

Image Geometry Through Multiscale Statistics

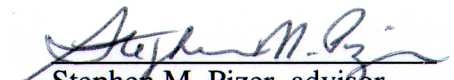
by
Terry Seung-Won Yoo

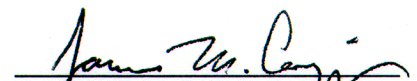
A Dissertation submitted to the faculty of the University of North Carolina at Chapel Hill
in partial fulfillment of the requirements for the degree of Doctor of Philosophy in the
Department of Computer Science.


Chapel Hill

1996

Approved by:


Stephen M. Pizer, advisor


James M. Coggins, reader


J. S. Marron, reader


George D. Stetten, reader

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Abstract

This study in the statistics of scale space begins with an analysis of noise propagation of multiscale differential operators for image analysis. It also presents methods for computing multiscale central moments that characterize the probability distribution of local intensities. Directional operators for sampling oriented local central moments are also computed and principal statistical directions extracted, reflecting local image geometry. These multiscale statistical models are generalized for use with multivalued data.

The absolute error in normalized multiscale differential invariants due to spatially uncorrelated noise is shown to vary non-monotonically across order of differentiation. Instead the absolute error decreases between zeroth and first order measurements and increases thereafter with increasing order of differentiation, remaining less than the initial error until the third or fourth order derivatives are taken.

Statistical invariants given by isotropic and directional sampling operators of varying scale are used to generate local central moments of intensity that capture information about the local probability distribution of intensities at a pixel location under an assumption of piecewise ergodicity. Through canonical analysis of a matrix of second moments, directional sampling provides principal statistical directions that reflect local image geometry, and this allows the removal of biases introduced by image structure. Multiscale image statistics can thus be made invariant to spatial rotation and translation as well as linear functions of intensity.

These new methods provide a principled means for processing multivalued images based on normalization by local covariances. They also provide a basis for choosing control parameters in variable conductance diffusion.

Acknowledgements

I am indebted to many people for the successful completion of this document. I am grateful for the generous support of my advisor, Dr. Stephen M. Pizer, who has been with me throughout the years as mentor, colleague, editor, and friend. I also extend special thanks to Dr. James M. Coggins who has been an unending source of advice.

I thank the other members of my committee, Dr. Jonathan A. Marshall, Dr. George D. Stetten, Dr. Benjamin Tsui, and especially Dr. J. S. Marron.

This research has been funded in part through the National Institutes of Health, grant number P01 CA 47982 and the National Science Foundation, NSF ASC-89-20219.

There are many other people without whom I would never have made it to the end of a successful graduate career. Thanks to Murray Anderegg for many cups of coffee and countless sanity-preserving hands of cribbage. Thanks to David Harrison who added breadth to my education in important ways. Thanks to Dr. Richard L. Holloway for the many needed distractions, his sharing of life's little important things, but most of all for his valued friendship. Thanks to my life friends Dr. Don and Claire Stone, without whom I would never have reached the home stretch. Thanks to Matt Fitzgibbon, Dr. Mary McFarlane and Dr. Greg Turk for being there when I've needed them the most.

During my graduate student tenure, I have had many gifted advisors and mentors. For their friendship and guidance I would like to thank Dr. Henry Fuchs, Dr. Frederick Brooks, Kathy Tesh, and Linda Houseman. For their contributions as colleagues, co-authors, conspirators, and confidants I thank David T. Chen, Marc Olano, Steven Aylward, Rob Katz, Dr. Bryan Morse, and Dr. Ross Whitaker.

Finally, I owe my greatest debts to my family. I thank my parents for life and the strength and determination to live it. Special thanks to my son, Ross, who reminds me daily that miracles exist everywhere around us. Most of all, I thank my beloved wife, Penny, who shares my burdens and my joys. This dissertation is equally her achievement. For me, it is she who makes all things possible.

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List of Symbols

Symbols are listed in order of their appearance in the text.

\mathbb{R}	real numbers
\mathbb{R}^n	n-space of real numbers
\rightarrow	maps onto
p	bold designates vector or tensor quantities
$I(\mathbf{p})$	function I of p
\in	is an element of
\mathbb{N}	natural numbers, (i.e., 0, 1, 2, 3, ...)
\mathbb{N}^n	n-space of natural numbers
\subset	is a subset of
<i>L</i>	italics designate set notation
\vec{V}	a vector field V
\bullet	dot or inner product operator
∇I	gradient of I
θ	Greek lower-case theta, used to designate an angular value
\otimes	the convolution operator
$G(\sigma, \mathbf{p})$	Gaussian with spatial scale σ .
\int	integral
$\frac{\partial^n}{\partial x^n}$	n-th partial derivative with respect to x
Σ	Greek upper-case sigma, without subscripts, denotes a covariance matrix
σ	Greek lower-case sigma, used as the scale parameter
e	the transcendental value e, the natural logarithm
π	Greek lower-case pi, the transcendental value, pi
\mathbf{p}^T	transpose of the tensor value p
$\nabla \bullet F$	“del-dot,” the divergence of F
$I(\mathbf{p} \sigma)$	multiscale measurement: function I(p) at scale σ
\tilde{u}	tilde over a character denotes random variable
μ	Greek lower-case mu, used to designate moments
$\langle \rangle$	the expectation operation
$N_{\mu, \mu^{(2)}}(\tilde{u})$	Standard Normal distribution of \tilde{u} , with mean μ and variance $\mu^{(2)}$.
$\mu^{(n)}$	n-th central moment
$\sum_{j=1}^n f(j)$	summation of $f(0) + f(1) + \dots + f(n)$
i	imaginary value $\sqrt{-1}$
$\overline{M}(u, v)$	the u-v spatial moment of an image function
$L_{x^n y^m}$	multiscale partial derivative $\frac{\partial^n}{\partial x^n} \frac{\partial^m}{\partial y^m} G(\sigma, \mathbf{p}) \otimes I(\mathbf{p})$
$\hat{L}_{x^n y^m}$	normalized multiscale partial derivative $\sigma^{n+m} \frac{\partial^n}{\partial x^n} \frac{\partial^m}{\partial y^m} G(\sigma, \mathbf{p}) \otimes I(\mathbf{p})$
\mathbb{Z}	integers

\forall	for all
$V(\tilde{u})$	variance of \tilde{u}
$M(\tilde{u})$	mean of \tilde{u}
$\text{Cov}(\tilde{u})$	covariance of \tilde{u}
$\prod_{k=1}^n f(k)$	product of $f(0) + f(1) + \dots + f(n)$
\underline{F}	stochastic process F
$f(x) \xrightarrow{a \rightarrow \infty} c$	$f(x)$ approaches c as a goes to infinity
$\lim_{d \rightarrow 0} P(d)$	limit of $P(d)$ as d approaches 0
$\text{erf}(x)$	standard error function: $\text{erf}(x) = \int_{-\infty}^x G(1, \tau) d\tau$
$ \mathbf{p} $	norm of \mathbf{p} : $\mathbf{p} \bullet \mathbf{p}$
\perp	perpendicular to
$\max_{-\infty < x < \infty} (S(x))$	global maximum of $S(x)$ over the interval $-\infty < x < \infty$