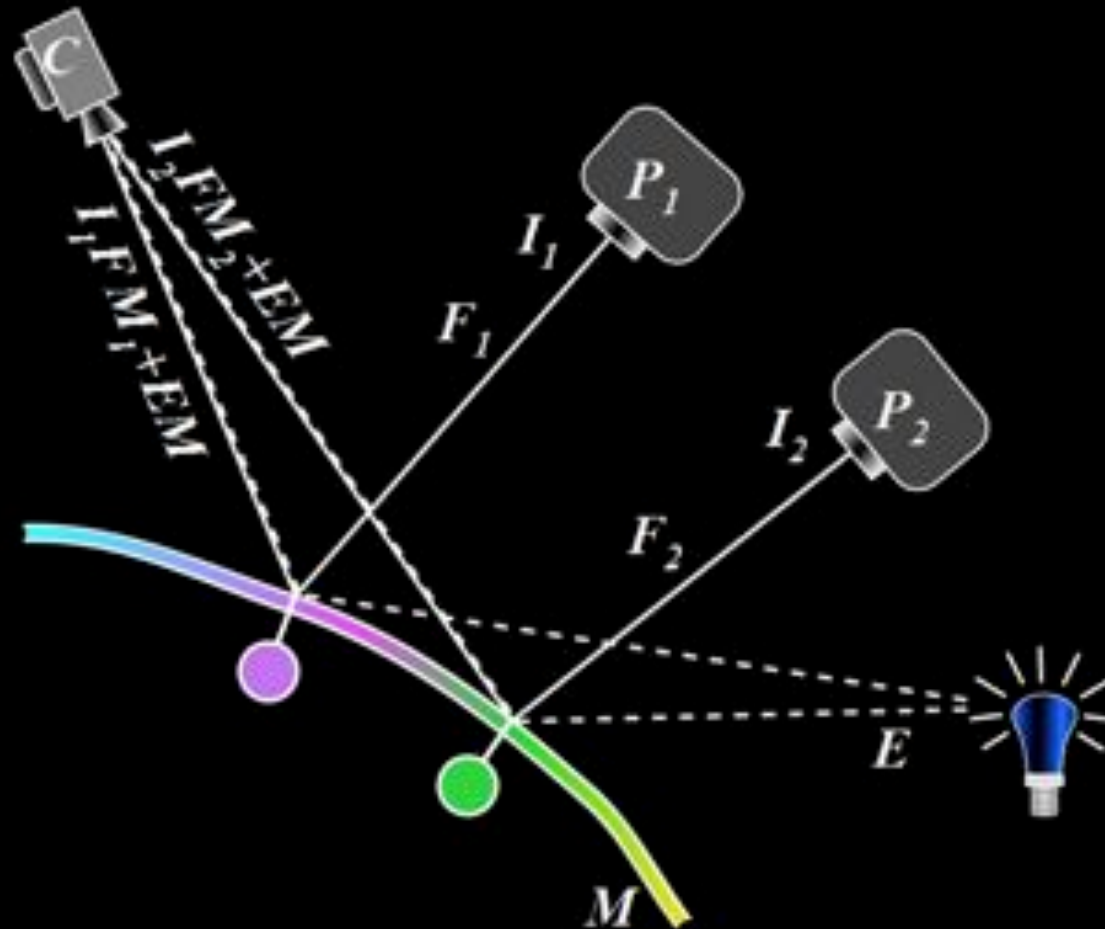
Radiometric Compensation

Photometric Calibration

- regions of display surfaces that are illuminated by multiple projectors simultaneously appear brighter
- projectors can have different brightness and can cover a different color space
- result: inconsistent image (intensity and color)
- humans can perceive 2% difference in brightness and a color variation of $2nm$
- variations in brightness is more critical than variation in color
- solutions: intensity blending and color space mapping
- **these techniques are not explained here!**
- **we assume that projectors and cameras are linearized and color mapped**



Compensating Local Light Effects





Single Projector

determining parameters (textures):

- (1) turn off environment light and project black flood image

$$I=0, E=0 \rightarrow \text{BFM}$$

- (2) turn on environment light and project black flood image

$$I=0, E=1 \rightarrow \text{EM (incl. BFM !)}$$

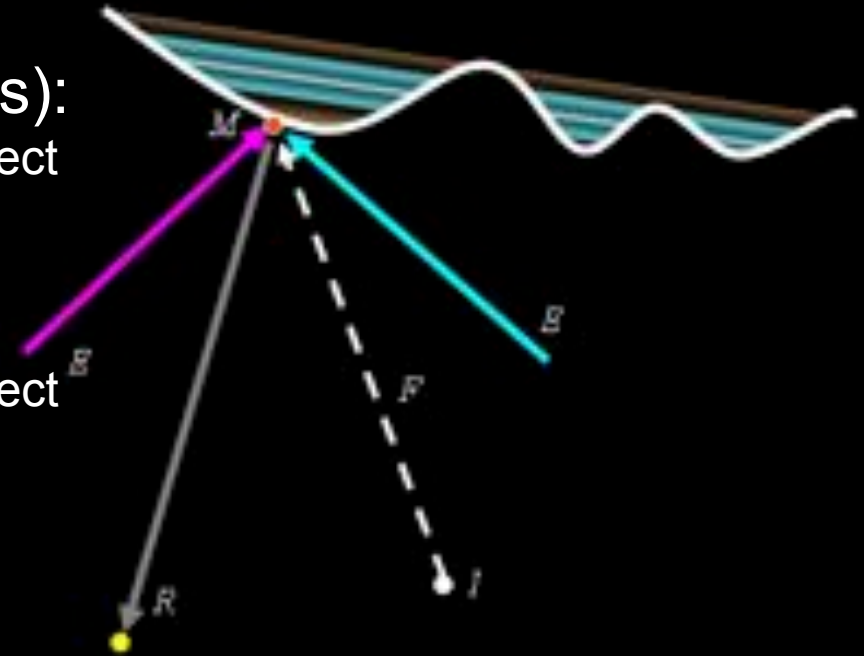
- (3) turn off environment light and project white flood image

$$I=1, E=0 \rightarrow \text{FM (incl. BFM !)}$$

$$\rightarrow \text{FM} = \text{FM} - \text{BFM}$$

compensation (per pixel):

$$I = (R - EM) / (FM)$$



$$R = IFM + EM$$

I → projected image

B → black-level

F → projector-2-surface form factor

E → environment light

M → surface reflectance (diffuse)

Color Mixing

determining color mixing matrix V :

for un-normalized matrix: (camera and projector response must be known and linearized):

capture 9+ images \rightarrow least squares

for normalized matrix (camera response must be known, projector response can be unknown):

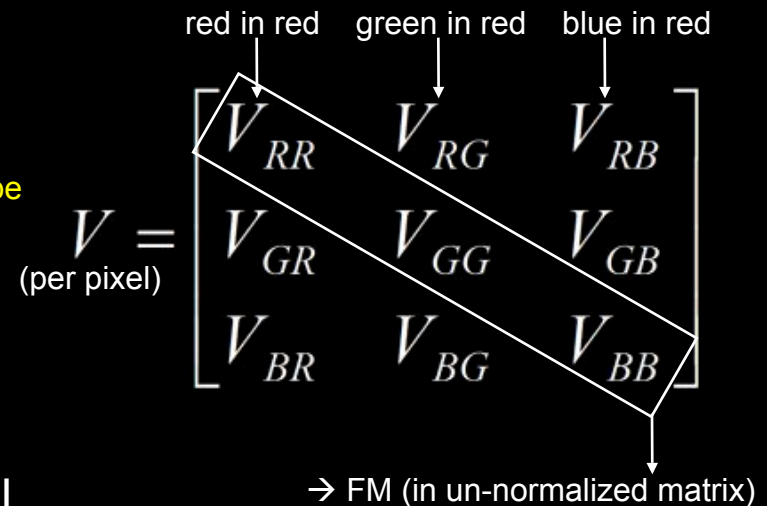
diagonals are 1 (unknown scaling)

off-diagonals are $V_{ij} = \Delta C_j / \Delta I_i = \Delta C_j / \Delta R_i$
(since $V_{ii} = 1$, $\Delta I_i = \Delta C_i$)

capture 6 images C (2 per color channel to determine deltas)

compensation (per pixel):

$I = V^{-1} * R$ (does not consider environment light!)



$$R = V * I$$

$I \rightarrow$ projected image

$V \rightarrow$ color mixing matrix
(projector/camera/reflectance)



Dynamic Adaptation

determining color mixing matrix V_0 :

similar as before: $V_{ij} = \Delta C_j / \Delta I_i$
(un-normalized!)

determine reflected environment

light $E_0 * M_0$ at $t=0$:

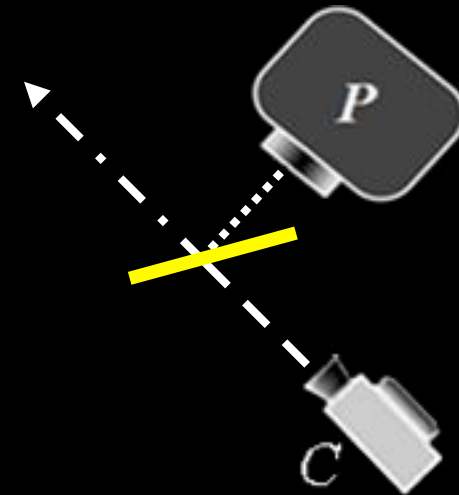
$E_0 * M_0 = C - V_0 * I$ (project
arbitrary I and capture C)

compensation (per pixel at t):

$$I_t = V_0^{-1} * (R * M_0 / M_{t-1} - E_{t-1} * M_0)$$

$$\rightarrow E_{t-1} * M_0 \text{ approx. } E_0 * M_0$$

$$\rightarrow M_0 / M_{t-1} = C_0 / C_{t-1}$$



$$R_t = M_t / M_0 * (E_t * M_0 + V_0 * I_t)$$

t → time index

I_t → projected image at t

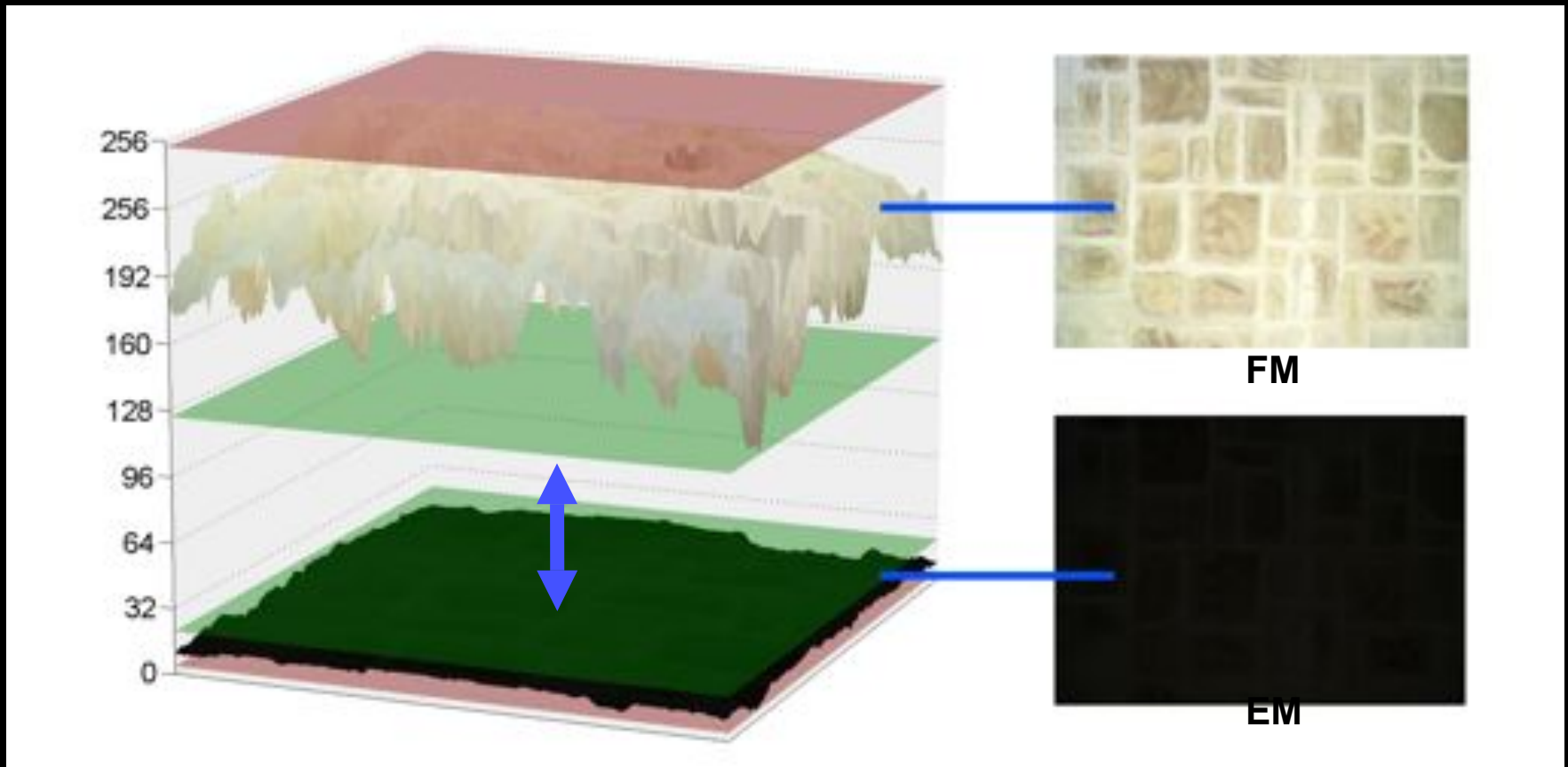
V_0 → un-normalized color mixing
matrix at $t=0$ (const.)

M_t → material at t

M_0 → material at $t=0$

E_t → environment light at $t=0$

Limited Dynamic Range and Brightness



Multiple Projectors

strategy: balance intensity load

- assume: total intensity is equally balanced among multiple low-capacity units

$$I_i = I_1 = I_2 = \dots = I_N$$

- this is equivalent to the assumption that a single high capacity projector produces the total intensity arriving on the surface virtually

$$R = EM + I_1(F_1M + F_2M + \dots + F_NM)$$

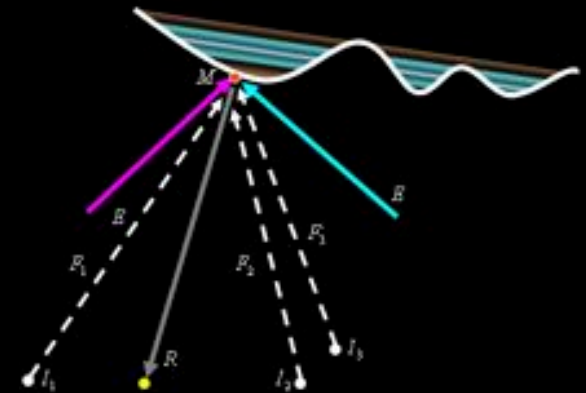
$$\rightarrow EM + I(F_1 + F_2 + \dots + F_N)M$$

compensation (per pixel):

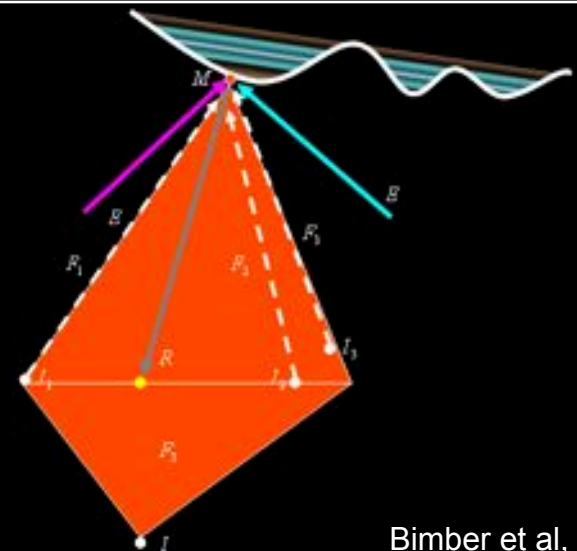
$$I_i = (R - EM) / (F_1M + F_2M + \dots + F_NM)$$

remember: $F_iM = F_iM - B_iF_iM$!

$$\text{or } BFM = B_1F_1M + \dots + B_iF_iM$$



$$R = EM + I_1F_1M + I_2F_2M + \dots + I_NF_NM$$



Bimber et al, IEEE Computer 2005

Considering Human Visual Perception

threshold map (Ramasubramanian et al. Siggraph'99)

- computes for every pixel of an image R the amount of luminance difference that is imperceptible
- considers *contrast*, *luminance* and *spatial frequency* in local neighborhood



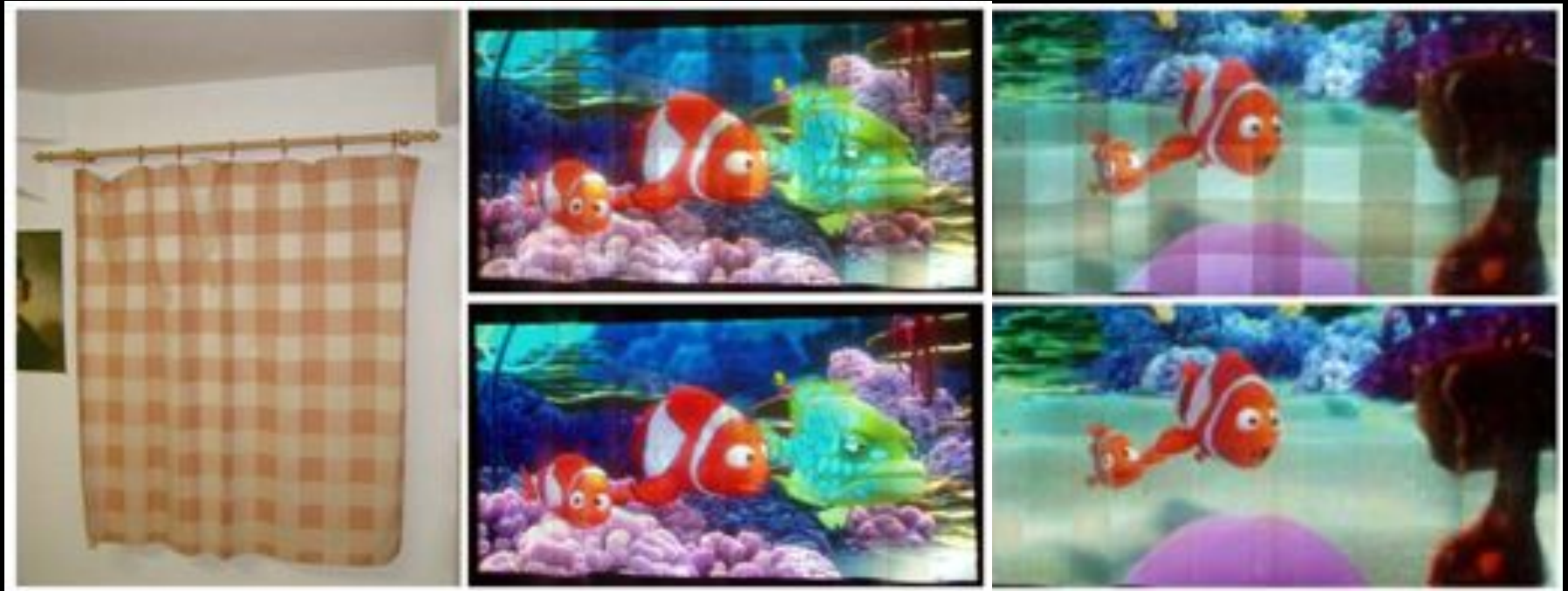
adaptation of un-compensated (original) image R :

- compute and apply a single (global) scaling factor $R' = R * \alpha$ that minimizes the perceived error (Wang, et al. 2005, only monochrome, not real-time, single projector)
- coming soon: color, real-time, global and local adaptation, potentially multiple projectors





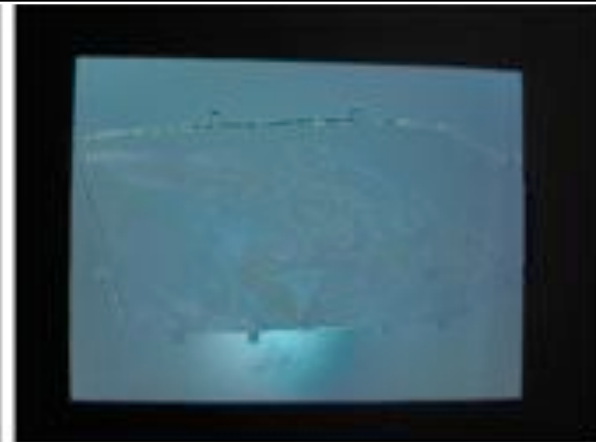
Example: Curtain



Bimber et al, IEEE
Computer 2005

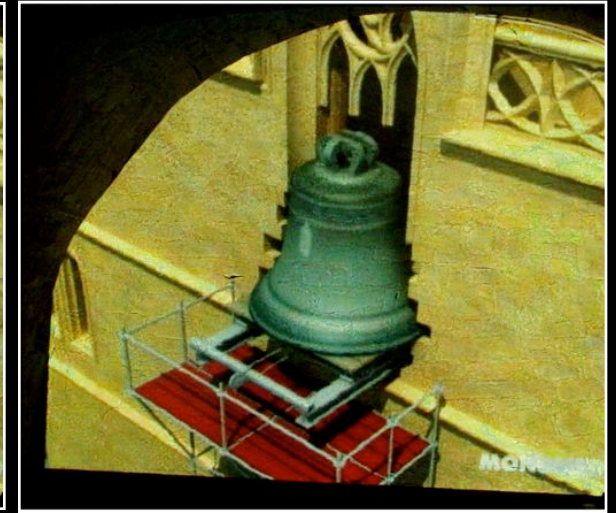


Example: Fossil



In coop. with Senckenberg Museum

Example: Natural Stone Wall



Bimber et al, IEEE Computer 2005
In coop. with Bennert Group

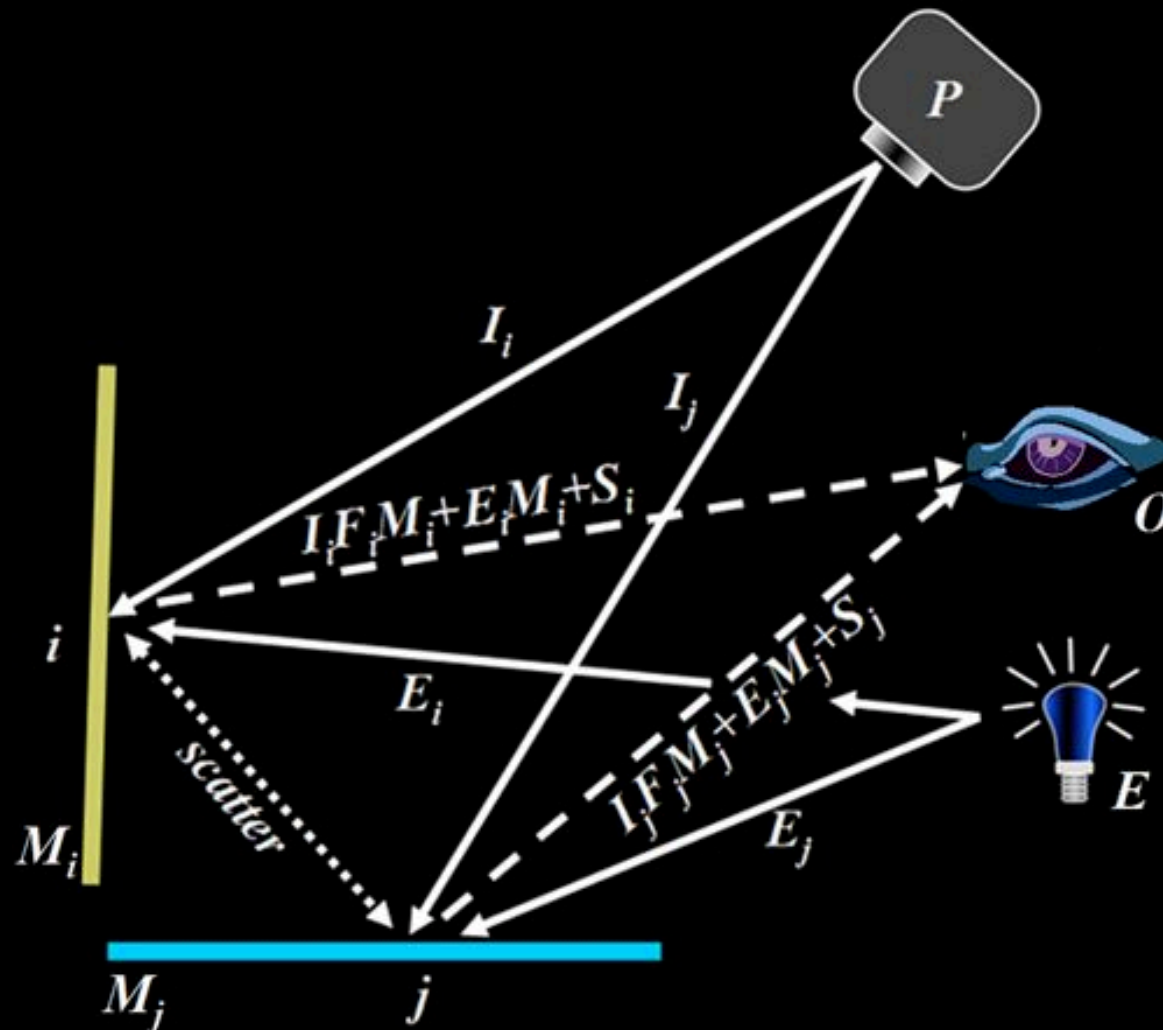


Example: Wallpaper



Bimber et al,
IEEE Computer
2005

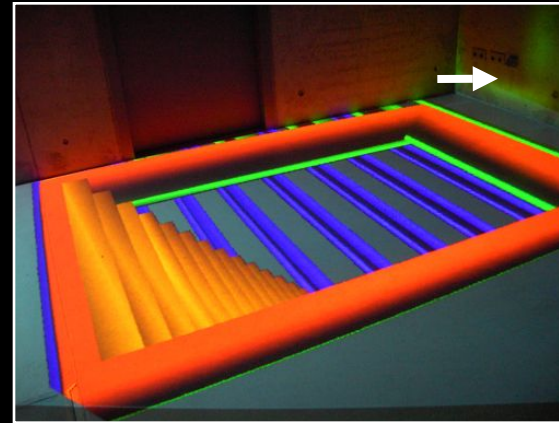
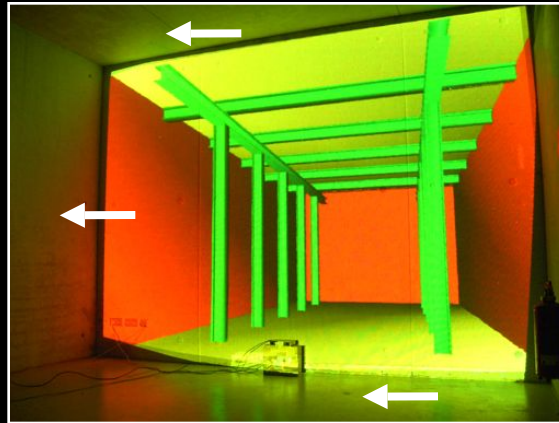
Compensating Global Light Effects



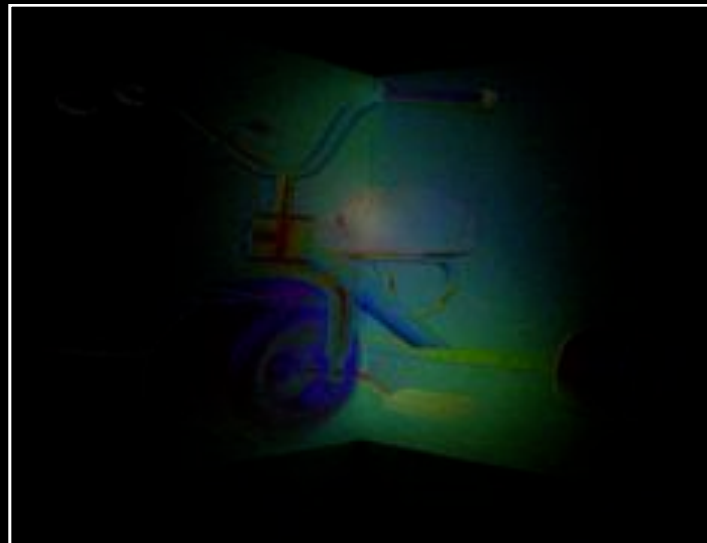


Compensating Diffuse Scattering

Bimber et al,
IEEE/ACM
ISMAR 2005



Bimber et al,
IEEE VR, 2006



details:
IEEE VR talk on
Wednesday morning
(8:30am), session on
tracking and projection
displays

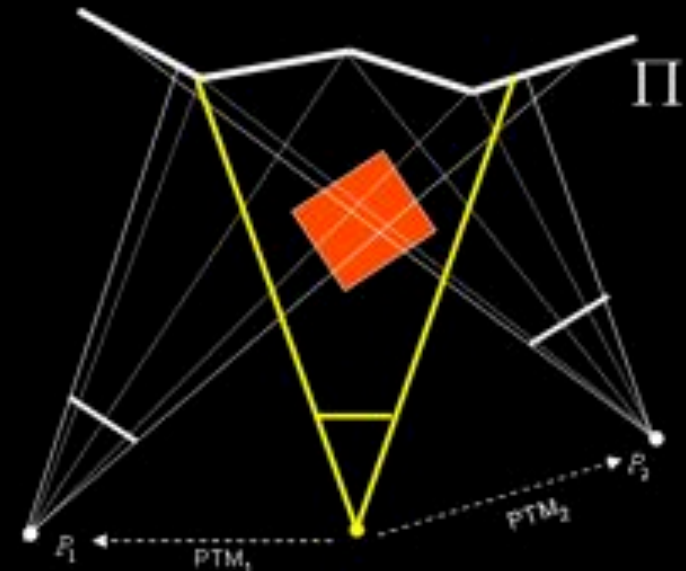
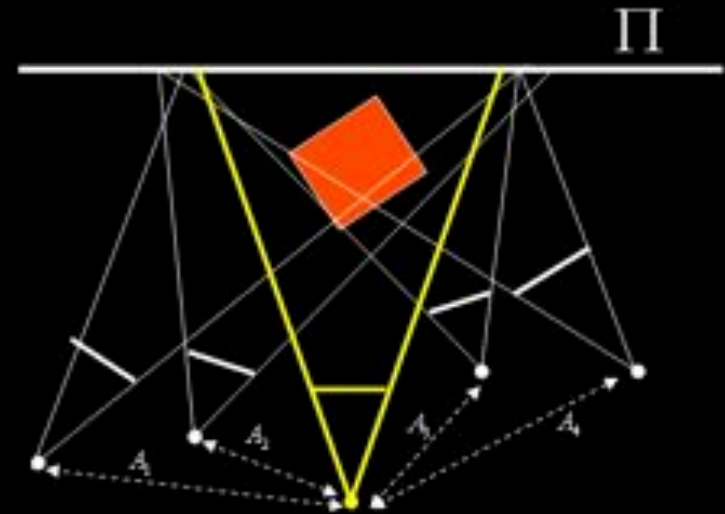
see demo!



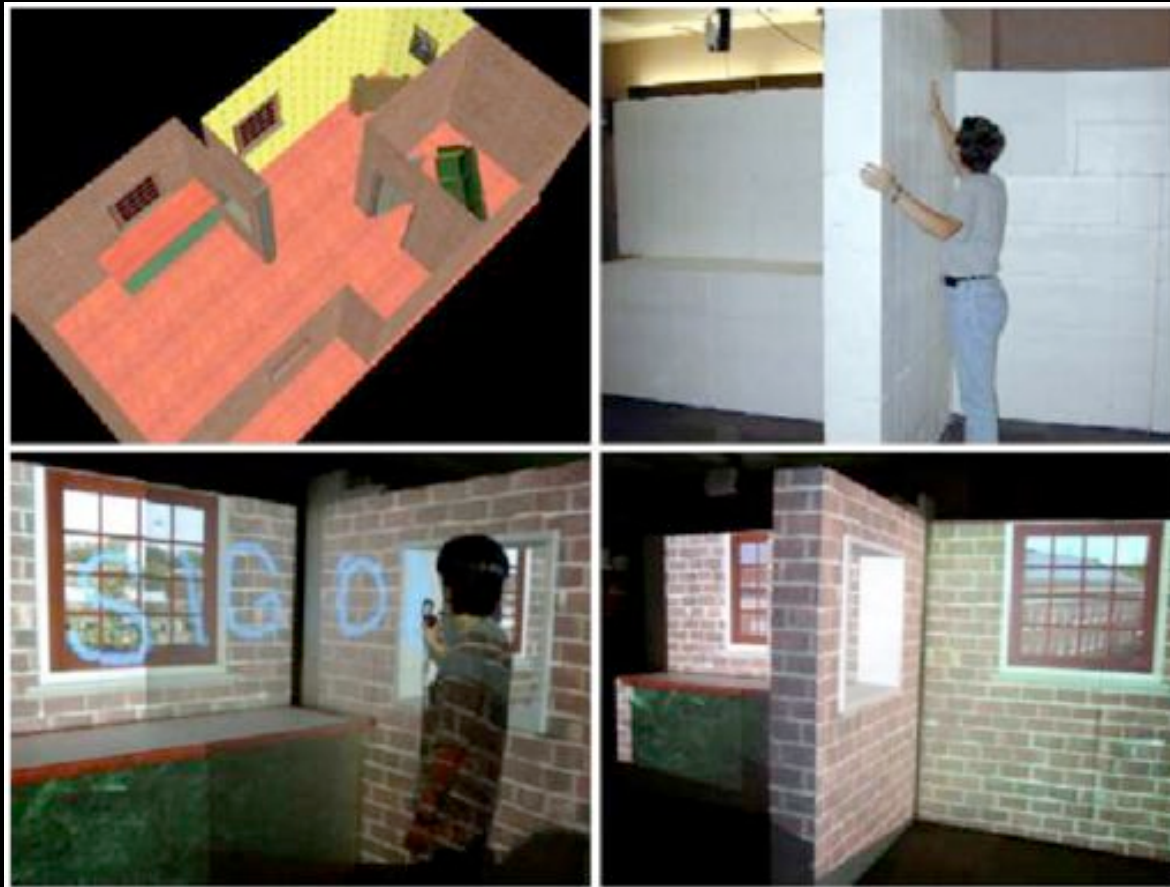
Advanced Techniques **View-Dependence**

Non-Complex Surfaces

- view-dependent geometry correction can be compute if geometry is known
- for example:
 - **planar/multi-plane:** off-axis projection
 - **parametric:** warping via parametric description
 - **scanned/modelled:** projective texture mapping



Example: Life-Sized Projector-Based Dioramas



Courtesy: Low, et al., 2001

Complex Surfaces

- if geometry is unknown image-based rendering helps
- sample geometric and radiometric parameters from multiple (source) camera (perspective)
- for novel (destination) camera

- compute weighted penalties:

$$p_j = \alpha a_j + (1 - \alpha) b_j$$

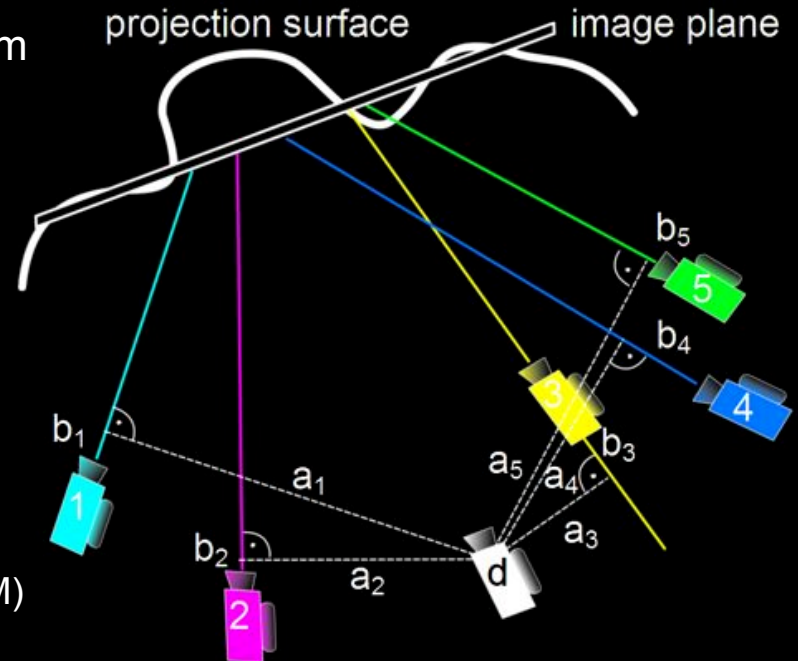
- select k best perspectives (lowest penalties) and normalize them:

$$w_j = \left(1 - \frac{p_j}{\max_{p^k} p_j} \right) \frac{1}{p_j}$$

- interpolate new parameter textures (P_i2C_j , $F_{ij}M$, $E_{ij}M$) and direction vector for destination perspective to render new IP:

$$t_d = \sum_j^k w_j t_j$$

- lookups in $F_{ij}M$, $E_{ij}M$, ... interpolated P_i2C_j
- lookups in IP with interpolated P_i2C_j



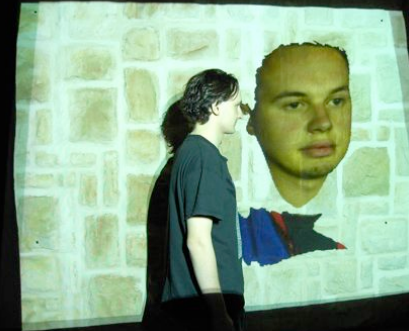
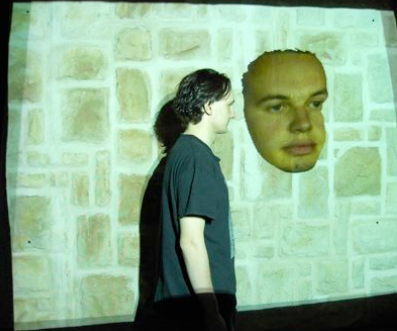
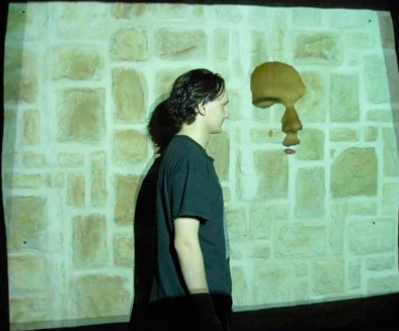
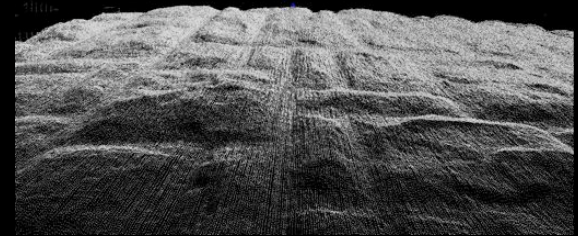
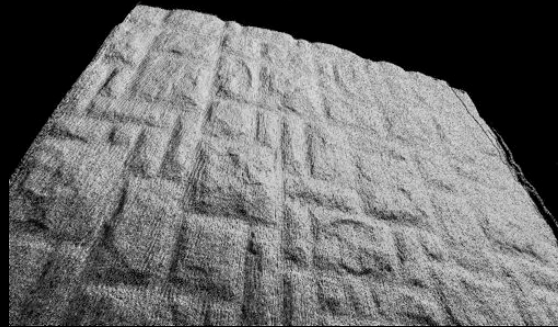
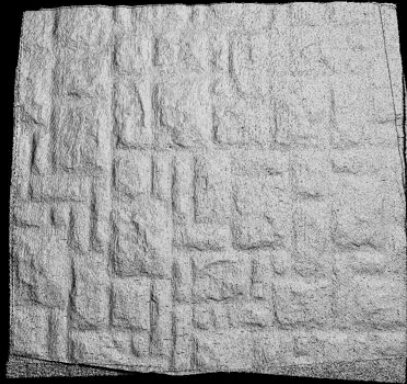


Example: Tracking and Stereo

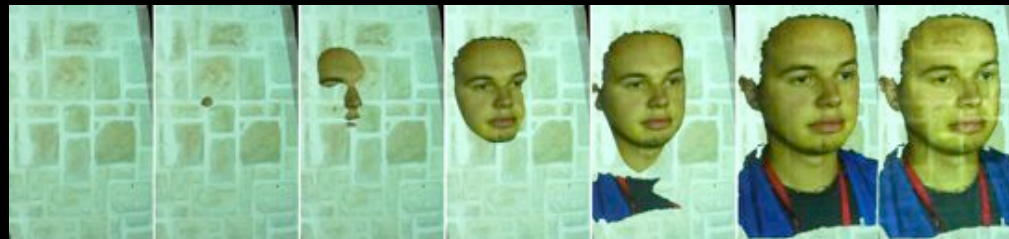


Bimber et al, IEEE/ACM ISMAR 2005

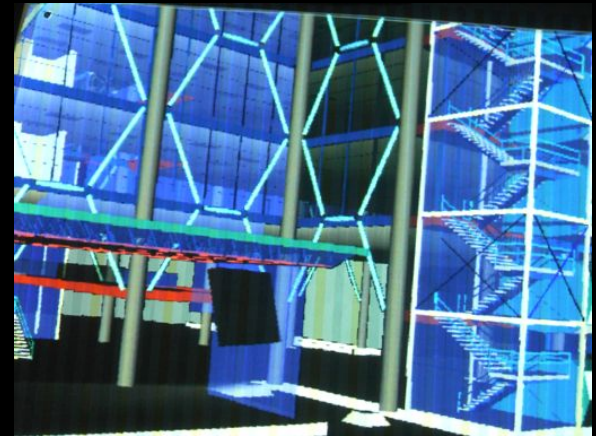
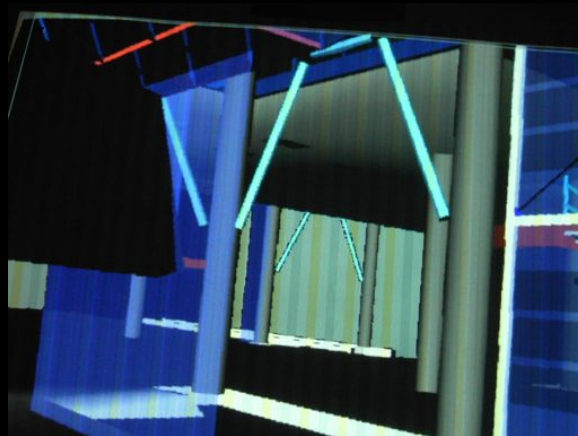
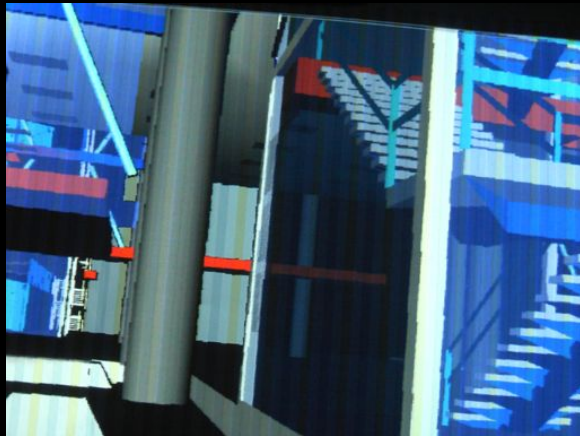
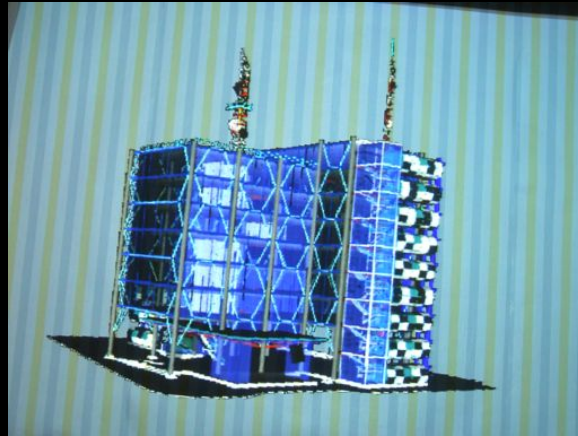
Depth and Occlusion



Bimber et al,
IEEE/ACM
ISMAR 2005



Example: Stereo on Wallpaper



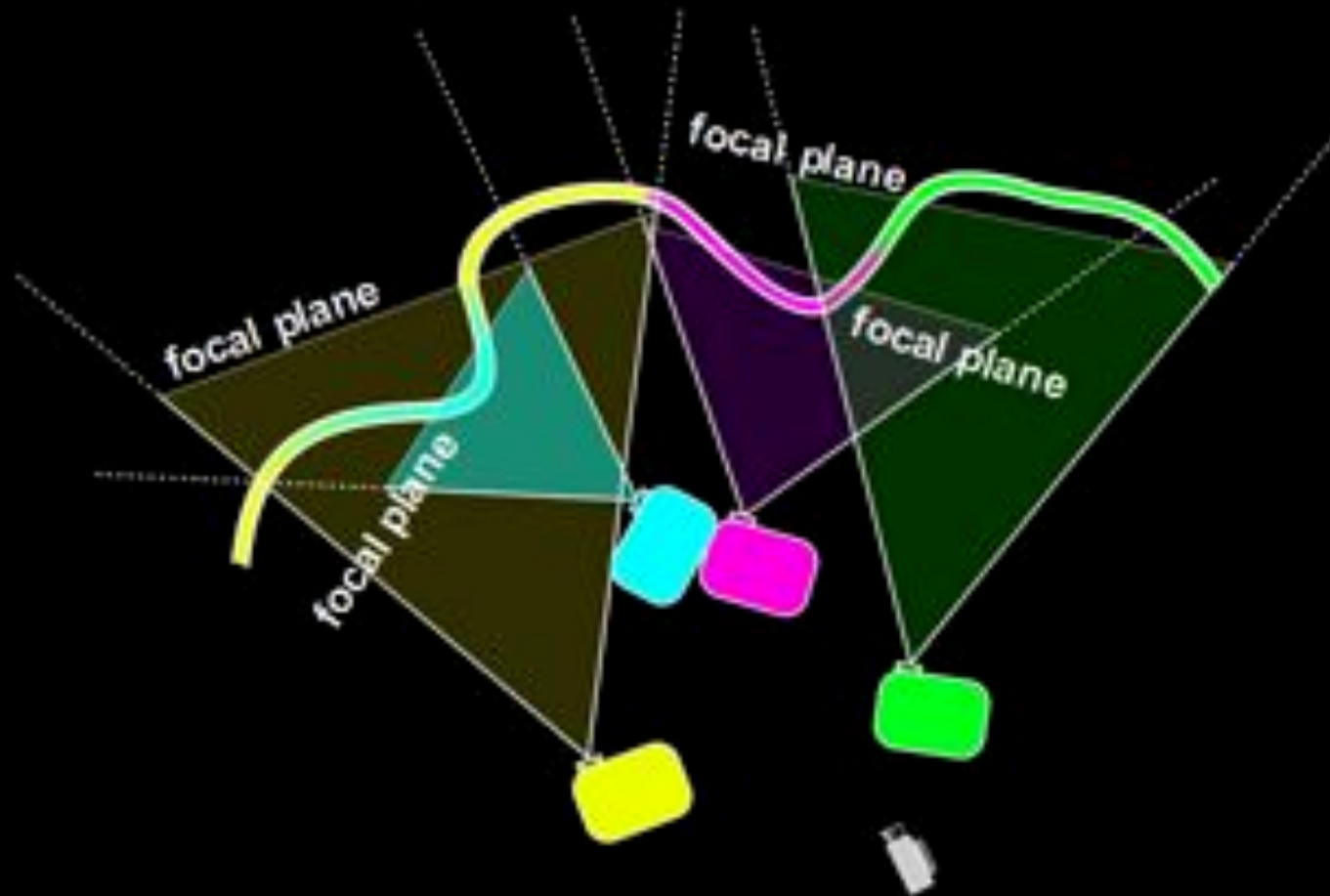
Bimber et al, IEEE/ACM
ISMAR 2005



Advanced Techniques

Multi-Focal Projection

Multi-Projector-Camera Technique that Increases Focal Depth



Determining Defocus

- structured light projection of grid point samples (2- dimensional phase shift)
 - pre-correction:** geometric and radiometric correction (corrected grid points must be observed in camera)

$$I_{x,y} = (R_{x,y} - EM_{x,y}) / FM_{x,y}$$

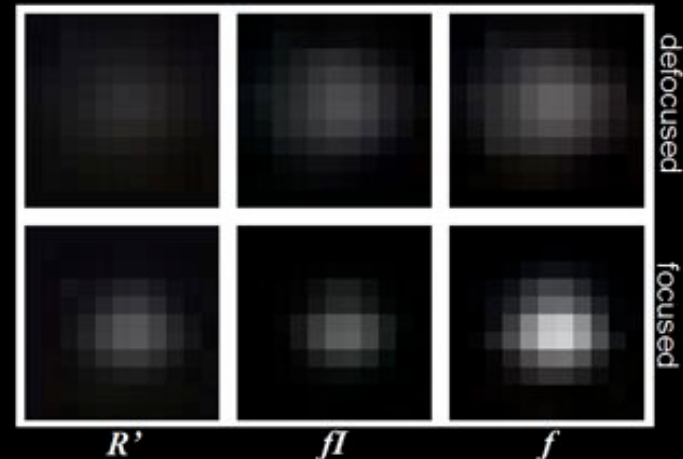
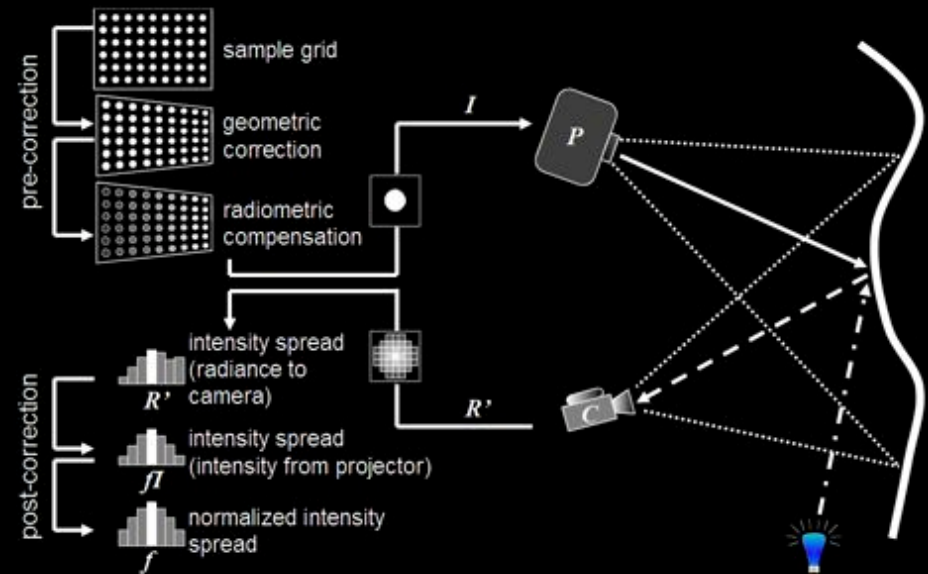
- post-correction:**

$$R'_{x',y'} = fI_{x,y} FM_{x',y'} + EM_{x',y'}$$

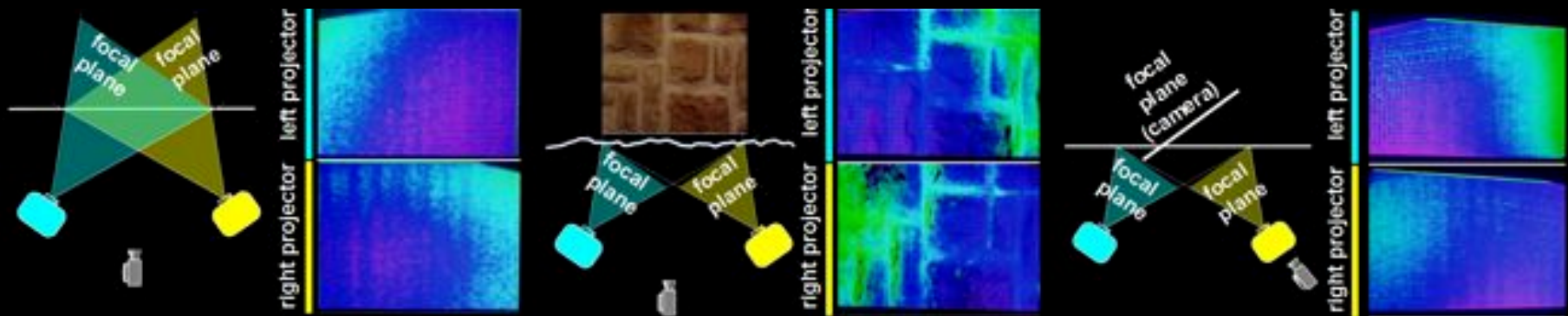
$$fI_{x,y} = \frac{(R'_{x',y'} - EM_{x',y'})}{FM_{x',y'}}$$

$$f = \frac{fI_{x,y}}{I_{x,y}}$$

- the normalized intensity spread texture f serves as basis to estimate focus measures (e.g., via FFT/DCT, intensity loss, point spread, etc.)

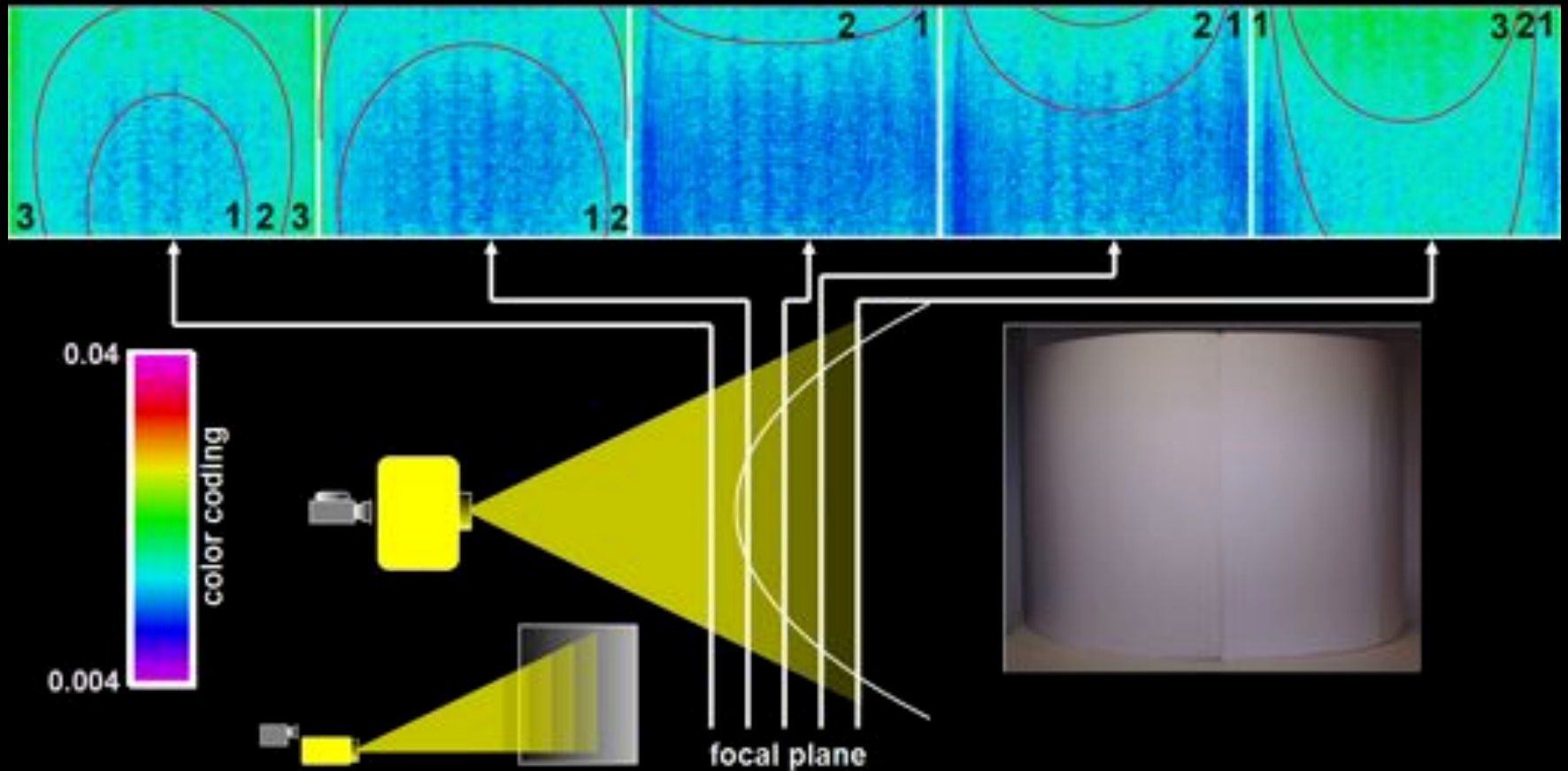


Example: Different Configurations



Bimber et al, IEEE TVCG 2006

Example: Shifting Focal Plane



Bimber et al, IEEE TVCG 2006

Image Composition

- using the focus values of each projector's pixels ($\Phi_{i,x,y}$), compose an image with minimal total defocus
 - exclusive composition:** surface point is covered by a single projector pixel (the one with highest $\Phi_{i,x,y}$)

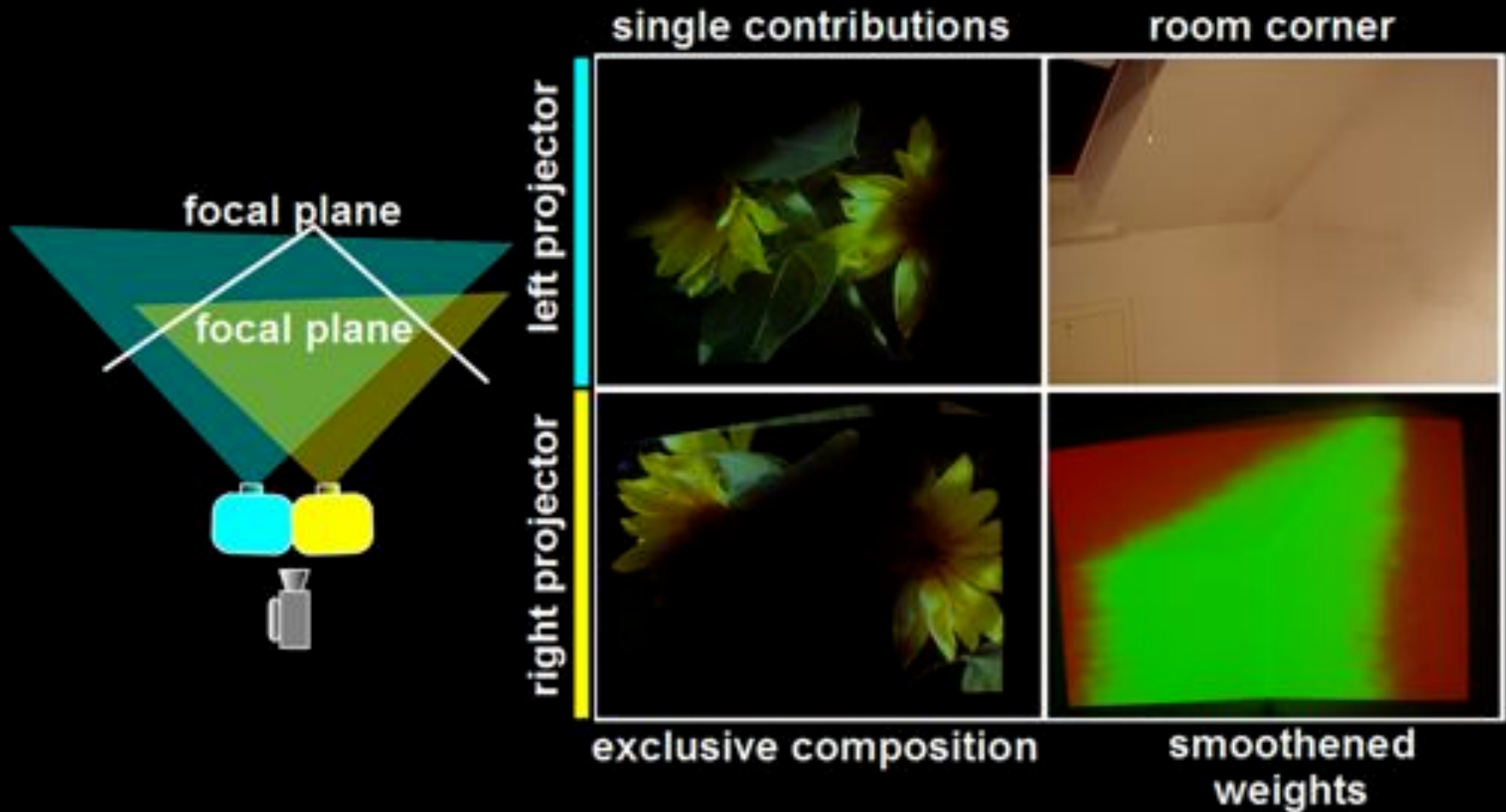


$$I_i = \frac{w_i(R - EM)}{\sum_j^N w_j FM_j} \quad w_{i,x,y} = \frac{\Phi_{i,x,y}}{\sum_j^N \Phi_{j,x,y}}$$

- weighted composition:** compute normalized weight and multiply it with FM and I

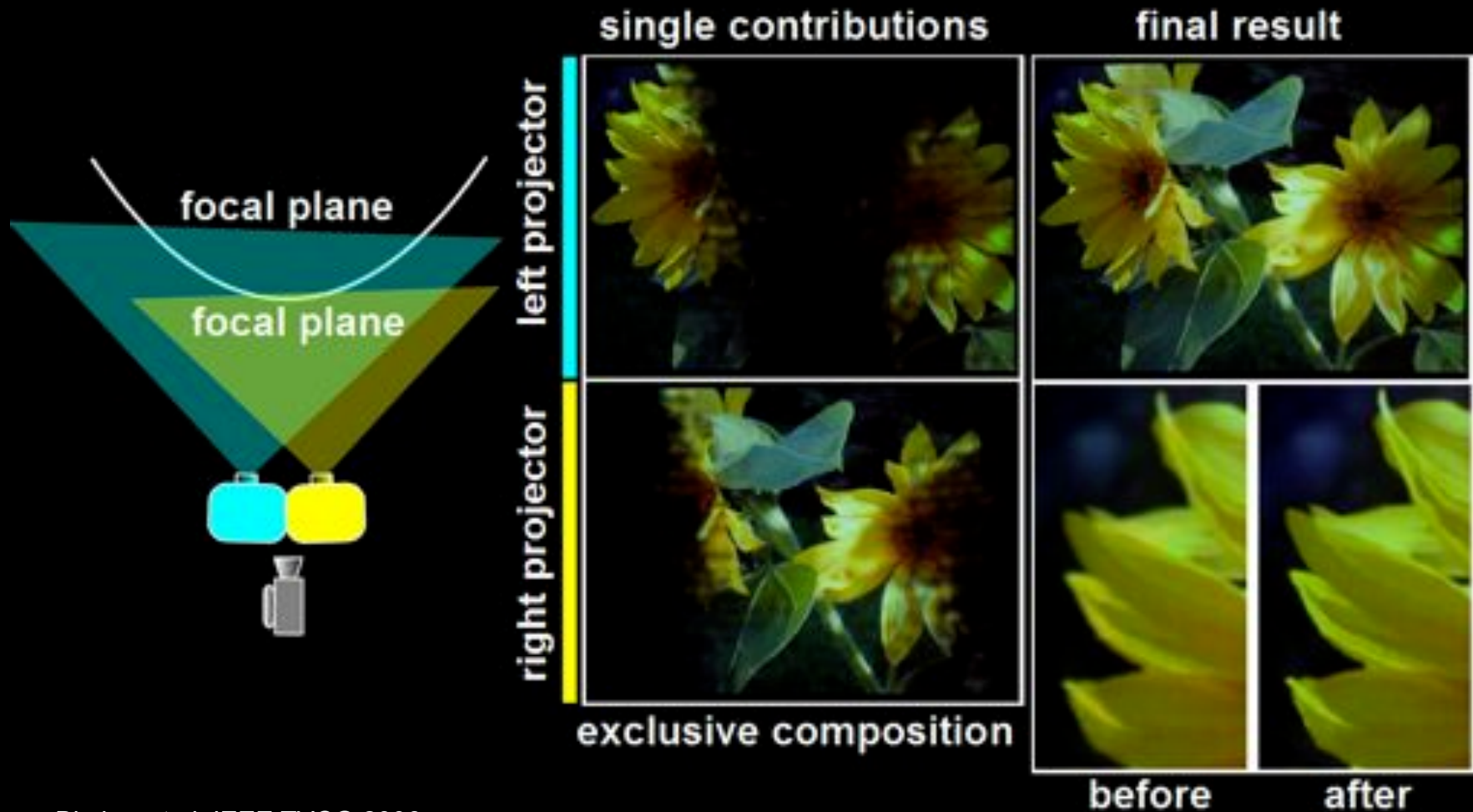
$$I_i = w_i(R - EM) / FM_i, \quad w_i = \begin{cases} 1 & \Phi_{i,x,y} \geq \Phi_{j,x,y} \\ 0 & \text{else} \end{cases}$$

Example: Room Corner



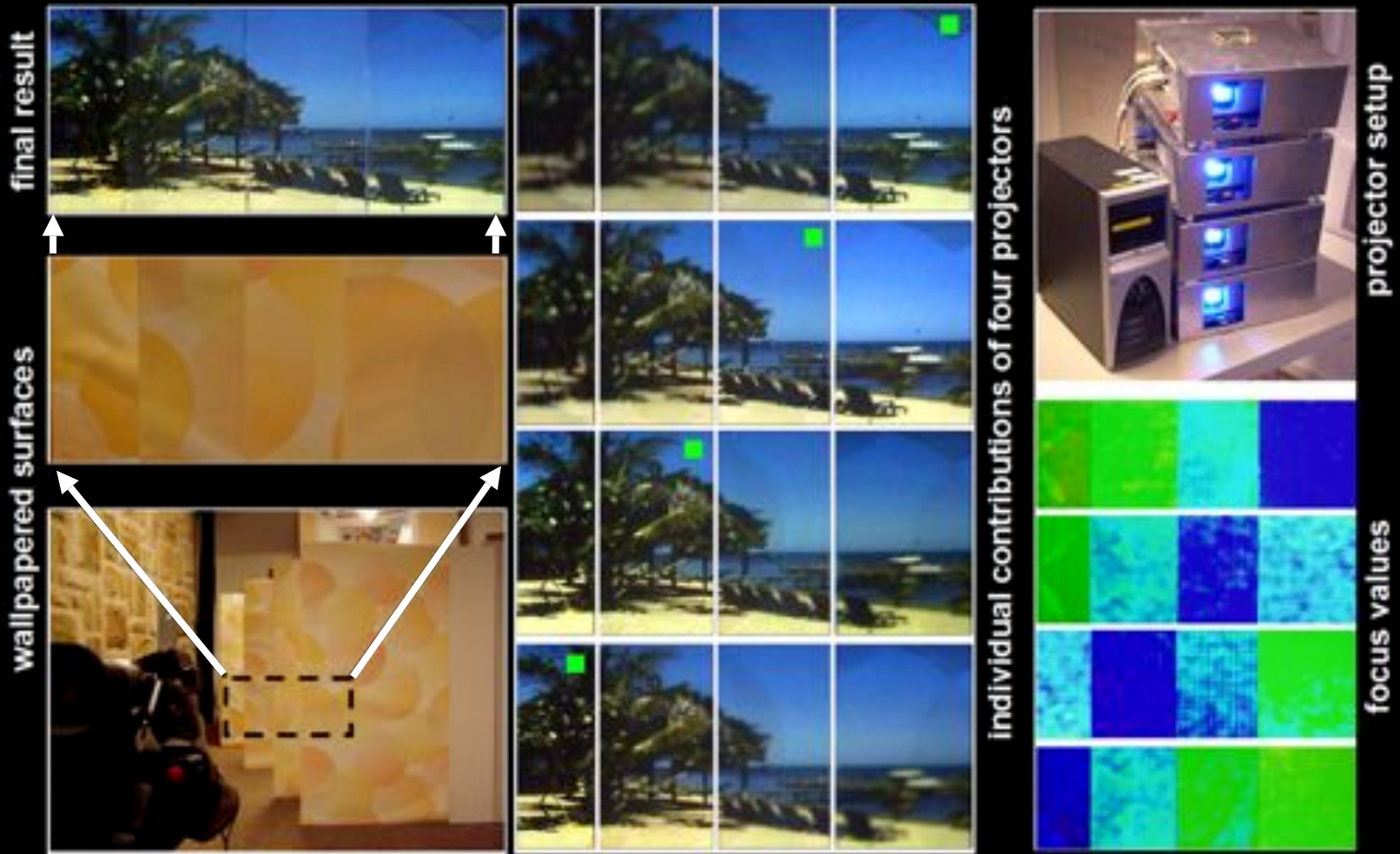
Bimber et al, IEEE TVCG 2006

Example: Cylindrical Surface



Bimber et al, IEEE TVCG 2006

Example: Large Focal Depth



Bimber et al, IEEE TVCG 2006

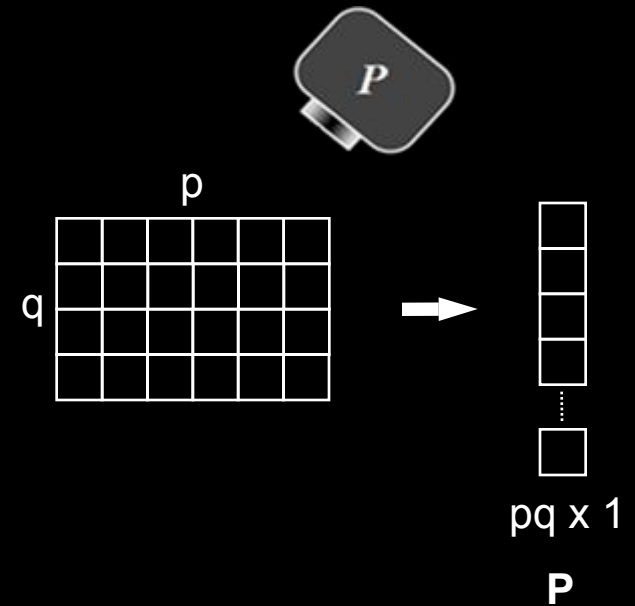
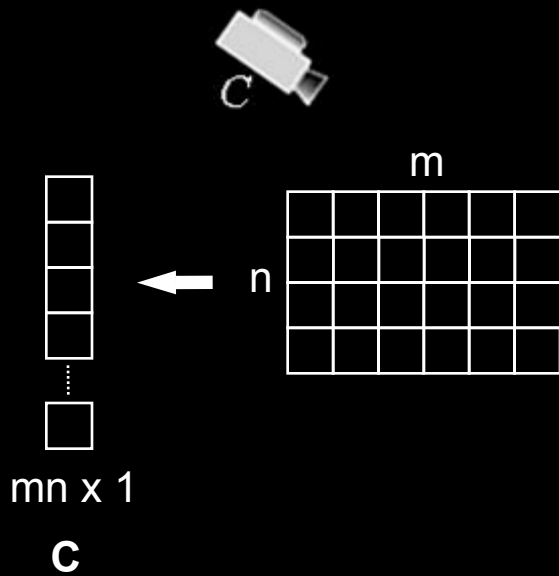


Advanced Techniques

Light Transport

Acquisition

single camera & projector capture 4D slice of 8D reflectance field



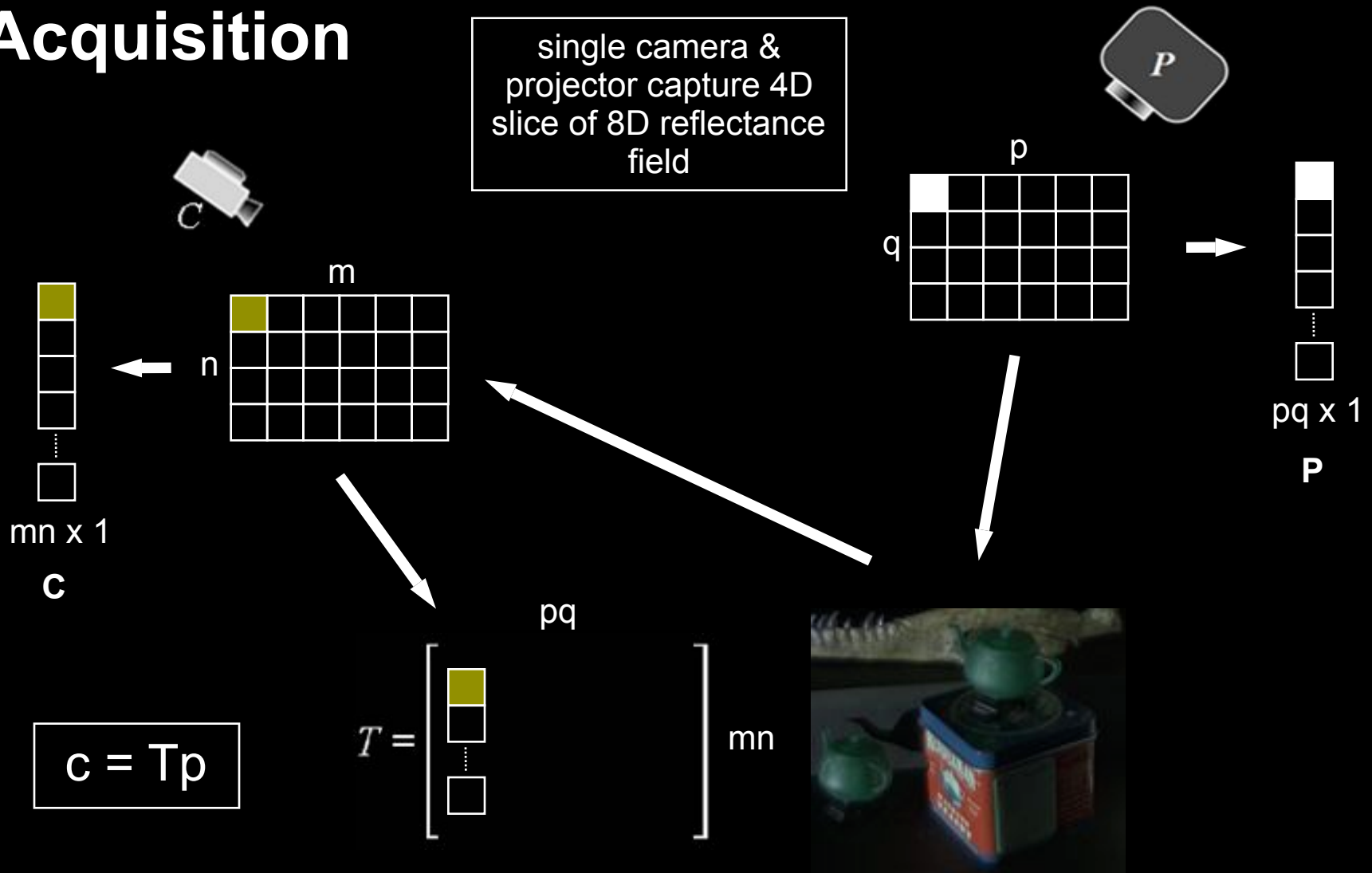
$$c = Tp$$

$$T = \begin{bmatrix} & pq \\ & \\ & \\ mn \end{bmatrix}$$



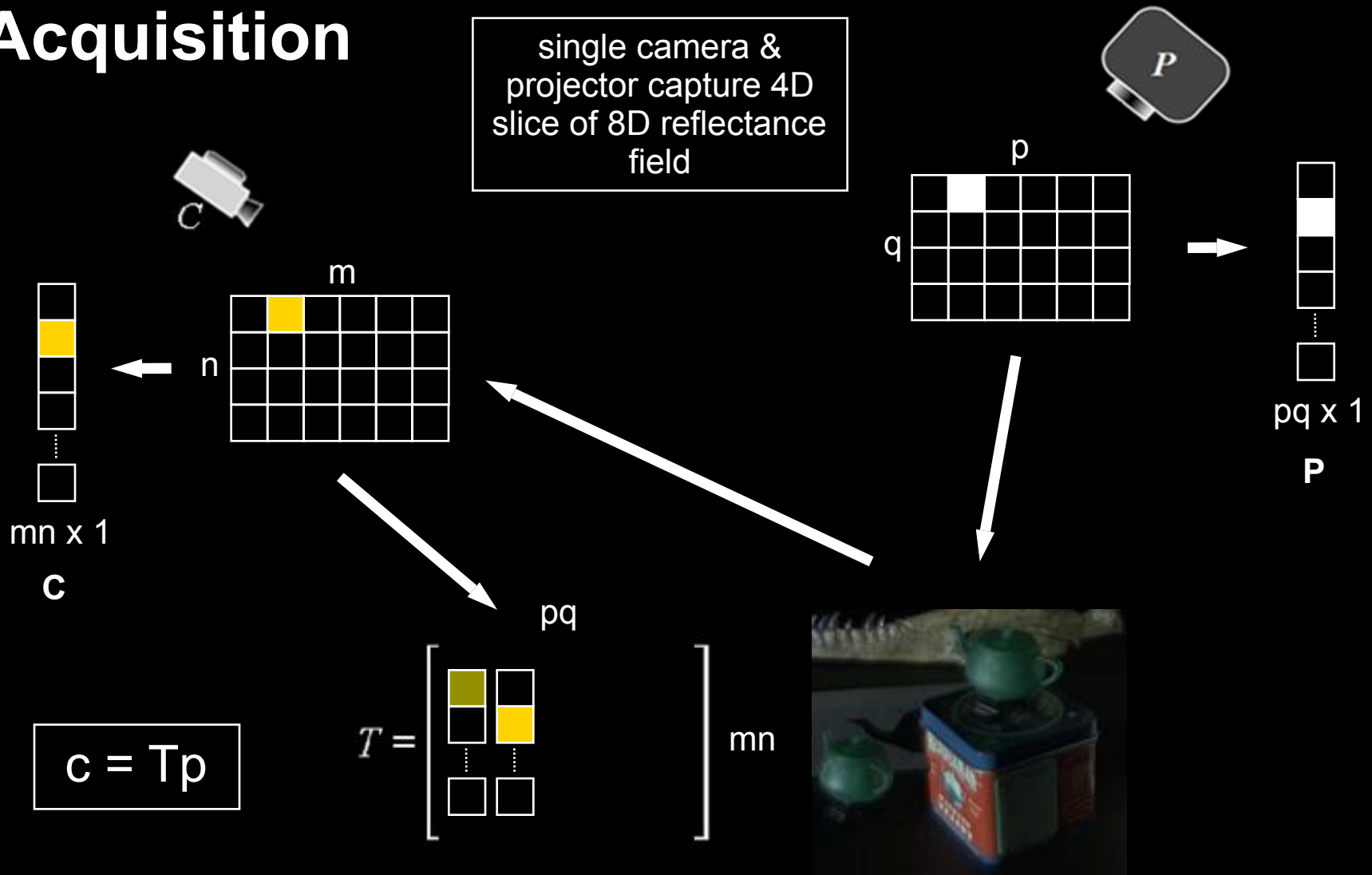
Acquisition

single camera & projector capture 4D slice of 8D reflectance field



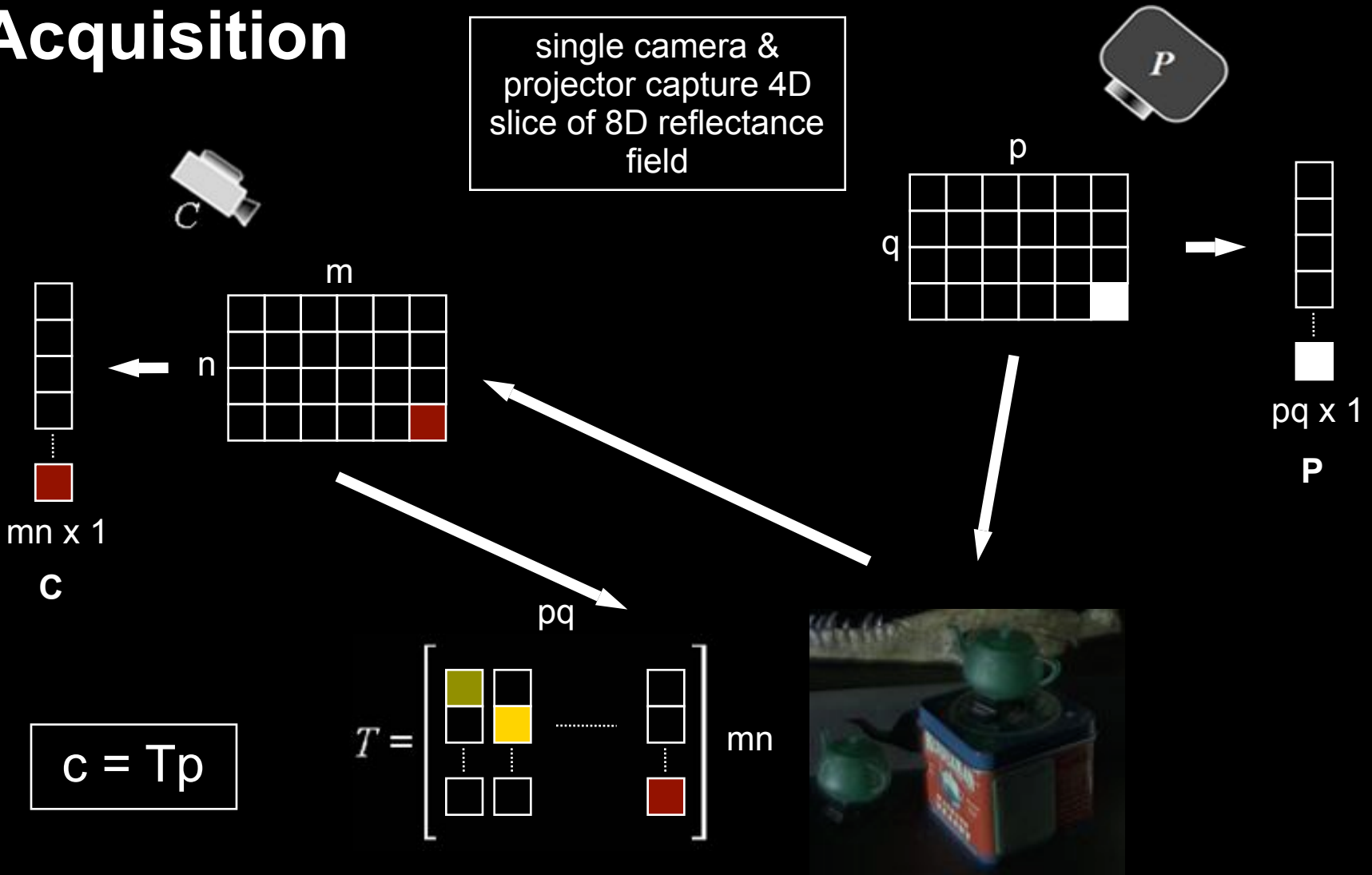
Acquisition

single camera & projector capture 4D slice of 8D reflectance field



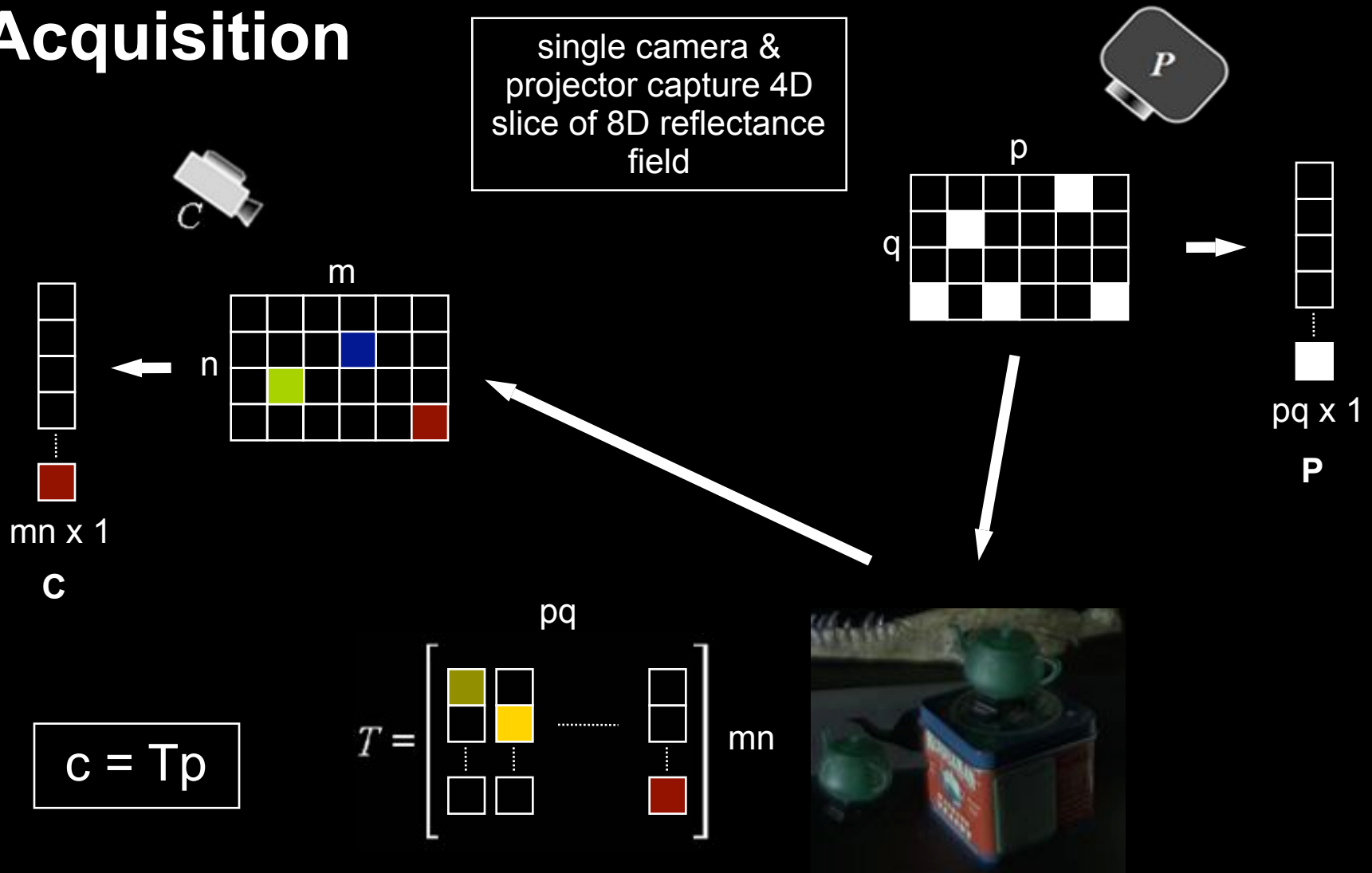
Acquisition

single camera & projector capture 4D slice of 8D reflectance field

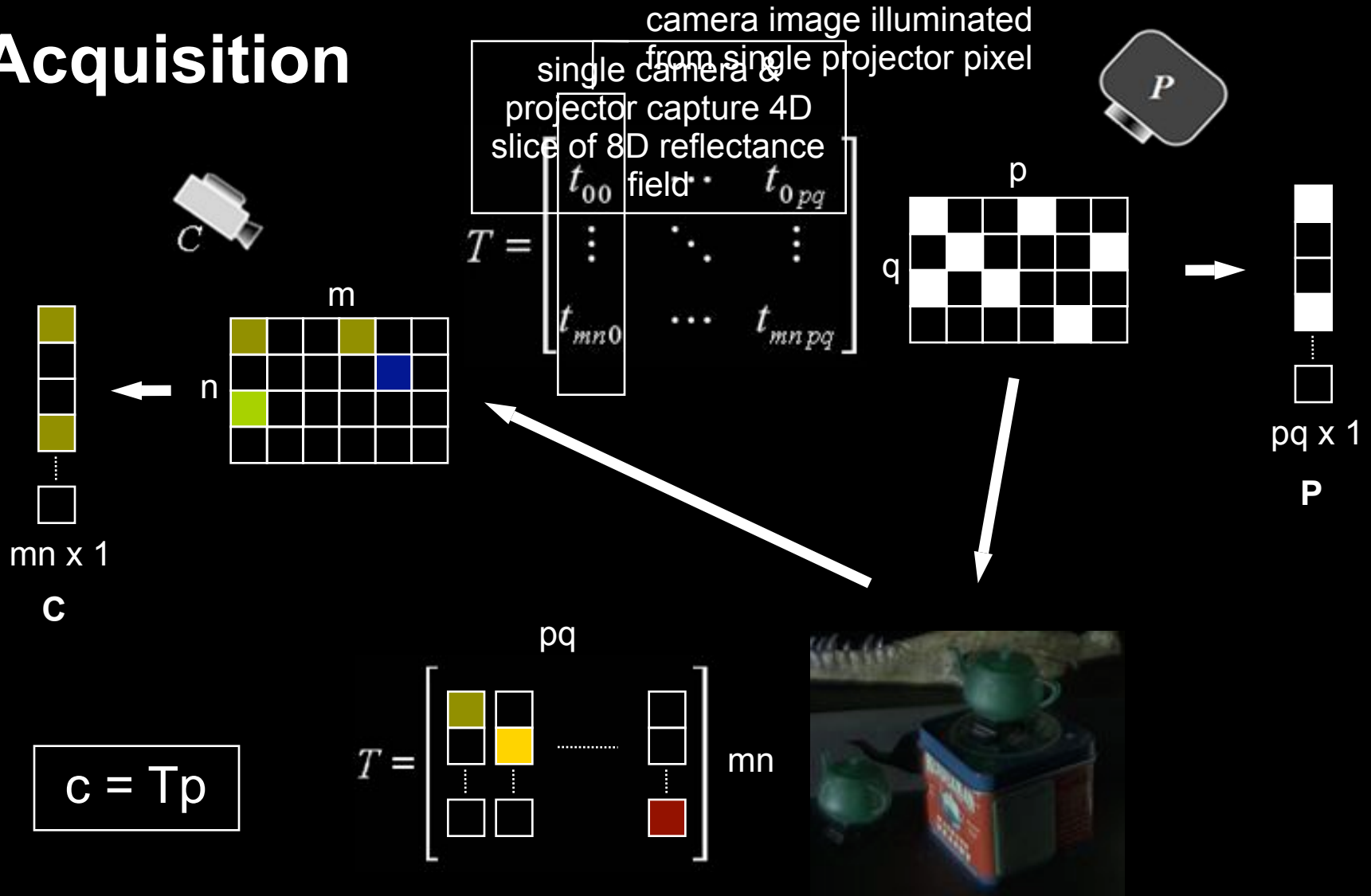


Acquisition

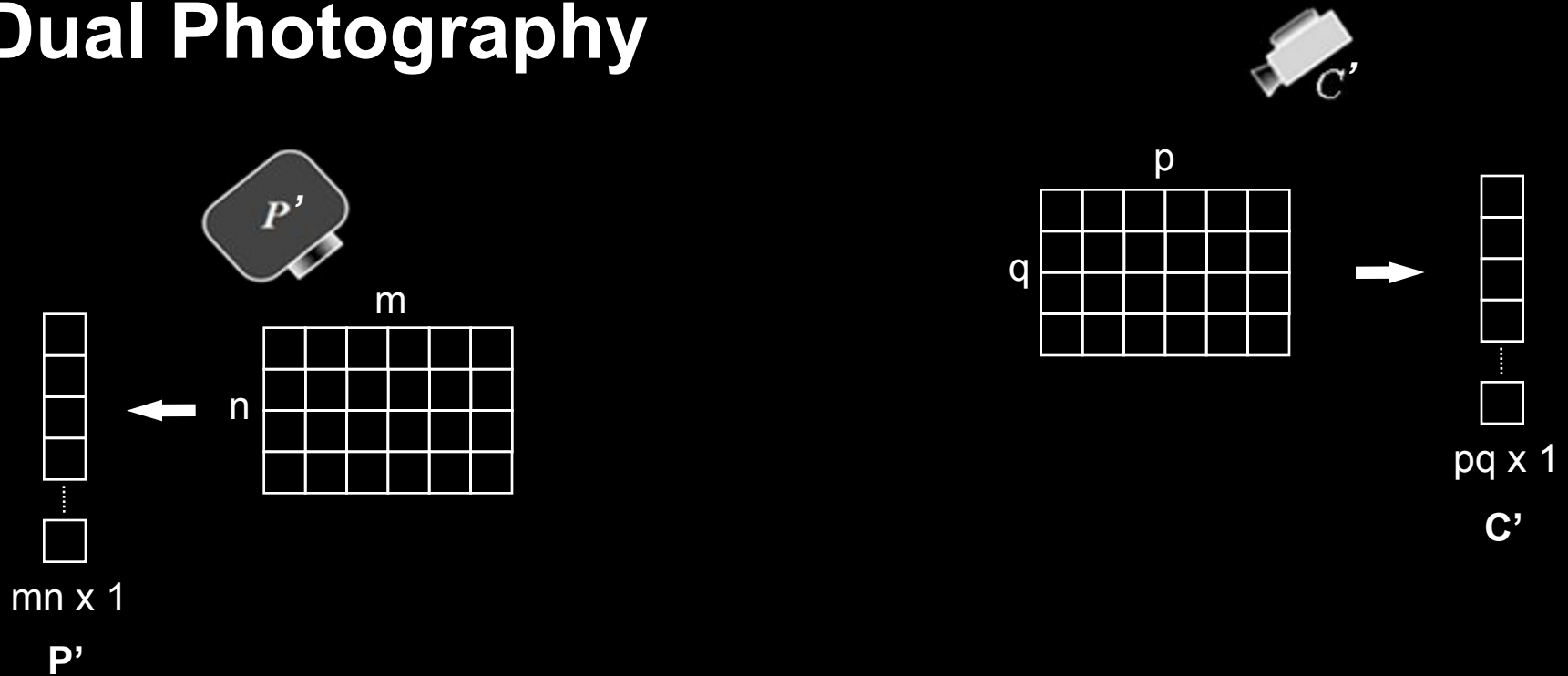
single camera & projector capture 4D slice of 8D reflectance field



Acquisition



Dual Photography



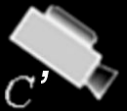
$$c = Tp$$

$$c' = T^T p'$$

$$T = \begin{bmatrix} \text{green} & \text{yellow} & \dots & \text{white} \\ \text{black} & \text{yellow} & \dots & \text{white} \\ \vdots & \vdots & \ddots & \vdots \\ \text{white} & \text{white} & \dots & \text{red} \end{bmatrix} \quad mn$$

$$T^T = \begin{bmatrix} \text{green} & \text{white} & \dots & \text{white} \\ \text{black} & \text{yellow} & \dots & \text{white} \\ \vdots & \vdots & \ddots & \vdots \\ \text{white} & \text{white} & \dots & \text{red} \end{bmatrix} \quad pq$$

Dual Photography



dual image



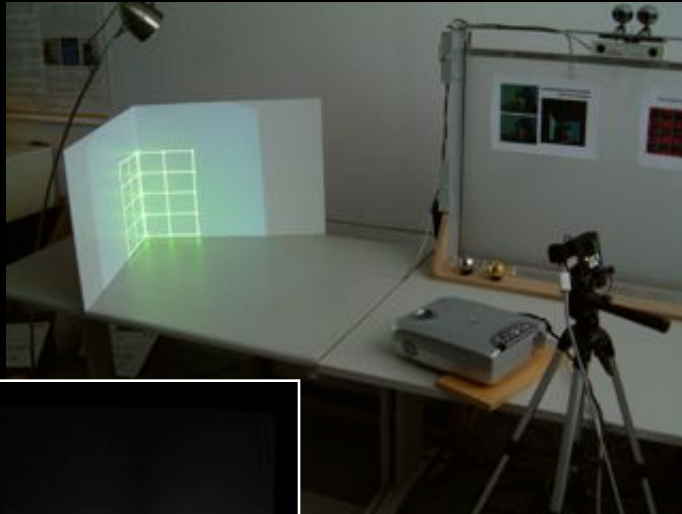
floodlight camera image



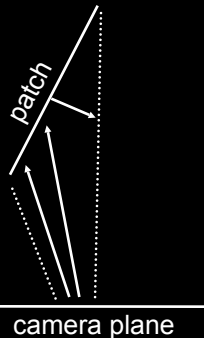
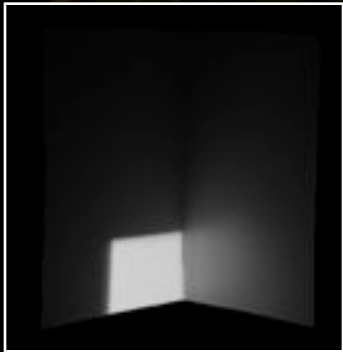
projected structured light

more information on
dual photography:
Sen, et al.,
Siggraph'05

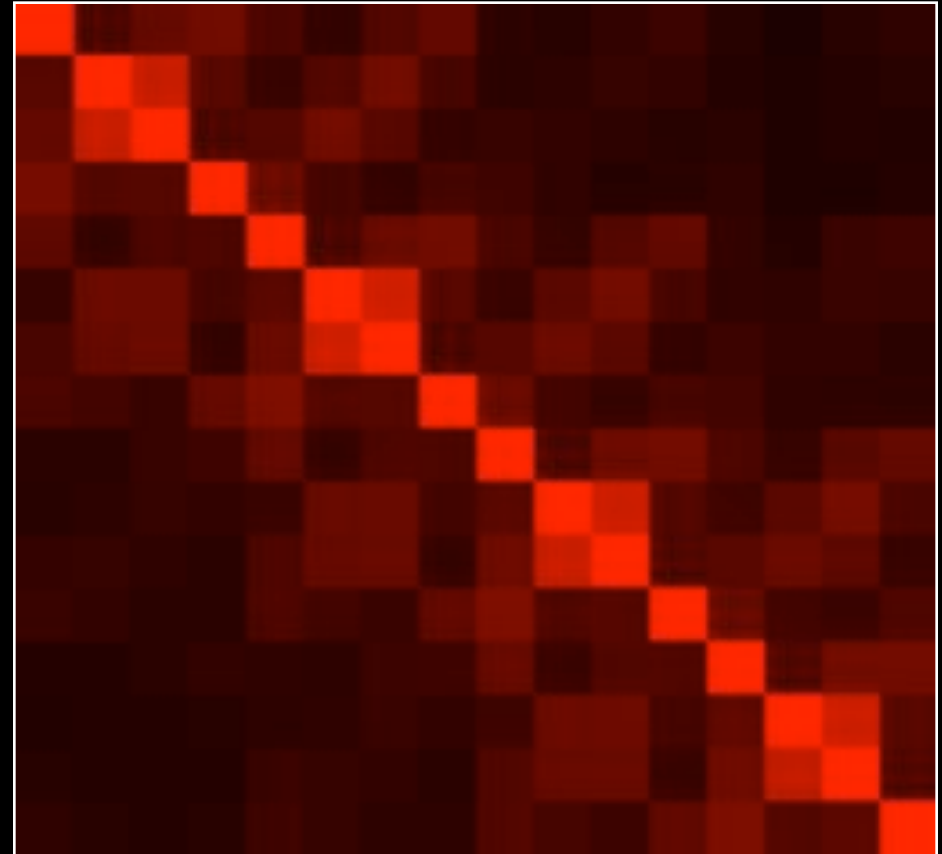
Form-Factors from Light Transport Matrix



experimental setup



$$M_e = \frac{d^2}{\cos \alpha} L_e$$



measured 16x16 form-factor matrix
(computed from light transport matrix)

Global Radiometric Compensation

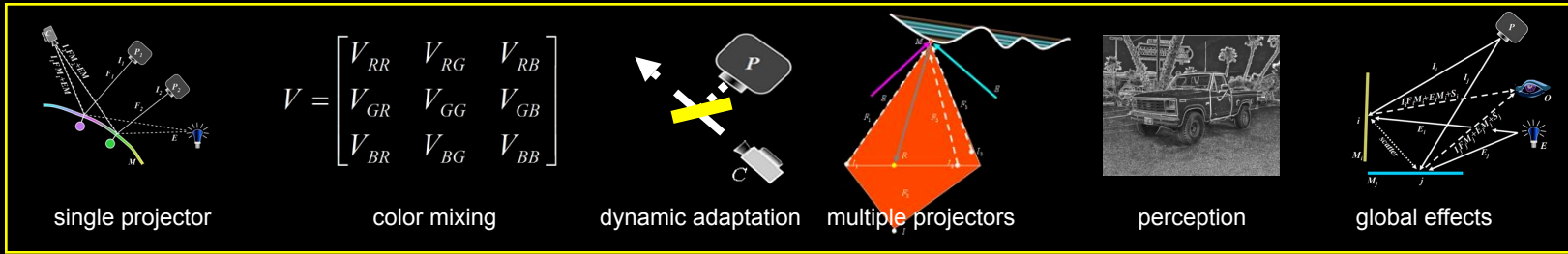
- traditional radiometric compensation requires direct projector-camera pixel correspondence
- include arbitrary global illumination effects using T
- apply inverse light transport $T^{-1}C=P$
- since T is huge, decompose it into clusters and solve in real-time on GPU

$$\begin{bmatrix} c_{10} \\ c_{11} \\ c_{12} \\ c_{13} \\ c_{14} \end{bmatrix} = \begin{bmatrix} t_{10}^4 & t_{10}^3 \\ t_{11}^4 & t_{11}^3 \\ t_{12}^4 & t_{12}^3 \\ t_{13}^4 & t_{13}^3 \\ t_{14}^4 & t_{14}^3 \end{bmatrix} \begin{bmatrix} p_4 \\ p_3 \end{bmatrix} \quad \rightarrow \quad \begin{bmatrix} t_{10}^4 & t_{11}^4 & t_{12}^4 & t_{13}^4 & t_{14}^4 \\ t_{10}^3 & t_{11}^3 & t_{12}^3 & t_{13}^3 & t_{14}^3 \end{bmatrix}^{-1} \begin{bmatrix} c_{10} \\ c_{11} \\ c_{12} \\ c_{13} \\ c_{14} \end{bmatrix} = \begin{bmatrix} p_4 \\ p_3 \end{bmatrix}$$

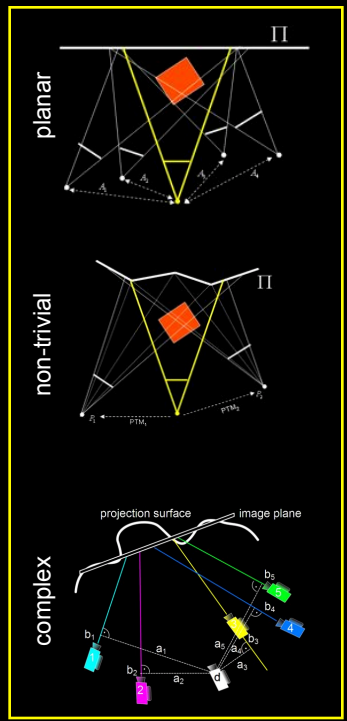
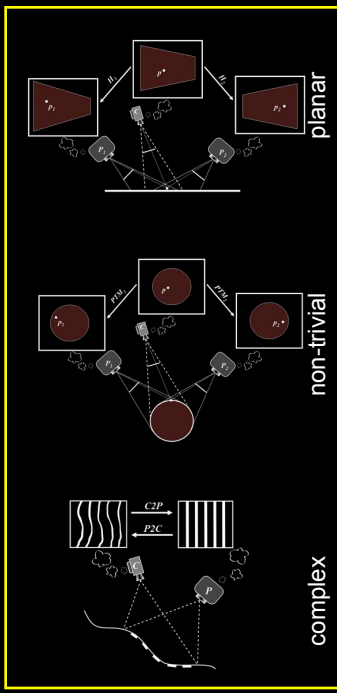




Outlook



radiometric compensation

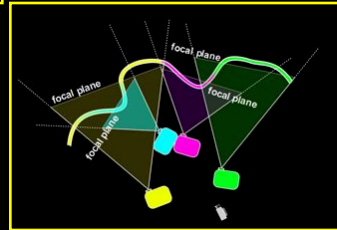


geometric warping

view-dependence

multi-focal projection

light transport



$$T = \begin{bmatrix} t_{00} & \dots & t_{0pq} \\ \vdots & \ddots & \vdots \\ t_{mn0} & \dots & t_{mnpq} \end{bmatrix}$$

global parameters

dual image

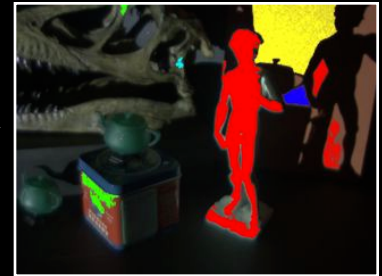
global rad. comp.

Limitations

- **technological limitations of projectors:**
 - brightness, resolution, focal depth
 - black-level and dynamic range
 - size, cost, portability
- **technological limitations of cameras:**

Future Work

- **new techniques:**
 - consider human visual perception
 - spent computational power only on overcoming limitations that can actually be perceived
 - consider global effects
 - inter-reflections, scattering, etc.



Selected Papers on Geometric Correction

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Selected Papers Other and Related Techniques

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Cotting, D., Naef, M., Gross, M., & Fuchs, H. (2004). Embedding Imperceptible Patterns into Projected Images for Simultaneous Acquisition and Display. *Proc. of IEEE/ACM International Symposium on Mixed and Augmented Reality (ISMAR'04)*, 100-109.

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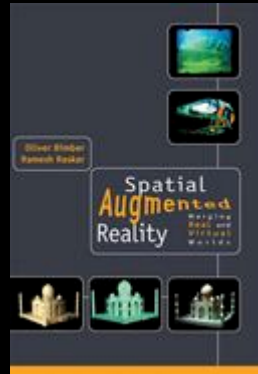
Sen, P., Chen, B., Garg, G., Marschner, S.R., Horowitz, M., Levoy, M., and Lensch, H.P.A (2005)., Dual Photography, *Proc. of ACM*

Siggraph, pp. 745-755

O. Bimber

Multi-Projector Techniques for Real-Time
Visualizations in Everyday Environments

04/01/06



Bimber, O. & Raskar, R.
*Spatial Augmented Reality: Merging
Real and Virtual Worlds*. A K Peters
LTD (publisher), ISBN:
1-56881-230-2.

Thank you!

www.uni.weimar.de/medien/AR



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