TR 79-021

A THREE-DIMENSIONAL DISPLAY FOR MEDICAL IMAGES FROM SLICES

Henry Fuchs
Stephen M. Pizer
Jon S. Cohen
Frederick P. Brooks, Jr.

University of North Carolina
Chapel Hill, North Carolina, U.S.A.

Even early versions of our system are expected to have a combination of capabilities not previously found in such displays; its 3-D image resolution will be significantly higher than CRT-based systems; it will have flexible slice positioning, illumination control and completely stationary film and projection components not found in any previous film-based systems. Later systems will also allow the user to modify the image interactively.

1. INTRODUCTION

In the past decade medical imaging has been improved by the invention and clinical use of modalities which map a physical parameter in three dimensions. This is in contrast to the previously available modalities such as radiography and scintigraphy which project the three-dimensional distribution of a parameter onto two dimensions. Examples of the new three-dimensional modalities are transmission computed tomography, in which the parameter mapped is x-ray attenuation; emission computed tomography, in which the parameter mapped is radioactivity; and B-scan ultrasound echography, in which the parameter mapped is echo amplitude.

Most current sensing systems collect the data slice-by-parallel-slice and display the slices individually on film or on a computer-controlled CRT. In many cases it is difficult to comprehend the 3-D object structure without viewing the images in their appropriate 3-D relation; indeed some users transfer the individual slice-images onto a transparent substrate and physically stack these transparencies, using various frames or holders to maintain the appropriate distance between adjacent images. In crystallography, this is often done on a large (meter-sized) scale to display electron density distributions--the device is commonly known as a Richards Box [Richards, 1968].

In order to take full advantage of such 3-D information, the user should be able to view it in its full 3-D context rather than as isolated measurements. Unfortunately display devices for effectively presenting full 3-D information-- not simply a 2-D projection of a 3-D distribution--have not been available.

1.1 REQUIREMENTS

1.1.1 Data Handling

The requirements for a 3-D display for medical imaging, determined by the new data collection devices, are considerable. Current CT scanners and other similar imaging devices synthesize images of approximately 256 X 256 pixels or more. Each pixel may be significant to 12 bits, although for display 8 is adequate [Pizer and Chan, 1979]. The number of slices forming a complete study varies widely, but up to 20 can be expected. Presenting this number of images with sufficient rapidity is taxing for displays which must be refreshed.

1.1.2 3-D Viewing

Human perception of 3-D structure is based on a number of interacting cues. Among these are stereopsis, kinetic depth effect, head motion parallax, perspective, and obscuration of distant parts by closer ones. For our application, perspective is a relatively weak cue since our objects do not have rectilinear edges.

1.1.2.1 Parallax Effects

Among the parallax cues, head motion parallax and the kinetic depth effect are much stronger than stereopsis. Further, head motion parallax tends to be stronger than the kinetic depth effect because of the strong coupling between user movements and alterations in the viewpoint. In addition, "true" 3-D displays inherently provide head-motion parallax, whereas the kinetic depth effect always requires extra computation.

1.1.2.2 Obscuration

Obscuration as a strong depth cue depends upon the 3-D space being composed of opaque objects. In our application, however, the objects are partially transparent, confusing the perception of objects in front and in back of them -- making this asset a liability. This difficulty is not due to limitations of a particular viewing device but rather is fundamental to the display of any 3-D distribution of continuous partially transparent data.

For images which consist mostly of edges, straightforward transparent display may be adequate. However, in cases in which most locations have non-zero intensity, as in computed tomography and in ultrasonography, intensities in front and in back of the region of interest may obscure structures in the region. To solve this problem, Szilard [1973], de Montebello [1977b], and we have suggested that certain regions of interest can be viewed while other regions are blanked. The region of interest will still be seen in three dimensions and in a three-dimensional context but without being obscured by other regions. The works of Szilard and de Montebello has indicated the usefulness of this idea.

We speculate that it will be very useful to allow interactive control of not only the slice(s) which are to be illuminated but more generally the size, shape, and (3-D) position of a region of interest, as well as the intensity of remaining regions. With such controls the user could easily (and we believe naturally) a) explore various parts of the image to determine areas of interest, b) examine various regions to different levels of detail, and c) examine closely different regions at different times -- all the time maintaining a strong feel for the 3-D context of the examined regions in the image.

1.2 ALTERNATIVE 3-D DISPLAYS

The potential utility of 3-D devices, for medical as well as for other applications, has long been recognized; the resulting interest has led to the development of a variety of designs and prototypes. Besides *varifocal mirrors*, which is the approach we have chosen, most of the previous systems for 3-D display of grey-scale images have been based on

a) stereo pairs [Valyus, 1966],

b) smoothly rotating 3-D models on 2-D screens [Sutherland, 1968; Rougelot & Schumacker, 1969; Shohat & Florence, 1977; Evans & Sutherland 1977a, 1977b; Newman & Sproull, 1979; Vector General, 1979],

c) holograms [Lesem & Hirsch, 1968; Huang, 1971; Benton, 1977],

d) vibrating or rotating mirrors or screens [Withey, 1958; *Aviation Week*, 1960; Goldberg, 1962; *Electronics*, 1962; *Space/Aeronautics*, 1962; Ketchpel, 1963; Harper, 1965; Szilard, 1973, 1977; Mark & Hull, 1977; Simon, 1977; de Montebello, 1977b]

e) arrays of lenses [Lippman, 1908; Ives, 1931; Pole, 1967; Burckhardt, 1968; Chutjian & Collier, 1968; de Montebello, 1977a].

Each of these systems have been burdened with one or more limitations which have kept it from general use. Among these limitations have been:

a) insufficient resolution,

b) necessity for rapid movement of bodies of significant mass or moments of inertia,

c) inconvenient production of images,

d) insufficient depth cues,

e) need for powerful computational facilities, and

f) restrictions to a single observer.

2. VARIFOCAL MIRROR DISPLAYS: PRINCIPLES

With varifocal mirror displays, as with all systems based on mirrors and screens, the perceived position of an image on a screen is determined by the position of the screen, or if viewed through a mirror, by the position of the mirror and the screen. This perceived position may be altered by changing the position of the screen or by keeping the screen stationary and moving the mirror. If the position of the moving element is varied in synchrony with the presentation of a number of images, and if this is done with sufficiently high repetition rate, the viewer will perceive all the images in their associated positions in 3-space.

A disadvantage of moving the screen is that large displacements are needed to achieve large 3-D image depth [Szilard, 1973; de Montebello, 1977]. The major advantage of a moving (planar) mirror with a stationary screen is that large displacements of apparent ("virtual") image can be achieved with relatively small displacement of the mirror -- the "multiplier effect".

Systems based on varifocal mirrors, in which a flexible mirror changes from concave through planar to convex and back, have a multiplier effect caused by the changing optical properties of a curved mirror [Muirhead, 1961; Traub, 1967, 1968; Rawson, 1968, 1969; Hobgood, 1969; Science News, 1977; Bolt Beranek and Newman, 1978]. Our systems use this display method.

The varifocal mirror display was invented by Muirhead in 1961 and developed by Traub in 1967, by Rawson in 1968, and by Hobgood in 1969 (in the Computer Science Department at UNC under the direction of one of the authors (FPB)). It has been further developed recently at Bolt Beranek and Newman Inc. [Sher, 1978] and at the University of Utah [Baxter, 1978]. The system consists of a varifocal mirror and its driving assembly, an image presentation device, and system control for synchronization (see figure 1). The system control, via the driving assembly, causes the mirror to vibrate at a certain rate and mode. When the screen is reflected at a particular depth within the image volume, the image presentation component displays the appropriate cross-section of the image on the screen. For every cycle of mirror vibration, the entire sequence of images is presented (see figure 2).

Previous work has shown that the varifocal mirror is useful in the display of three-dimensional slice-type distributions and has the following advantages over other schemes mentioned above:

a) It requires no substantial computer nor expensive optics.

b) It requires small movement of little mass to achieve the 3-D image.

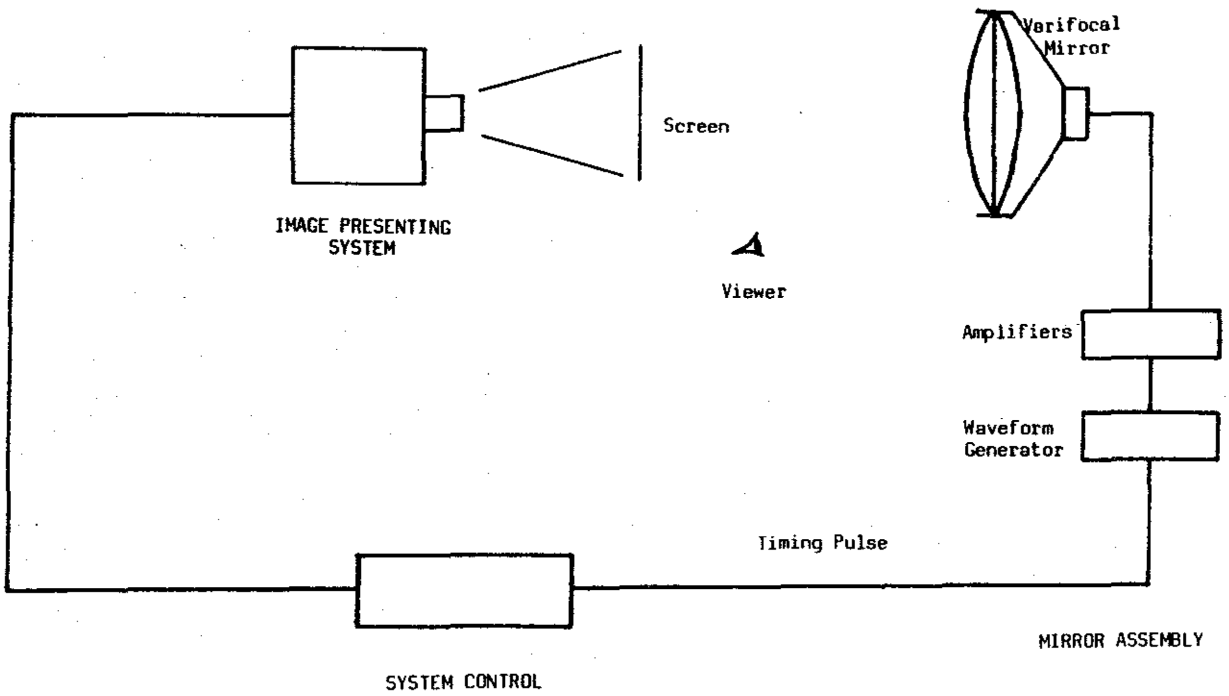

Figure 1: General Configuration

c) It allows flexible modification of the 3-D image being viewed, if suitable image-presentation is used.

d) It produces large images (a 20 cm. cube is straightforward), viewable from many angles by more than one observer at a time.

e) It is mechanically stable.

The varifocal mirror has the following limitations:

a) The data must be presented in order of depth within the image volume.

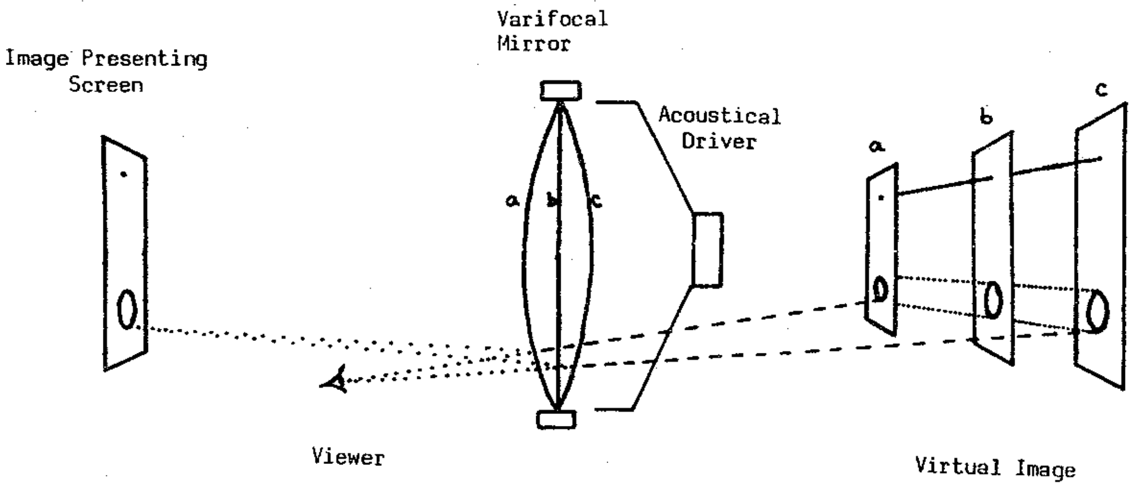

Figure 2: General Concept

b) The images can be viewed only from directions within approximately 45° from perpendicular to the planar mirror surface. Since the presentation can easily be reversed front to back by changing the mirror synchronization by one half mirror cycle (assuming anomalous perspective correction is made), the range of possible viewing angles is doubled -- to approximately one-third of all possible angles. It is not clear whether the restriction on viewing angle is important, but if so, this problem can be overcome, with appropriate computing resources, by interpolating coronal and sagittal slices and displaying these.

c) The device produces a hum, which can be disturbing, though all but insignificant noise can be avoided by vibrating a carefully designed mirror strictly sinusoidally.

d) There is a problem of variable magnification with depth (sometimes called anomalous perspective) [Traub, 1967; Rawson, 1969; Hobgood, 1969]. This is easily corrected by magnifying the collected slices before presentation.

3. IMAGE PRESENTATION FOR VARIFOCAL MIRROR DISPLAYS

3.1 PRESENTATION RATES AND METHODS

In schemes based on rotating or vibrating mirrors or screens, the system must be provided with successive planar images making up the three-dimensional intensity distribution. This matches well with ultrasonography and computed tomography, for these produce images of parallel slices. To form an apparently continuous three-space image, different sections must be displayed at a rate above the motion fusion frequency of the human eye, about twelve views per second. To form a flicker-free image, a complete image must be refreshed at a rate above the eye's flicker frequency, about 30-40 images per second. Assuming 10 to 20 different sections, the system must be capable of presenting 300 to 800 two-dimensional images in a second.

Various system considerations may demand considerably higher image presentation rates for certain portions of the mirror cycle. As indicated above, noise considerations make sinusoidal mirror motion desirable. This motion implies a nonlinear relation between time and the perceived position of image in the 3-D image-volume. Also, we do not wish to restrict image collection to a particular spacing scheme. Furthermore, since the shape of the mirror may be slightly different during the convex-to-concave phase from the concave-to-convex phase, it is often convenient to utilize only half of each mirror cycle for display. In addition, the organization of image presentation is simplified by using only one half-cycle. All of the above considerations combine to require instantaneous image presentation rates far in excess of the average. For instance, with only one half cycle used, if two images in the middle of a 20 cm image volume are 5 mm apart, they have to be presented within 250 microseconds of each other, equivalent to a rate of about 4000 images per second.

Two methods of image presentation have previously been explored: a) fast CRT displays supported by high-speed memory, and b) fast movement of film. The CRT displays require no moving parts and allow flexibile image modification. Their technology, however, has not yet developed to the point where many hundred high-quality video images can be drawn in a second. CRT displays for these applications consequently have largely been restricted to presentation of line drawings. An exception is the system developed by Mark and Hull [1977]. This system, with a specially built CRT using the latest technology, is still restricted to three-dimensional images consisting of a small number of raster elements (128 X 128), grey levels (4 bits), and slices (12). In addition, all CRT's for this application need special, short-persistence phosphor to prevent images from smearing in depth.

Systems based on fast movement of film have been developed by Rawson [1968], Szilard [1973, 1977], and de Montebello [1977b]. Film is cheaper and produces higher image quality than CRT's. Unless one resorts to rephotographing, however, modifications of the images are severely limited with these systems. Further complications are incurred by the need for fast mechanical movement of the film and control of the light projecting the image. Moreover, because rapid nonlinear movement of film is prohibitively difficult, these systems severely limit slice spacing.

Developers of three-dimensional displays based on parallel collected slices disagree on the number of slices required to produce the impression of continuity in depth. Some authors, e.g. Szilard [1973], have stated that around ten is enough, while others, e.g. Mark and Hull [1977] and de Montebello [1977b], have stated that many tens are required. The authors preferring few slices seem to have had systems which displayed each slice with long persistence, whereas those preferring many slices seem to have had systems which displayed them each for a few microseconds only. Systems with short persistence require many slices to produce fusion, while systems with long persistence limit the number of slices. Clearly, control of persistence is desirable.

3.2 OBSCURATION

As discussed previously one of the major problems of comprehending a 3-D grey-scale image is the obscuration of detail by parts of the image in front or in back of the region of interest. We propose interactive specification of the region of interest to be illuminated or highlighted.

Szilard and de Montebello have developed mechanisms to implement this approach. Szilard's approach is to vary the brightness of the illuminating source with the slice being projected. This allows relative intensification only of slabs parallel to the collected slices. De Montebello's approach involves interposing a movable filter between the film and the screen. This allows the relative intensification of any slab perpendicular to the slice planes.

As indicated before, the solution which we are developing (the systems to be described in later sections of the report) involves enhanced illumination ("highlighting") of a region of interest and interactive specification of its location and size. With such a capability the user would "roam" the space (with a 3-D joystick of some kind) in a manner roughly analogous to a miner or a spelunker exploring a cave with only the light on his helmet as a guide. (The analogy, in fact, is more limited than our solution; a miner's lamp has to illuminate everything in its path, while our "highlighting" control clearly does not.)

Another attack on the problem of one region obscuring another is with the use of a pseudo-color representation of intensity [Szilard, 1973]. Use of color may also it does increase the perceived dynamic range of the display, as in two-dimensional display. Unless these colors are chosen carefully, however, the overlapping of colors may cause confusion.

4. UNC VARIFOCAL-MIRROR 3-D DISPLAY SYSTEMS

Our development of 3-D display systems based on varifocal mirrors is planned to encompass several phases, each phase producing a different system. The systems each consist of the three major components outlined earlier: the mirror and its driving assembly, the image presentation unit, and the system control. In each phase we enhance one or more of these components.

4.1 LINE-DRAWING SYSTEM

Our first system consisted of an existing image presentation unit, a real-time line-drawing graphic system (a Vector General Model

3) the computer host of the line-drawing system (a DEC PDP-11/45) as the system control, and an aluminized Mylar mirror driven sinuisodally by a loud speaker (see figure 3). The principal purpose of this simple system was to develop and to test the mirror and its driving assembly.

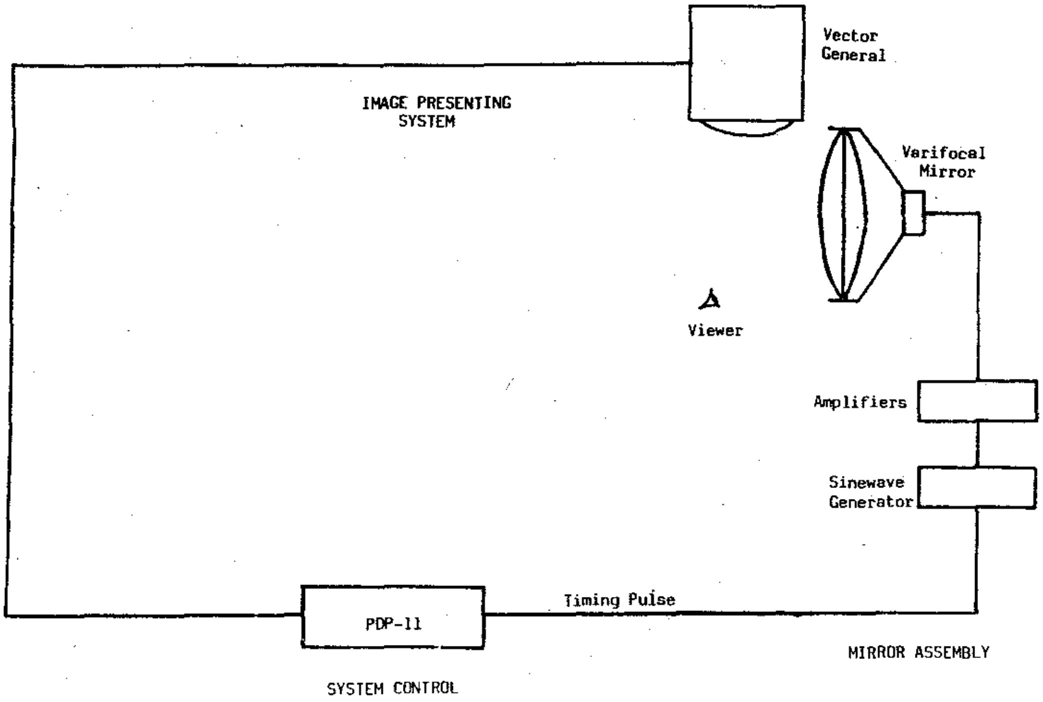

Figure 3: Line-drawing System Configuration

The mirror was made of .001 inch thick silvered polyester mounted in an aluminum frame such that the vibrating surface was 31 cm. in diameter (see figure 4). With the help of L. Sher, the mirror mount was especially designed to produce even surface tension without wrinkles. (See figures 5 and 6 for mount details.) Such care is necessary if the vibrating mirror shape is always to be spherical, thus producing undistorted images.

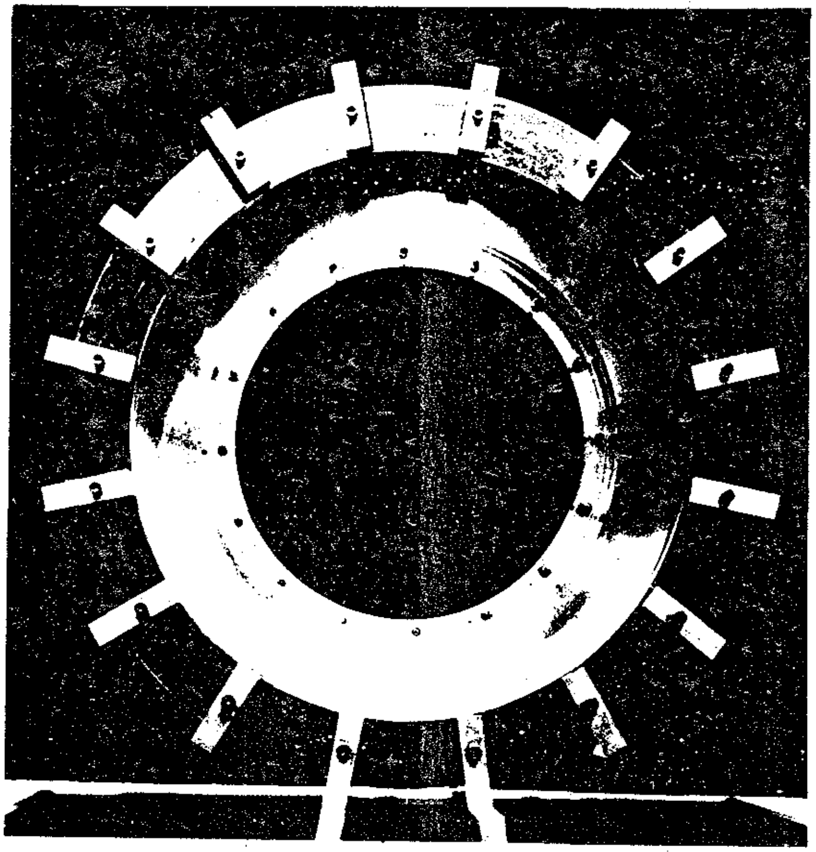

Figure 4: Mirror Mount, Front View

Because this kind of a mirror can be made to vibrate over a wide range of modes and frequencies, it is ideal for experimentation. However, such a mirror is more prone to undesirable modes of vibration than a semi-rigid self-resonant mirror, as developed by L. Sher at Bolt Beranek and Newman. Nevertheless, with our 31 cm diameter mirror

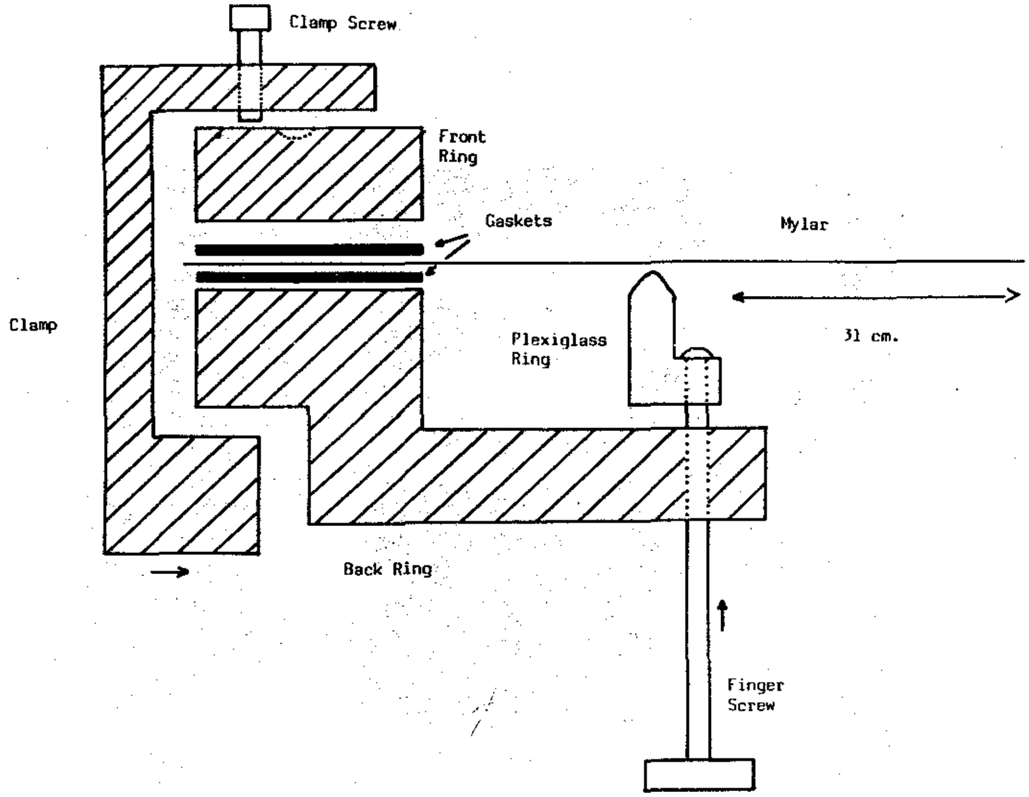

Figure 5: Mirror Mount Design

was, we were able to obtain no discernible distortion in 3-D images up to 30 cm in depth.

The mirror was vibrated by placing a 10-inch woofer behind the mirror surface but within the mount (see figure 7). The woofer was driven sinusoidally at 40 Hz to match the frame rate of the Vector General image presentation unit. An interrupt from this unit at the beginning of each frame was used to determine the frequency and phase of a sinewave generator, the output of which was amplified to drive the speaker.

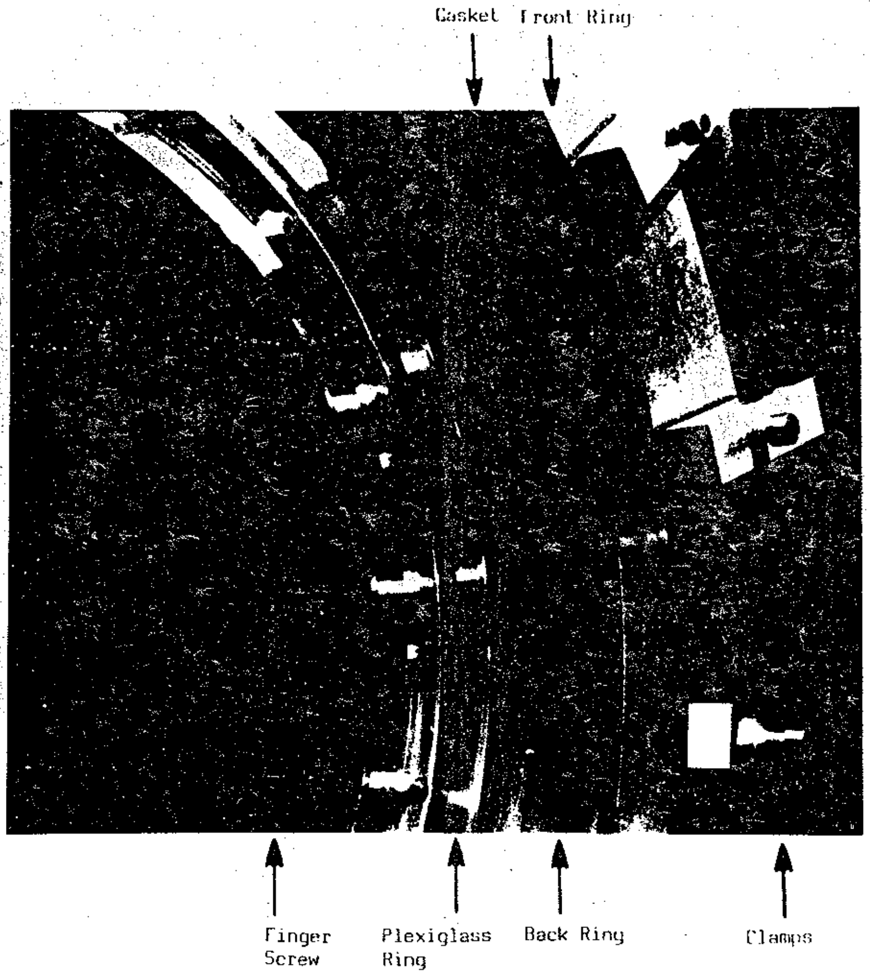

Figure 6: Mirror Mount Detail

The mirror should ideally be positioned a) to allow the user a maximum number of comfortable viewing angles, in which case the mirror should face the viewer, and b) to minimize keystone distortion, in which case the mirror should face the image presenting screen. Since these two conditions are hard to satisfy simultaneously, an orientation in between the two extremes is usually adopted.

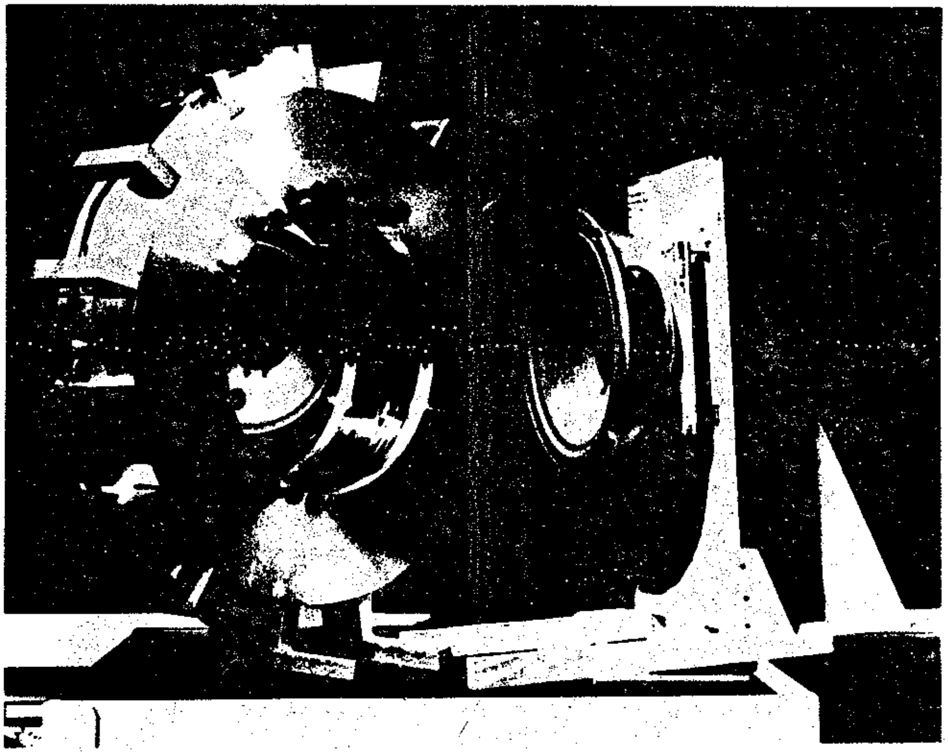

Figure 7: Speaker and Back of Mount (with Mirror in Place)

Figure 8 shows the mirror and image presenting unit of the line-drawing version of our system. Their orientation was selected for maximum ease of experimentation, but it produced a significant keystone effect. Tests using this system were done by generating a rectangular prism with identified front and back faces composed of approximately 100 image slices.

The distance between the screen and the mirror affects both the range of viewing angles and the image depth. Details of image position as a function of mirror center excursion, mirror diameter, and mirror-to-screen distance can be found in Hobgood [1969] and in Cohen [1979]. For our mirror, an excursion of ±6 mm, combined with a

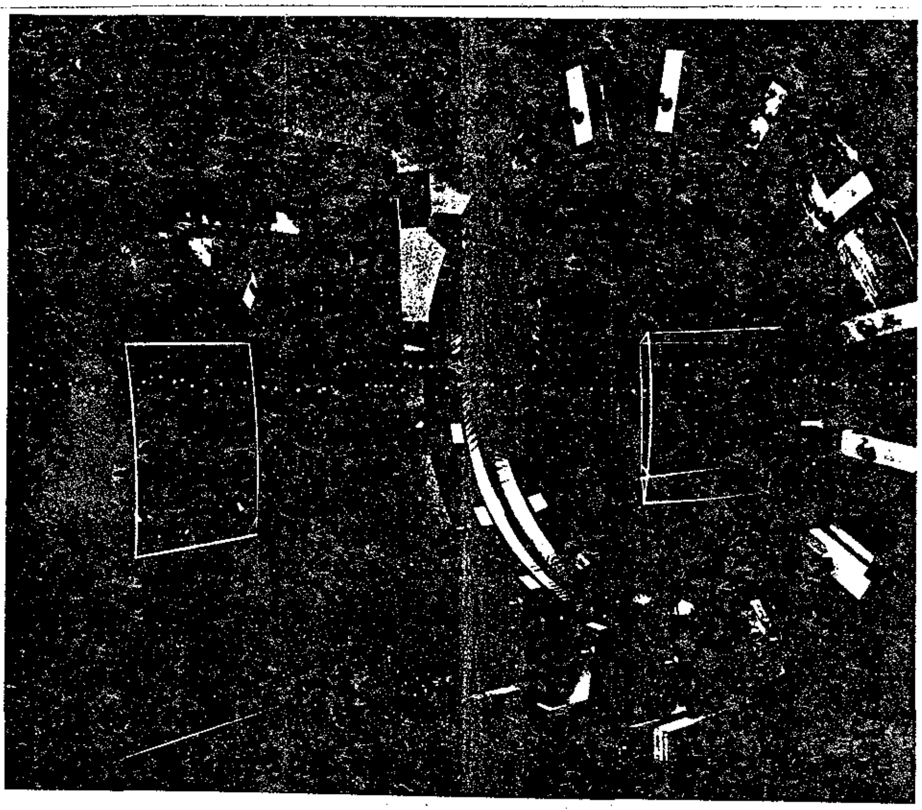

Figure 8: Line Drawing System Showing Uncorrected Cube

30 cm mirror-to-screen distance produced our desired volume of about 20 cm cubed.

4.2 SYSTEM BASED ON FLASH-TUBE PROJECTION

The second version of our system has as its objective the demonstration of high-resolution grey-scale 3-D display and the investigation of the properties of such display. In order to achieve such resolution, the slice images are photographed and presented via high-speed projector. This version utilizes the mirror and driving assembly from the first version and uses a microprocessor (Heathkit 6800) for system control, and a specially built projector described below.

Although the image presentation unit of this system does not permit interactive modification of the 3-D image, it is well suited to the display of CT and ultrasound images, which are normally recorded as parallel slices on film and which normally contain less than 20 slices. In addition we shall discuss later in this paper ways in which this approach can be extended to uses with interactive requirements.

In order to avoid the constraints imposed by high speed motion picture projection, our system uses a special projector, a modified version of Picker Corporation's Image Tunnel Projector [Sano et al, 1978]; it allows full choice of the time and duration for the display of each slice and involves no mechanical motion. The projector consists in effect (but not in fact) of many slide projectors with a) optics to focus the images on a common screen and b) control of which slice (or slices) is displayed at any time.

The optics consist principally of a mirror tunnel of square cross-section with a lens at the end closest to the array of images and a small back-projection screen at the other end (see figure 9).

Figure 10 shows how these optics project an image from the array onto the screen by reflecting the image within the mirror tunnel. Figure 11 shows how the optics project each image from the array onto the same screen, with the number of reflections in the mirror tunnel, and thus the orientation of the projected image, dependent on the position of the frame in the array. The photographs in the array are mounted so that the images will all be projected in the same orientation.

(Since this system's image presentation rate is flexible, 30 hz -- rather than the previous 40 -- was chosen to allow more relaxed control programming and less audible speaker hum.) Our prototype

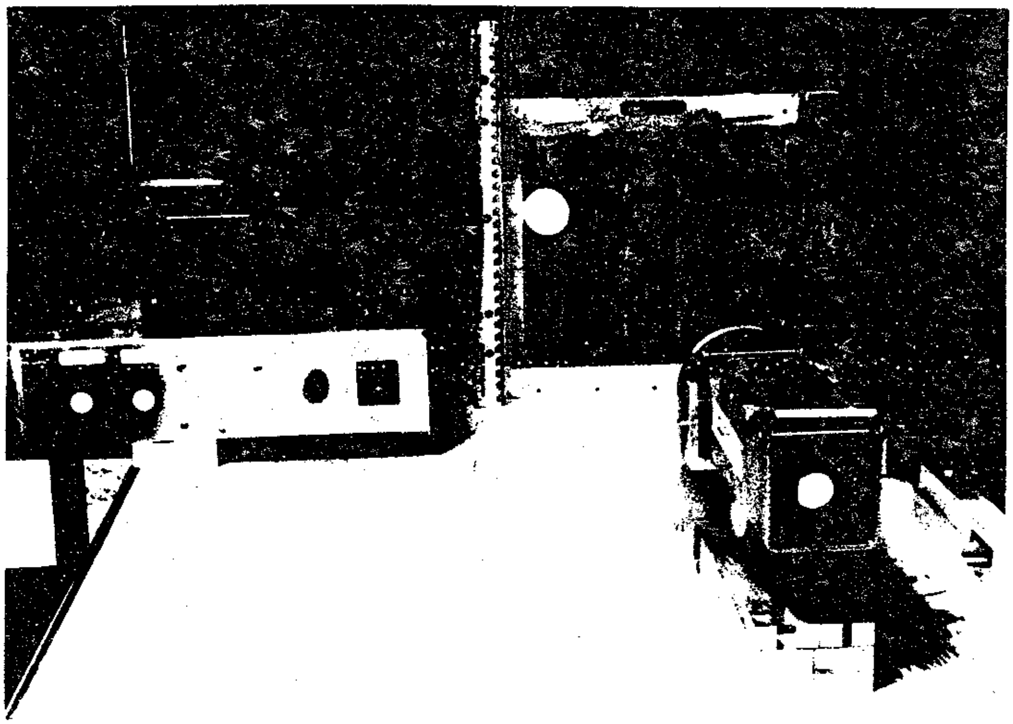

Figure 9: Picker Image Tunnel Projector Prototype with Array Element #6 Illuminated

uses the straightforward approach of illumination of each slice by a xenon flash tube. The microprocessor generates pulses which trigger each flash tube at the appropriate time (see figure 12). Each slice image must be separately illuminated with a duration limited to a few hundred microseconds or less and repeatable at rate of approximately 30 Hz.

The various components of this system have recently been completed; the overall system is currently being integrated.

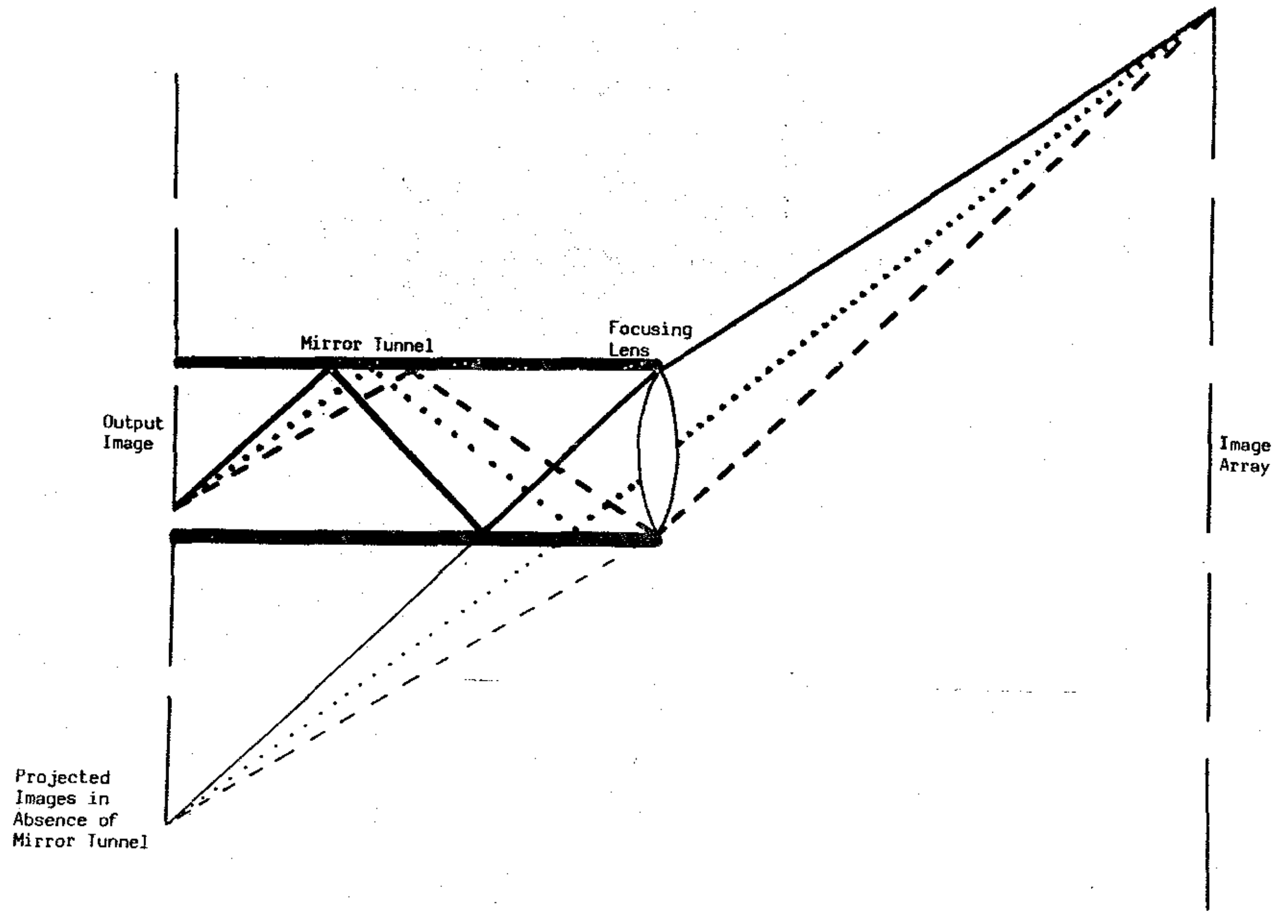

Figure 10: Mirror Tunnel Folding Rays from Single Point

4.3 SYSTEM BASED ON VARIABLE SLICE-TO-SLICE ILLUMINATION

The planned next version of our 3-D display will attempt to provide increased comprehension of the 3-D image by allowing more flexible illumination control. Specifically in this version the intensity and the duration of each slice will be independently controlled, thus allowing a) dim illumination, rather than blanking, of slices around the current region of interest, and b) variable effective thickness of

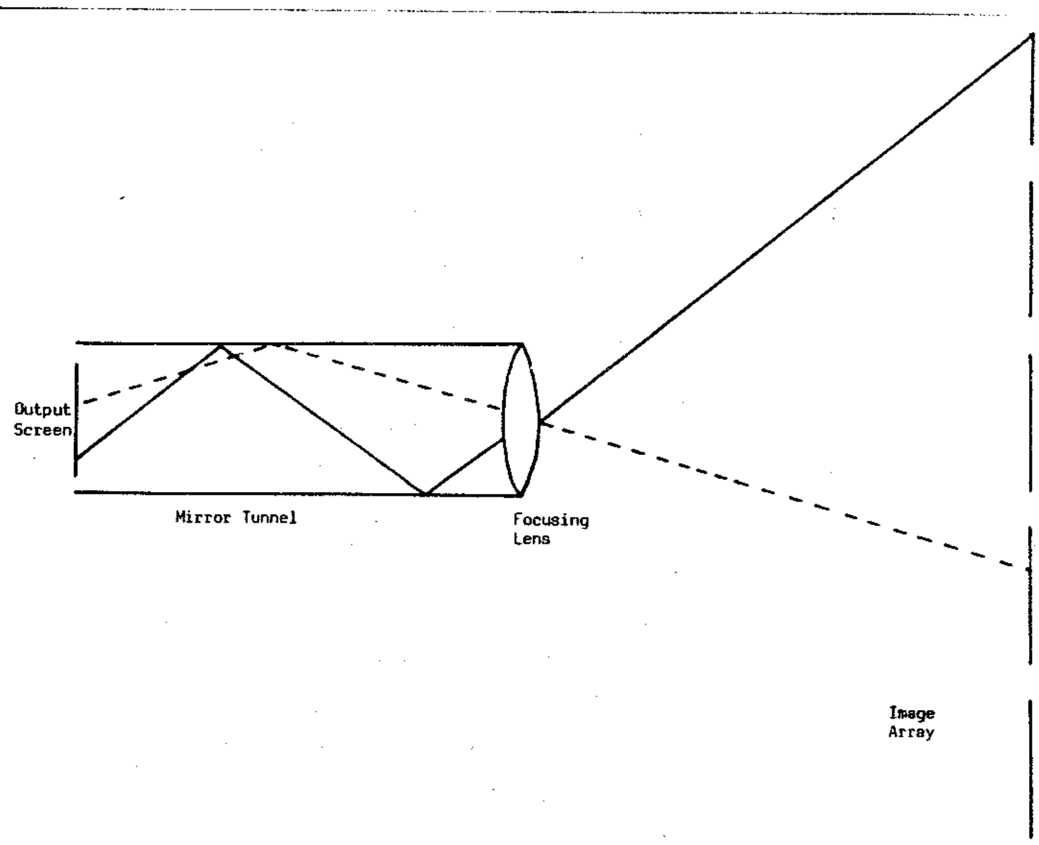

Figure 11: Projecting Multiple Images onto Single Screen

a slice within the 3-D image. These capabilities are to be achieved by replacing the previous version's flash tube illuminations with constantly lit bulbs each capable of variable intensity and high-speed electronic shutters in front of each element of the array (see figure 13).

This version will incorporate our present microprocessor controller, with appropriate software modifications, and our present mirror and mirror driving assembly, (although we are considering the acquisition of a semi-rigid, self-resonant mirror). Due to the varying light loss in the mirror tunnel and the difficulty of

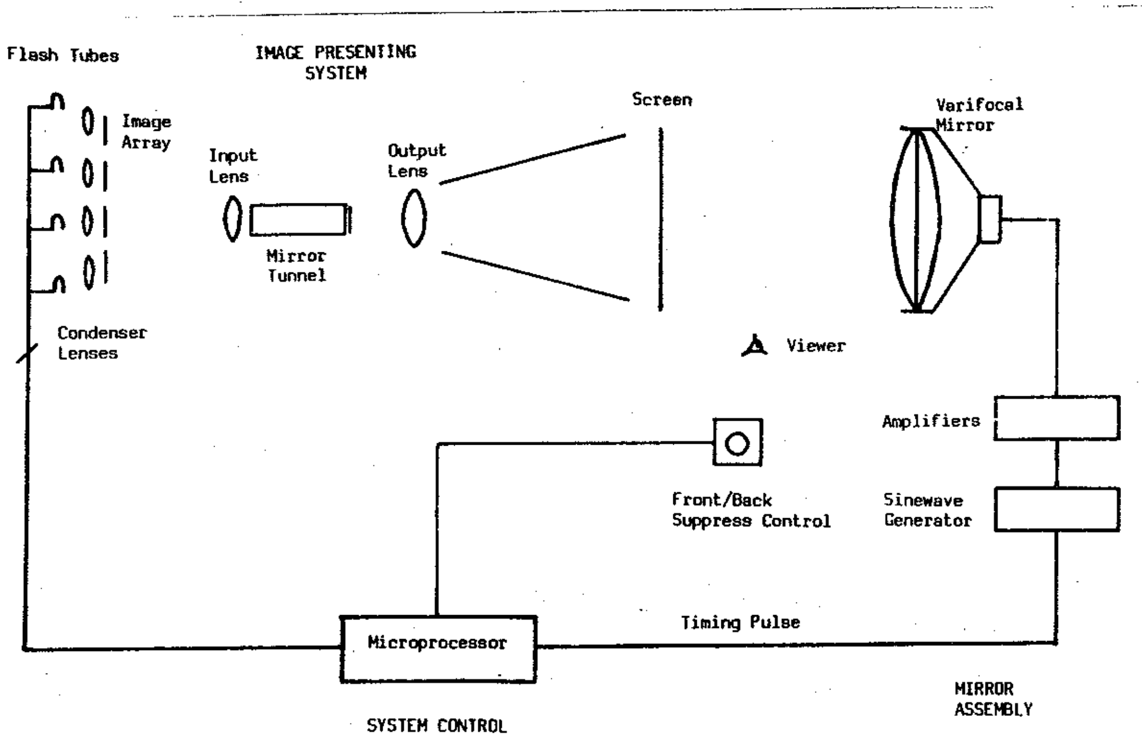

Figure 12: System Configuration with Flash Tube Projection

alignment, it is possible that an array of individual lenses will replace the present mirror tunnel, although the alignment and matching of twenty lenses seem to have their own complexities. The planned configuration of this system is shown in figure 14.

4.4 FULLY INTERACTIVE SYSTEM

The final system we are currently planning will use multiple CRT's in place of the film projection unit. This modification will allow flexible control of

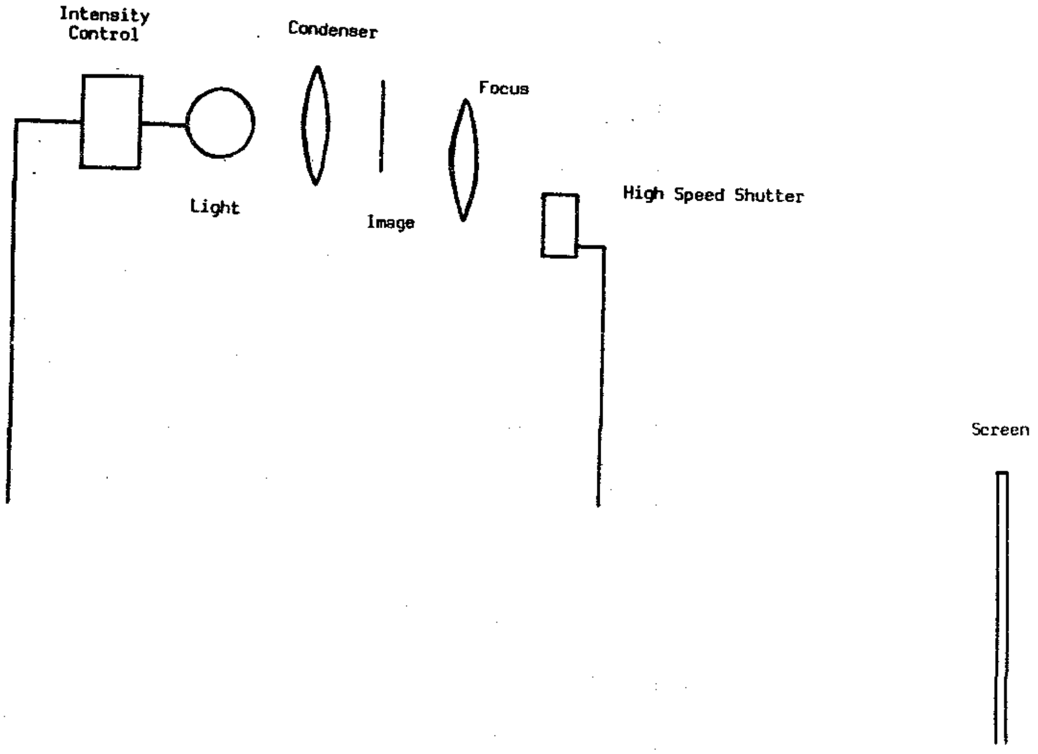

Figure 13: Projector Detail for Variable Slice-to-Slice Illumination

a) the size, shape, position, and illumination characteristics of the highlighted region of interest,

b) interactive image modifications such as intensity mapping and edge enhancement,

c) input of non-parallel slices and, indeed, arbitrary 3-D input,

d) rotation of the 3-D image to allow viewing from any orientation,

e) 3-D dynamic images (e.g. gated studies, animation),

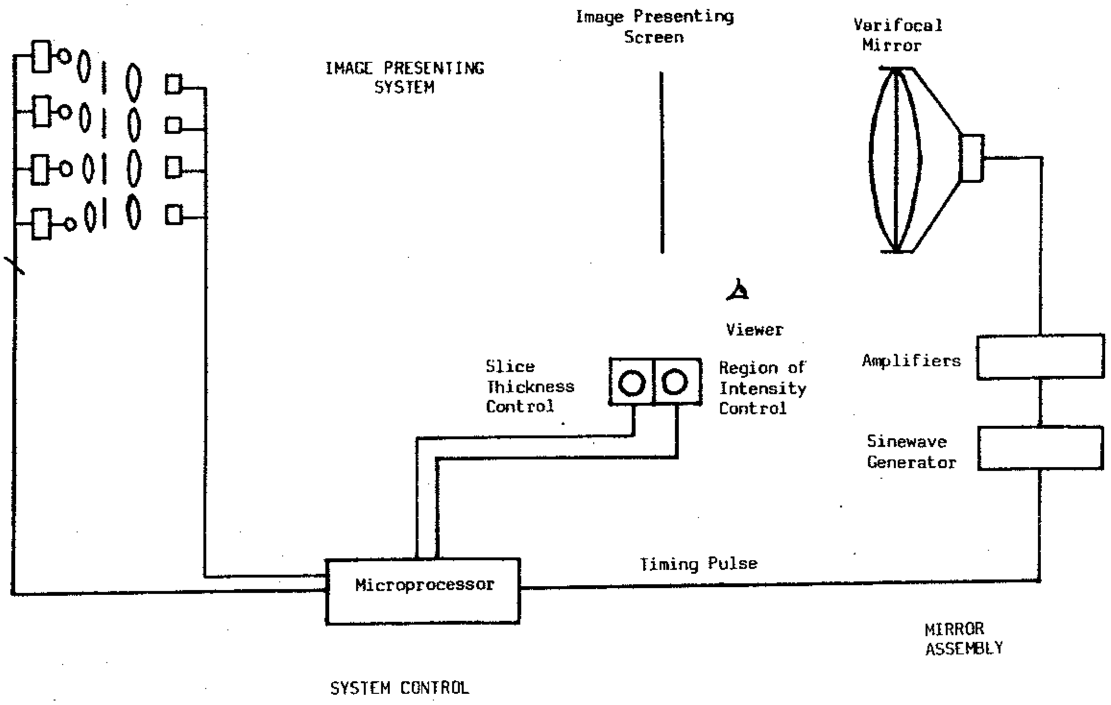

Figure 14: System Configuration for Variable Slice-to-Slice Illumination

f) accurate correction of distortion under program control, and

g) easy measurement of distances within virtual volume.

We note that the system's organization can be viewed either as a modification of the previous version with CRT's replacing film, or alternatively, as multiple connected, co-operating copies of our initial line drawing projection system. We note further that the computational requirements for such a system are not only increased in proportion to the volume of data but more sharply due to interaction among various parts of the data. For example, were rotation of 3-D data attempted, not only would each of the 3-D pixels need to be transformed but also the order of display of these would have to be recomputed.

We propose to solve these problems with a number of semiautomonous processing modules each controlling a separate CRT. These modules would all receive data from a central broadcast controller and would have an additional system of interconnections to relieve congestion when one module became overloaded (see figure 15). This multiple CRT system is expected to have all the capabilities of single CRT systems while alleviating their most serious problem--that of presenting slice-iages at sufficiently high rates.

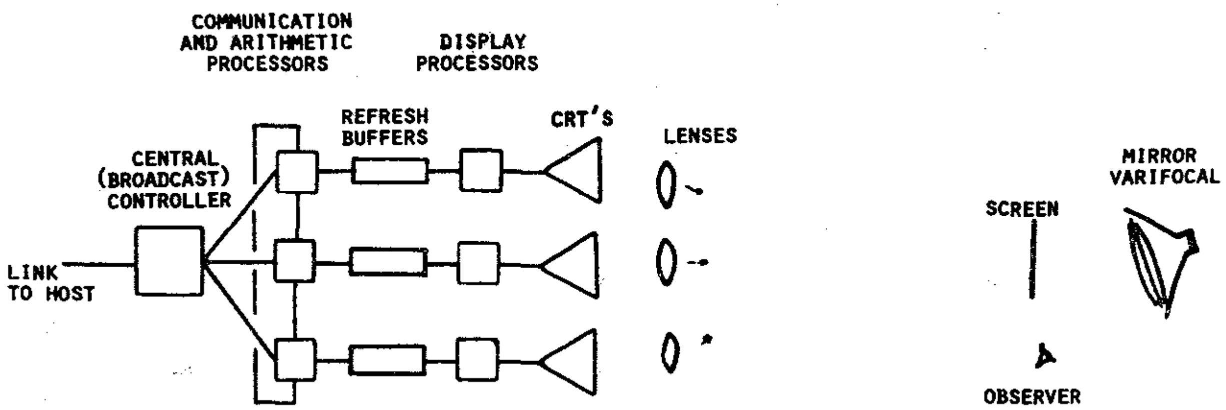

Figure 15: Tentative System Organization with Multiple CRT's and Processing Units

An issue which remains to be resolved is whether sufficient light intensity can be generated from CRT's, given the optical pathways that are required. We are encouraged in this regard by the availability of high output CRT's in consumer projection video systems and the rapid development of non-CRT (e.g. liquid crystal) graphic displays.

In conclusion, we speculate that part of the reason, in certain applications, for collecting 3-D information in parallel slices is the lack of an effective 3-D viewing device, since comprehension of a 3-D structure from a set of isolated slices in other than this simple arrangement would be exceptionally difficult. However, with an effective 3-D viewing device not biased toward slices, the collection process for 3-D data might become more flexible and effective. In ultrasonography, for example, freedom to explore in three dimensions by transducer angle and position would reduce problems presently associated with obstructions such as bone and gas.

5. SUMMARY

We have outlined the major issues of 3-D display and the technical development of a series of such displays which we are building for applications, such as medical imaging, for which data most often is collected as a set of cross-sectional slices. These displays, all based on varifocal mirrors, attempt to overcome the major limitation of all such displays--that of presenting image data with sufficient speed and flexibility. Parallel optical image presentation techniques have been found useful both to overcome this bottleneck and also to allow possibilities for parallel processing among semiautonomous digital display units. Affordable computational and display components to implement these systems are expected to become available in the near future.

We expect that a true 3-D display will significantly increase the ease of understanding 3-D data and thus the usefulness of 3-D imaging devices, just as 2-D displays (B-scans) of ultrasound data are generally more useful than a collection of one-dimensional A-scans.

6. ACKNOWLEDGEMENTS

We thank the Picker Corporation for the use of their Image Tunnel Projector prototype. We gratefully acknowledge the assistance of Francis Chan, Leon Partain, Richard Sano and Hobie Shackford of Picker Corporation, Ross Scroggs, and Lawrence Sher of Bolt Beranek and Newman. We especially want to thank Peter Rientjes for his work on the hardware implementations and Kathy Yount for text editing.

7. REFERENCES

__Aviation Week__, (October 1960), "New Display Gives Realistic 3-D Effect".

Baxter, Brent (1978), Personal Comunications, University of Utah, Salt Lake City, Utah.

Benton, S. A. (1977), ed., __Three-Dimensional Imaging__, __Proceedings of the Society of Photo-Optical Instrumentation Engineers__, Vol. 120, San Diego, California.

Bolt, Beranek and Newman Inc. (1978), "The Spacegraph Display: Images in True 3-D", Promotional literature.

Burckhardt, C. B. (1968), "Optimum Parameters and Resolution Limitations of Integral Photography", __Journal of the Optical Society of America__, Vol. 58.

Chutjian, A. and Collier, R. J. (1968), "Recording and Reconstructing 3-Dimensional Images of Computer-Generated Subjects by Lippman Integral Photography", _Applied Optics_, Vol. 7.

Cohen, Jon S. (1979), _Three-Dimensional Display of Parallel Sliced Images_, M.S. Thesis (in preparation), Department of Computer Science, University of North Carolina, Chapel Hill, N.C.

De Montebello, Roger L. (1977a), "Wide-Angle Integral Photography-The Integram System", _Proceedings of the Society of Photo-Optical Instrumentation Engineers_, Vol. 120, San Diego, California.

De Montebello, Roger I. (1977b), "The Synthalyzer for Three-Dimensional Synthesis, and Analysis by Optical Dissection", _Proceedings of the Society of Photo-Optical Instrumentation Engineers_, Vol. 120, San Diego, California.

Electronics (November, 1962), "CRT Provides Three-Dimensional Displays".

Evans and Sutherland Computer Corporation (1977a), Improved scene generation capability. Final report, NASA contract No. NAS 9-14010.

Evans & Sutherland Computer Corporation (1977b), _Picture System 2/PDP-11 Reference Manual_, Salt Lake City, Utah.

Goldberg. A. A. (December, 1962), "3-D Display System", _Proceedings of the Institute of Radio Engineers_, Vol. 50.

Harper, Paul V. (1965), "Three-Dimensional Image Reconstruction in Scanning and Radiography", _Proceedings of the XIth International Congress of Radiology_, Rome.

Hobgood, W. Sands (1969), A Three-Dimensional Computer Graphics Display Using a Varifocal Mirror. Master's Thesis, University of North Carolinâ, Chapel Hill.

Huang, T.S. (Septeber, 1971), "Digital Holography", Proceedings of the IEEE, Vol. 59.

Ives, H. E. (1931), "Optical Properties of a Lippann Lenticulated Sheet", Journal of the Optical Society of America, Vol. 21.

Ketchpel, R.D., (September 1963), "A Direct-View Three-Diensional Display Tube", IEEE Transactions on Electronic Devices, Vol 5, pp.@324-329.

Lesem, L.B., and Hirsch, P.M., (October, 1968), "Computer Synthesis of Holograms for 3-D Display," Communications of the ACM, XII, pp. 661-674.

Lippmann, G., (1908) Comptes-Rendus, Vol. 146, pp.@446-451

Mark, Howard and Hull, Fred (1977), "Three-Dimensional Viewing of Tomographic Data - The Tomax System", Proceedings of the Society of Photo-Optical Instrumentation Engineers, Vol. 120.

Muirhead, J.C., (1961), "Variable Focal Length Mirrors", Review of Scientific Instrumentation, Vol. 32, pp.@210.

Newman, W.M., and Sproull, R.F. (1979), Principles of Interactive Computer Graphics, McGraw-Hill, New York.

Pizer, Stephen M., and Chan, Francis H. Chan (1979), "Evaluation of the Number of Levels Produced by a Display", in these proceedings.

Pole, R.V., (1967), "3-D Imagery and Holograms of Objects Eliminated in White Light", *Appl. Phys. Letters, 10* Vol. 10.

Rawson, Eric G. (1969), "Vibrating Varifocal Mirrors for 3-D Imaging", *IEEE Spectrum*, Vol. 6, pp.@37-43.

Rawson, Eric G. (August 1968), "3-D Computer-Generated Movies Using a Varifocal Mirror", *Applied Optics*, Vol. 6, pp.@1505-1511.

Sano, R.M., Shackford, H., and Ioannou, B., (1978), "Pure Optical Projector for Cine Presentation of 8" X 10" Original Transparencies", *Abstracts, Second International Congress of the World Federation of* Nuclear Medicine and Biology, Washington.

Richards, F. M. (1968), *Journal of Molecular Biology*, Vol. 37, p. 225-230.

Rougelot, R. S. and R. Schumacker (1969), G.E. real time display. NASA Report NAS 9-3916. General Electric Co., Syracuse, NY.

Science News, (October, 1977), "3-D: It's All Done with Mirrors", Vol. 112.

Sher, Lawrence (1978), Personal Communications, Bolt Beranek and Newman, Boston, Mass.

Simon, William and Walters, Timothy (1977), "A Spinning Mirror Auto-Stereoscopic Display", *Proceedings of the Society of Photo-Optical Instrumentation Engineers*, Vol. 120.

Shohat, M. and J. Florence (1977), Application of digital image generation to the shuttle mission simulation. *Proc. 1977 Summer Computer Simulation Conference*.

Space/Aeronautics, (September, 1962), "3-D Display".

Sutherland, I.E., (1968), "A Head-Mounted Three-Dimensional Display", AFIPS Conference Proceedings 1968 FJCC, Vol. 33, pp.@757-764.

Szilard, J. (June 1973), "An Improved 3D Display System", Proc. Second World Congress on Ultrasonics in Medicine, Rotterdam, Netherlands.

Szilard, J. (1977), "A Further Improved 3D Display: Mark 2", Proc. Ultrasonics International, p. 131, I.P.C., Guilford, Surrey.

Traub, A.C., (1967), "Stereoscopic Display Using Rapid Varifocal Mirror Oscillations", Applied Optics, VI 1085-1087.

Traub, A.C., (1968), A New 3-Dimensional Display Technique (Bedford, Mass., The MITRE Corporation).

Traub, Alan C. (Feb. 3, 1970), "Three Dimensional Display", U.S. Patent 3,493,290, The Mitre Corporation Assignee, Bedford, Mass.

Valyus, N.A., (1966), Stereoscopy, New York: Focal Press

Vector General, Inc. (1979), Series 3300 Reference Manual, Woodland Hills, California.

Withey, E.L., (May 1958), "Cathode-ray Tube Adds Third Dimension", Electronics, 31: 21.