1.8(a)

Use the "geometric series" formula:

for $0 < A < 1$, $\sum_{i=0}^{\infty} A^i = \frac{1}{1-A}$

$$\sum_{i=0}^{\infty} \frac{1}{4^i} = \frac{1}{1-\frac{1}{4}} = \frac{4}{3} = 1\tfrac{1}{3}$$ ← ANSWER

1.8 (b)

Let $S = \sum_{i=0}^{\infty} i/4^i$

$$S = \frac{0}{1} + \frac{1}{4} + \frac{2}{4^2} + \frac{3}{4^3} + \dots$$

$$\frac{S}{4} = \frac{0}{4} + \frac{1}{4^2} + \frac{2}{4^3} + \dots$$

$$\Rightarrow \frac{3}{4} S = \underbrace{\frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots}$$

But this summation is the summation in 1.8 (a), divided by 4. Hence,

$$\frac{3}{4} S = \frac{4}{3 \cdot 4}$$

$$\equiv S = \frac{4}{9}$$ ANSWER

1.11 (a)

$F_0 = 1; \; F_1 = 1; \; F_2 = 2; \; F_3 = 3, \; F_4 = 5; \; F_5 = 8;$

$F_N = F_{N-1} + F_{N-2}$

BASE CASE. $N = 3$

$$\sum_{i=1}^{N-2} F_i = F_1 = 1$$

$$F_N - 2 = F_3 - 2 = 3 - 2 = 1$$

INDUCTION HYPOTHESIS: Suppose that the statement holds for $N-1$; ie,

$$\sum_{i=1}^{N-3} F_i = F_{N-1} - 2$$

Consider the LHS for N.

$$\sum_{i=1}^{N-2} F_i = \left(\sum_{i=1}^{N-3} F_i\right) + F_{N-2}$$

(By the Induction Hypothesis)

$$= F_{N-1} - 2 + F_{N-2}$$

$$= F_{N-1} + F_{N-2} - 2$$

$$= F_N - 2 \quad \text{(By definition of } F_N\text{)}$$

$$= \text{RHS}$$

PROVED

1.12 a

$$\sum_{i=1}^{N}(2i-1) = \sum_{i=1}^{N} 2i - \sum_{i=1}^{N} 1 = 2\sum_{i=1}^{N} i - N$$

$$= 2\,\frac{N(N+1)}{2} - N = N^2 + N - N = N^2 = RHS$$

PROVED

1.12 b

$$\sum_{i=1}^{N} i^3 = \left(\sum_{i=1}^{N} i\right)^2$$

PROVE BY INDUCTION

BASE: $N = 1$ — VERIFY!

I.H: Suppose $\sum_{i=1}^{N-1} i^3 = \left(\sum_{i=1}^{N-1} i\right)^2$

Now, $\sum_{i=1}^{N} i^3 = \left(\sum_{i=1}^{N-1} i^3\right) + N^3$

(By the IH)

$$= \left(\sum_{i=1}^{N-1} i\right)^2 + N^3$$

$$= \left(\sum_{i=1}^{N} i - N\right)^2 + N^3$$

(Using: $(a-b)^2 = a^2 - 2ab + b^2$)

$$= \left(\sum_{i=1}^{N} i\right)^2 - 2N.\sum_{i=1}^{N} i + N^2 + N^3$$

$$= \left(\sum_{i=1}^{N} i\right)^2 - 2N.\frac{N(N+1)}{2} + N^2 + N^3$$

$$= \left(\sum_{i=1}^{N} i\right)^2$$

PROVED