GEOMETRIC METHODS FOR ANALYSIS OF RIDGES IN N–DIMENSIONAL IMAGES

by

David Howard Eberly

A Dissertation submitted to the faculty of The University of North Carolina at Chapel Hill in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the Department of Computer Science.

Chapel Hill

1994

Approved by:

____________________________ Advisor

____________________________ Reader

____________________________ Reader

____________________________ Reader

____________________________ Reader

DAVID HOWARD EBERLY. Geometric Methods for Analysis of Ridges in N–Dimensional Images (Under the direction of Stephen M. Pizer.)

Abstract

Image segmentation is a process which identifies and labels objects in an image. The goals of this dissertation are to produce an algorithm for segmenting an image in the way that a front-end vision system does, using the local geometry induced by the intensity values of the image, to create multiscale representations of the objects that allow exploration of the details of the image via an interactive computer system, and to provide a formal geometric foundation for multiscale image analysis. The geometric concept of ridges is discussed. These structures are used to describe the shape properties of objects in an image. Various definitions are given for d- dimensional ridge structures of n-dimensional images. Ridges are used in conjunction with multiscale methods to segment an image. The output of the segmentation is a single tree and the objects in the image are represented as unions and differences of subtrees. The tree and image are used as input to a visualization tool which allows the user to explore the image interactively. A formal foundation for multiscale analysis is given which involves non- Euclidean geometry. Metric selection for scale space is naturally related to invariances required for a particular application. The anisotropic diffusion process for generating multiscale data is automatically determined by the metric. Moreover, the metric is used as an aid in developing fast, stable, and adaptive numerical algorithms for solving the nonlinear diffusion equations. This geometric foundation for multiscale analysis provides a natural set of tools for extracting information about objects in an image.

Acknowledgements. I wish to thank my advisor Stephen Pizer and committee member James Coggins for carefully reading the entire dissertation and for giving helpful suggestions. I also wish to thank my other committee members, Robert Gardner, David Beard, and Dinesh Manocha, for carefully reading portions of the dissertation. Many of the ideas presented in this dissertation were clarified through discussions with my colleagues Ross Whitaker, Bryan Morse, and Dan Fritsch. Thanks also go to Charles Kurak, Terry Yoo, and John Rhoades for listening to my ramblings and pointing out where my ideas needed improvement. Clearly the intellectual environment in our research group was an asset in helping me prepare the theoretical concepts of my dissertation. But just as important was the fantastic computer environment provided by the Facilities group headed by Bill Howell; the implementation of the theoretical concepts as computer programs certainly was made easier for me by Facilities support. The computer environment in the Computer Science Department is the best run and most reliable environment I have ever had the pleasure to work in. Finally, I wish to thank the funding agencies that supported me financially. Professor Stephen Pizer provided my regular salary through National Institutes of Health grant NIH P01–CA–47982. The National Science Foundation was kind enough to let me carry over a grant from my previous university. Grant NSF DMS–9003037 provided a summer salary for my first year in the UNC program.

Contents

1 Introduction **1**

1.1 The Need for Ridges in Image Analysis 2

1.2 Object Construction via Multiscale Methods 5

1.3 Scale Space: A Foundation for Image Analysis 6

2 Ridge Definitions **8**

2.1 Ridge Definitions 8

2.2 Directional Derivatives 12

2.3 Height Definition 15

2.3.1 Extreme Points 15

2.3.2 Relative Extreme Points 15

2.3.3 Crease Definitions 16

2.3.4 Examples 17

2.3.5 Invariance Properties 19

2.4 Principal Direction Definition 20

2.4.1 Creases on Graphs 22

2.4.2 Creases on Level Surfaces 23

2.4.3 Graph Examples 23

2.4.4 Level Surface Example 26

2.4.5 Invariance Properties 27

2.5 Level Definition 27

2.5.1 Creases on Level Surfaces 27

2.5.2 1–Dimensional Creases from Mean Curvature 29
2.5.3 Invariance Properties 31
2.6 Nonmetric Definition 32
2.6.1 General Dimensions 32
2.6.2 Invariance Properties 34
2.7 Experiments on Images 34
2.8 Discussion 36

3 Ridge Flow Segmentation **37**
3.1 Introduction 37
3.2 Ridge Flow Models 42
3.2.1 Height Ridges 43
3.2.2 Flow Regions 44
3.3 Ridge–Valley Flow Models 46
3.4 Image Hierarchy 49
3.5 Object Definition and Visualization 52
3.6 Discussion 67

4 The Geometry of Scale Space **68**
4.1 Introduction 68
4.2 The Structure of Scale Space 71
4.2.1 Metric Tensor 71
4.2.2 Christoffel Symbols and Covariant Derivatives 72
4.2.3 Riemann Tensor and Curvature 74
4.2.4 Isometries 74
4.3 Measurements in Scale Space 75
4.3.1 Gradient and Hessian 75
4.3.2 Curves 76
4.3.3 Geodesics 77
4.3.4 Distance Between Points 78
4.3.5 Volume Integrals, Hyperspheres 80

4.3.6 Curvature of Surfaces 81
4.4 Ridges in Scale Space 84
4.4.1 Height Ridges 85
4.4.2 Principal Direction Ridges 89
4.4.3 Level Ridges 91
4.4.4 Invariance Properties 91
4.5 Anisotropic Diffusion as a Consequence of the Metric 92
4.5.1 Linear Diffusion in Euclidean Space 92
4.5.2 Linear Diffusion in Non–Euclidean Space 93
4.5.3 Anisotropic Diffusion 95
4.6 Discussion 98

5 Numerical Issues **101**
5.1 Finite Differences 102
5.1.1 Derivatives of Univariate Functions 102
5.1.2 Derivatives of Multivariate Functions 105
5.2 Gaussian Blurring 106
5.2.1 An Approximation to Diffusion 107
5.2.2 Stability of the Method 109
5.2.3 Bounds on Approximation Error 110
5.2.4 Timing 111
5.3 Solving Eigensystems 112
5.3.1 Eigenstuff for $A\vec{v} = \lambda\vec{v}$ 112
5.3.2 Symbolic Tridiagonalization 116
5.3.3 Eigenstuff for $A\vec{v} = \lambda B\vec{v}$ 123
5.4 Skeletons of Objects 124
5.4.1 Distance Transforms 125
5.4.2 Component Labeling 127
5.4.3 Thinning Algorithm 130

6 Summary **138**

6.1 Ridges 138

6.2 Object Construction 140

6.3 Scale Space 141

6.4 Future Research 142

Bibliography **144**

List of Figures

1.1 Illustration of objects in an image 1

1.2 Edges as ridges of the magnitude of gradient of intensity 3

1.3 Binary object and medial axis, graph of distance function 4

1.4 MR image and graph of its intensity values 4

2.1 Height ridges of x^2y 18

2.2 Graphs of $f(x) = x^2$ and $f(x) = x^4$ 24

2.3 Principal direction ridges of x^2y 25

2.4 Level ridges of x^2y 29

2.5 Nonmetric ridges of x^2y 34

2.6 Ridges of MR head image 35

3.1 MR image, MR intensity surface, and ridges of intensity 42

3.2 Ridge segmentation of gray scale image 47

3.3 Comparison of flow regions 48

3.4 Hierarchy for an image 51

3.5 Hierarchy traversal and coloring 53

3.6 More hierarchy traversal and coloring 54

3.7 Visualization of hierarchy subtrees (ridge flow) 55

3.8 Visualization of hierarchy subtrees (ridge–valley flow) 57

3.9 Magic Crayon Interface with 3D head image 58

3.10 Colored regions after brain stem partially colored 59

3.11 Regions after add–more–all 1 60

3.12 Regions after add–more–all 2 61

3.13 Regions after add–more–all 3 62
3.14 Regions after add–more–all 4 63
3.15 Regions after add–more–all 5 64
3.16 Slices 12–15 after add–more–all 5 65
3.17 Slices 4–7 after add–more–all 5 66

4.1 Geodesic coordinate axes 78
4.2 Geodesics as shortest paths 79
4.3 Objects to be registered 98

5.1 Illustration of component labeling 128
5.2 Original image, marked image after pass 1 133
5.3 Trimmed image (not reset) after pass 3, marked image after pass 2 135
5.4 Trimmed image (not reset) after pass 2, marked image after pass 3 135
5.5 Trimmed image (not reset) after pass 3, marked image after pass 4 136
5.6 Trimmed image (not reset) after pass 4, result of first trimming 136
5.7 Result of first trimming, positive measure pixels removed 137

List of Tables

5.1 Timing for numeric versus symbolic tridiagonalization 117