GEOMETRIC METHODS FOR ANALYSIS OF RIDGES IN N-DIMENSIONAL IMAGES

by

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DAVID HOWARD EBERLY. Geometric Methods for Analysis of Ridges in N-Dimensional Images (Under the direction of Stephen M. Pizer.)

Abstract

Image segmentation is a process which identifies and labels objects in an image. The goals of this dissertation are to produce an algorithm for segmenting an image in the way that a front-end vision system does, using the local geometry induced by the intensity values of the image, to create multiscale representations of the objects that allow exploration of the details of the image via an interactive computer system, and to provide a formal geometric foundation for multiscale image analysis. The geometric concept of ridges is discussed. These structures are used to describe the shape properties of objects in an image. Various definitions are given for d-dimensional ridge structures of n-dimensional images. Ridges are used in conjunction with multiscale methods to segment an image. The output of the segmentation is a single tree and the objects in the image are represented as unions and differences of subtrees. The tree and image are used as input to a visualization tool which allows the user to explore the image interactively. A formal foundation for multiscale analysis is given which involves non-Euclidean geometry. Metric selection for scale space is naturally related to invariances required for a particular application. The anisotropic diffusion process for generating multiscale data is automatically determined by the metric. Moreover, the metric is used as an aid in developing fast, stable, and adaptive numerical algorithms for solving the nonlinear diffusion equations. This geometric foundation for multiscale analysis provides a natural set of tools for extracting information about objects in an image.
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# Contents

1 Introduction  
  1.1 The Need for Ridges in Image Analysis ........................... 2  
  1.2 Object Construction via Multiscale Methods ...................... 5  
  1.3 Scale Space: A Foundation for Image Analysis .................. 6  

2 Ridge Definitions  
  2.1 Ridge Definitions ............................................. 8  
  2.2 Directional Derivatives ........................................... 12  
  2.3 Height Definition .............................................. 15  
    2.3.1 Extreme Points ........................................... 15  
    2.3.2 Relative Extreme Points .................................. 15  
    2.3.3 Crease Definitions ....................................... 16  
    2.3.4 Examples ............................................... 17  
    2.3.5 Invariance Properties ..................................... 19  
  2.4 Principal Direction Definition .................................. 20  
    2.4.1 Creases on Graphs ....................................... 22  
    2.4.2 Creases on Level Surfaces ................................ 23  
    2.4.3 Graph Examples .......................................... 23  
    2.4.4 Level Surface Example ................................... 26  
    2.4.5 Invariance Properties .................................... 27  
  2.5 Level Definition ............................................... 27  
    2.5.1 Creases on Level Surfaces ................................. 27
4.3.6 Curvature of Surfaces ........................................... 81

4.4 Ridges in Scale Space ............................................. 84
4.4.1 Height Ridges ..................................................... 85
4.4.2 Principal Direction Ridges ..................................... 89
4.4.3 Level Ridges ..................................................... 91
4.4.4 Invariance Properties ........................................... 91

4.5 Anisotropic Diffusion as a Consequence of the Metric ................. 92
4.5.1 Linear Diffusion in Euclidean Space ............................. 92
4.5.2 Linear Diffusion in Non-Euclidean Space ...................... 93
4.5.3 Anisotropic Diffusion .......................................... 95

4.6 Discussion .......................................................... 98

5 Numerical Issues .................................................... 101
5.1 Finite Differences ................................................... 102
5.1.1 Derivatives of Univariate Functions ......................... 102
5.1.2 Derivatives of Multivariate Functions ...................... 105
5.2 Gaussian Blurring .................................................. 106
5.2.1 An Approximation to Diffusion ............................... 107
5.2.2 Stability of the Method ........................................ 109
5.2.3 Bounds on Approximation Error ............................. 110
5.2.4 Timing ............................................................ 111
5.3 Solving Eigensystems ............................................... 112
5.3.1 Eigenstuff for \( A \vec{v} = \lambda \vec{v} \) ............................. 112
5.3.2 Symbolic Tridiagonalization .................................. 116
5.3.3 Eigenstuff for \( A \vec{v} = \lambda B \vec{v} \) ............................. 123
5.4 Skeletons of Objects ............................................... 124
5.4.1 Distance Transforms .......................................... 125
5.4.2 Component Labeling ......................................... 127
5.4.3 Thinning Algorithm ......................................... 130
6 Summary

6.1 Ridges ................................................................. 138
6.2 Object Construction ............................................. 140
6.3 Scale Space .......................................................... 141
6.4 Future Research ..................................................... 142

Bibliography .......................................................... 144
List of Figures

1.1 Illustration of objects in an image .......................... 1
1.2 Edges as ridges of the magnitude of gradient of intensity ............ 3
1.3 Binary object and medial axis, graph of distance function ............. 4
1.4 MR image and graph of its intensity values ........................ 4

2.1 Height ridges of $x^2 y$ ........................................ 18
2.2 Graphs of $f(x) = x^2$ and $f(x) = x^4$ ............................. 24
2.3 Principal direction ridges of $x^2 y$ ............................... 25
2.4 Level ridges of $x^2 y$ ........................................... 29
2.5 Nonmetric ridges of $x^2 y$ ....................................... 34
2.6 Ridges of MR head image ......................................... 35

3.1 MR image, MR intensity surface, and ridges of intensity .................. 42
3.2 Ridge segmentation of gray scale image ............................. 47
3.3 Comparison of flow regions ....................................... 48
3.4 Hierarchy for an image .......................................... 51
3.5 Hierarchy traversal and coloring ................................... 53
3.6 More hierarchy traversal and coloring ................................ 54
3.7 Visualization of hierarchy subtrees (ridge flow) ......................... 55
3.8 Visualization of hierarchy subtrees (ridge–valley flow) ................. 57
3.9 Magic Crayon Interface with 3D head image .......................... 58
3.10 Colored regions after brain stem partially colored ..................... 59
3.11 Regions after add–more–all 1 .................................. 60
3.12 Regions after add–more–all 2 .................................. 61
3.13 Regions after add–more–all 3 ........................................ 62
3.14 Regions after add–more–all 4 ........................................ 63
3.15 Regions after add–more–all 5 ........................................ 64
3.16 Slices 12–15 after add–more–all 5 ................................. 65
3.17 Slices 4–7 after add–more–all 5 ................................. 66

4.1 Geodesic coordinate axes .............................................. 78
4.2 Geodesics as shortest paths ........................................... 79
4.3 Objects to be registered .............................................. 98

5.1 Illustration of component labeling ................................. 128
5.2 Original image, marked image after pass 1 ....................... 133
5.3 Trimmed image (not reset) after pass 3, marked image after pass 2 .......................... 135
5.4 Trimmed image (not reset) after pass 2, marked image after pass 3 .......................... 135
5.5 Trimmed image (not reset) after pass 3, marked image after pass 4 .......................... 136
5.6 Trimmed image (not reset) after pass 4, result of first trimming .......................... 136
5.7 Result of first trimming, positive measure pixels removed .......................... 137
List of Tables

5.1 Timing for numeric versus symbolic tridiagonalization .................. 117