

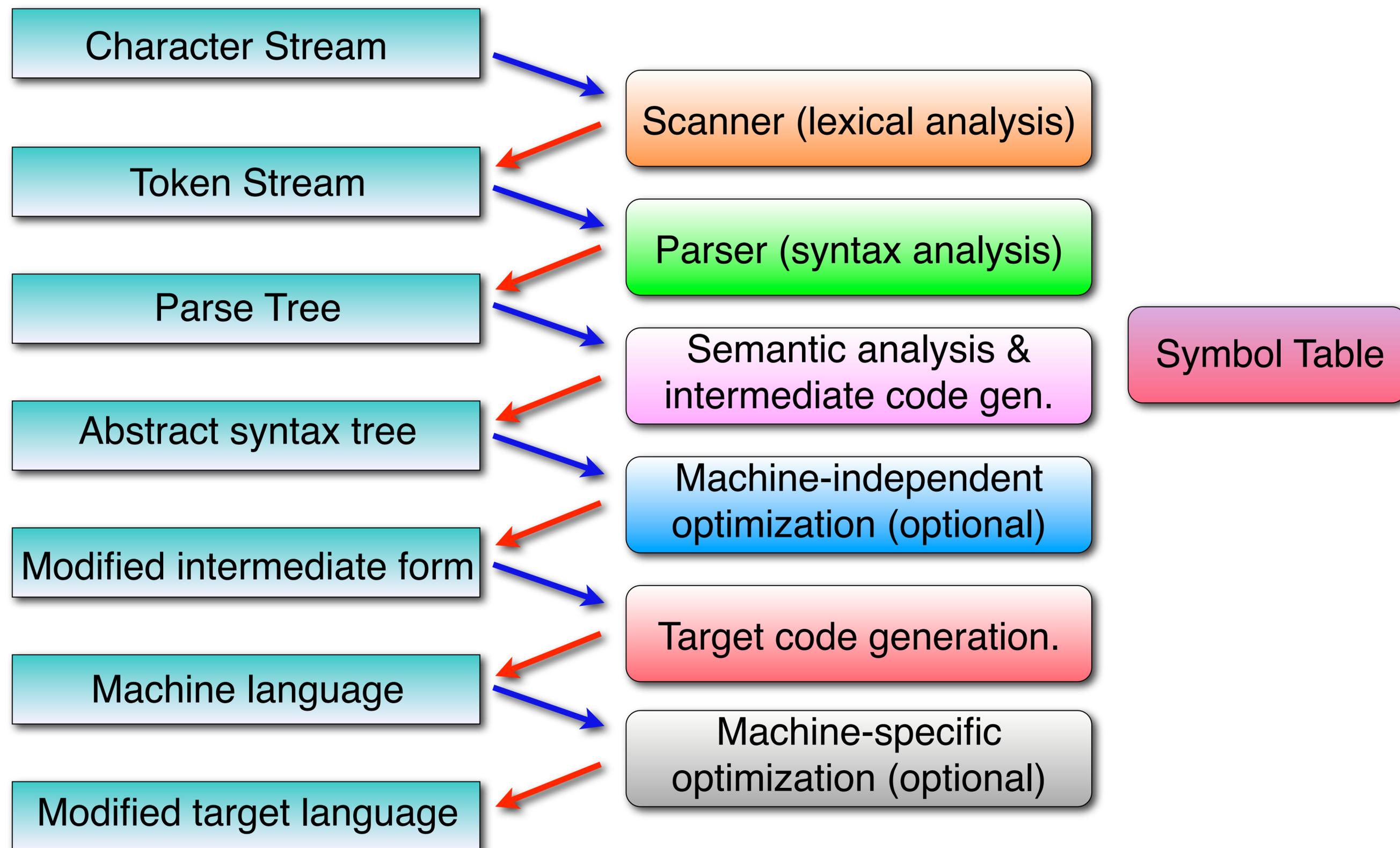
Syntax Analysis



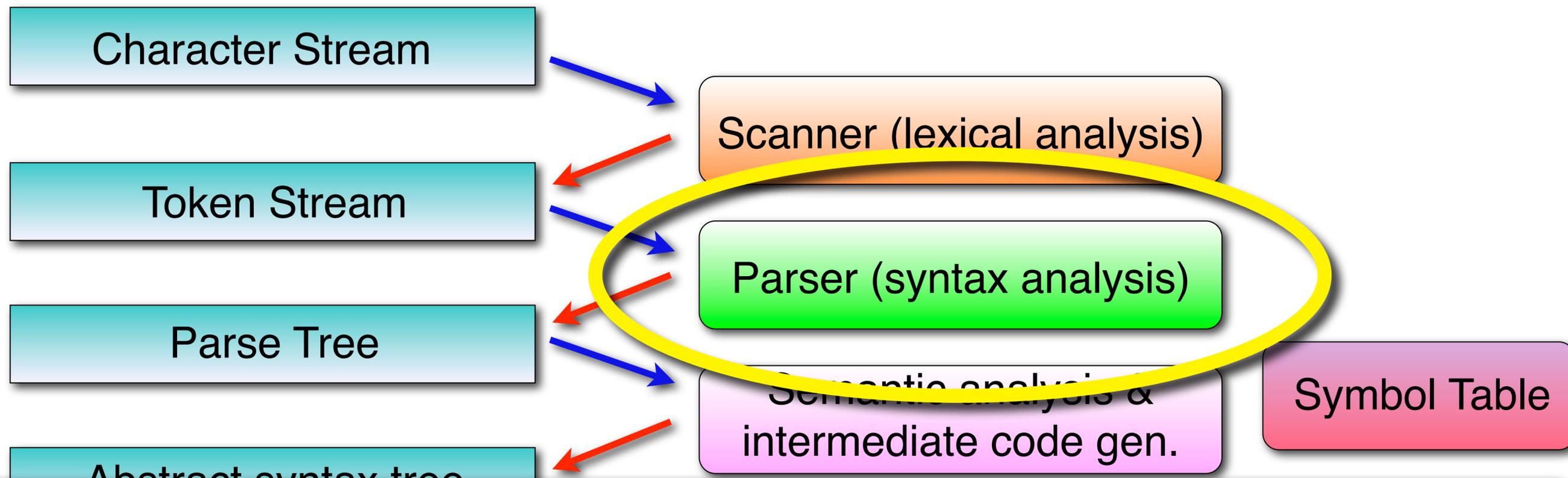
COMP 524: Programming Language Concepts
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The University of North Carolina at Chapel Hill

The Big Picture



The Big Picture



Syntax Analysis: **Discovery of Program Structure**

Turn the stream of individual input tokens into a **complete, hierarchical representation** of the program (or compilation unit).

Syntax Specification and Parsing

Syntax Specification

How can we **succinctly** describe the structure of legal programs?

Syntax Recognition

How can a compiler discover if a program **conforms** to the specification?

Context-free Grammars

LL and LR Parsers

Context-Free Grammars

regular grammar + recursion

Review: grammar.

- Collection of **productions**.
- A production **defines** a non-terminal (on the left, the “**head**”) in terms of a string terminal and non-terminal symbols.
- Terminal symbols are elements of the **alphabet** of the grammar.
- A non-terminal can be the head of **multiple productions**.

Example: Natural Numbers

digit → 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

non_zero_digit → 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

natural_number → *non_zero_digit digit**

Context-Free Grammars

regular grammar + recursion

Regular grammars.

- Restriction: **no unrestricted recursion**.
- A non-terminal symbol cannot be defined in terms of itself.
(except for special cases that equivalent to a Kleene Closure)
- **Serious limitation**: e.g., cannot express matching parenthesis.

Example: Natural Numbers

digit → 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

non_zero_digit → 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9

natural_number → *non_zero_digit digit**

Context-Free Grammars

regular grammar + recursion

Context-free Grammars (**CFGs**) **allow** recursion.

Arithmetic expression with parentheses

$$expr \rightarrow id \mid number \mid '-' expr \mid '(' expr ')' \mid expr op expr$$
$$op \rightarrow '+' \mid '-' \mid '*' \mid '/'$$

Context-Free Grammars

regular grammar + recursion

Recursion

“An **expression** is a a minus sign followed by an **expression**.”

allow recursion.

Arithmetic expression with parentheses

$expr \rightarrow id \mid number \mid '-' expr \mid (' expr ') \mid expr op expr$

$op \rightarrow '+' \mid '-' \mid '*' \mid '/'$

Context-Free Grammars

regular

Context-free Gram

Can express **matching parenthesis** requirement.

Arithmetic expression with parentheses

$expr \rightarrow id \mid number \mid '-' expr \mid '(' expr ') \mid expr op expr$

$op \rightarrow '+' \mid '-' \mid '*' \mid '/'$

Context-Free Grammars

regular grammar + recursion

Key difference to lexical grammar:

terminal symbols are tokens,
not individual characters.

recursion.

Arithmetic expression with parentheses

$expr \rightarrow id \mid number \mid '-' expr \mid '(' expr ') \mid expr op expr$

$op \rightarrow '+' \mid '-' \mid '*' \mid '/'$

Context-Free Grammars

One of the non-terminals, usually the first one, is called the **start symbol**, and it defines the construct defined by the grammar.

on.

Arithmetic expression with parentheses

$expr \rightarrow id \mid number \mid '-' expr \mid '(' expr ') \mid expr op expr$

$op \rightarrow '+' \mid '-' \mid '*' \mid '/'$

BNF vs. EBNF

Backus-Naur-Form (**BNF**)

- Originally developed for ALGOL 58/60 reports.
- Textual notation for context-free grammars.

```
expr → id | number | '-' expr | '(' expr ')' | expr op expr
```

is written as

```
<expr> ::= id | number | - <expr> | ( <expr> ) |  
          <expr> <op> <expr>
```

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<expr> ::= id | number | - <expr> | ( <expr> ) |  
          <expr> <op> <expr>
```

Strictly speaking, it does not include the Kleene Star and similar “notational sugar.”

BNF vs. EBNF

Extended Backus-Naur-Form (**EBNF**)

- Many authors **extend** BNF to simplify grammars.
- One of the first to do so was Niklaus Wirth.
- There exists an **ISO standard for EBNF** (ISO/IEC 14977).
- **But many dialects exist.**

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- There exists an **ISO standard for EBNF** (ISO/IEC 14977).
- **But many dialects exist.**

Features

- Terminal symbols are quoted.
- Use of '=' instead of ': :=' to denote →.
- Use of ',' for concatenation.
- **[A]** means *A* can occur **optionally** (zero or one time).
- **{A}** means *A* can occur repeatedly (**Kleene Star**).
- Parenthesis are allowed for grouping.
- And then some...

BNF vs. EBNF

Extended Backus-Naur-Form (EBNF)

→ Many authors **extend** BNF to simplify grammars.

We will use mostly BNF-like grammars with the addition of the **Kleene Star**, ϵ , and **parenthesis**.

→ Use of '=' instead of '::=' to denote \rightarrow .

→ Use of ',' for concatenation.

→ **[A]** means *A* can occur **optionally** (zero or one time).

→ **{A}** means *A* can occur repeatedly (**Kleene Star**).

→ Parenthesis are allowed for grouping.

→ And then some...

Example: EBNF to BNF Conversion

$$id_list \rightarrow id(, id)^*$$

is equivalent to

$$id_list \rightarrow id$$
$$id_list \rightarrow id_list, id$$

(Remember that non-terminals can be the head of multiple productions.)

Derivation

A grammar allows programs to be **derived**.

→ Productions are **rewriting rules**.

→ A program is **syntactically correct** if and only if it can be derived from the start symbol.

Derivation Process

→ Begin with string consisting only of **start symbol**.

while string contains a non-terminal symbol:

Choose one **non-terminal** symbol **x**.

Choose production where **x** is the head.

Replace **x** with right-hand side of production.

Derivation

If we **always** choose the **left-most non-terminal symbol**, then it is called a **left-most derivation**.

ived.

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Derivation Process

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Derivation

A grammar allows programs to be **derived**.

→ Productions are **rewriting rules**.

→ A program is **sy**
be derived from

If we **always** choose the **right-most non-terminal symbol**, then it is called a **right-most** or **canonical** derivation.

Derivation Process

→ Begin with string

while string contains a non-terminal symbol:
 Choose one **non-terminal** symbol **x**.
 Choose production where **x** is the head.
 Replace **x** with right-hand side of production.

Example Derivation

Arithmetic grammar:

$expr \rightarrow id \mid number \mid '-' expr \mid '(' expr ') \mid expr op expr$

$op \rightarrow '+' \mid '-' \mid '*' \mid '/'$

Program

slope * x + intercept

Example Derivation

Arithmetic grammar:

$expr \rightarrow id \mid number \mid '-' expr \mid '(' expr ') \mid expr \ op \ expr$
 $op \rightarrow '+' \mid '-' \mid '*' \mid '/'$

Program

slope * x + intercept

$expr \Rightarrow expr \ op \ expr$

Example Derivation

Arithmetic grammar:

$expr \rightarrow id \mid number \mid '-' expr \mid '(' expr ') \mid expr op expr$

$op \rightarrow '+' \mid '-' \mid '*' \mid '/'$

Program

slope * x + intercept

\Rightarrow denotes “derived from”

$expr \Rightarrow expr op expr$

Example Derivation

Arithmetic grammar:

expr → **id** number | '-' *expr* | '(' *expr* ')' | *expr* *op* *expr*
op → '+' | '-' | '*' | '/'

Program

slope * x + intercept

expr ⇒ *expr* *op* *expr*

⇒ *expr* *op* id

Example Derivation

Arithmetic grammar:

expr → *id* | *number* | '-' *expr* | '(' *expr* ')' | *expr op expr*

op → '+' | '-' | '*' | '/'

Program

slope * x + intercept

expr ⇒ *expr op expr*

⇒ *expr op id*

⇒ *expr + id*

Example Derivation

Arithmetic grammar:

$expr \rightarrow id \mid number \mid '-' expr \mid '(' expr ') \mid expr op expr$
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Program

slope * x + intercept

$\Rightarrow expr op expr + id$

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Arithmetic grammar:

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Program

slope * x + intercept

$expr \Rightarrow expr op expr$

$\Rightarrow expr op id$

$\Rightarrow expr + id$

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$\Rightarrow expr op id + id$

$\Rightarrow expr * id + id$

Example Derivation

Arithmetic grammar:

$expr \rightarrow id \mid number \mid '-' expr \mid '(' expr ') \mid expr op expr$
 $op \rightarrow '+' \mid '-' \mid '*' \mid '/'$

Program

slope * x + intercept

$expr \Rightarrow expr op expr$

$\Rightarrow expr op id$

$\Rightarrow expr + id$

$\Rightarrow expr op expr + id$

$\Rightarrow expr op id + id$

$\Rightarrow \mathbf{expr} * id + id$

$\Rightarrow id * id + id$

Example Derivation

Arithmetic grammar:

$expr \rightarrow id \mid number \mid '-' expr \mid '(' expr ') \mid expr op expr$

$op \rightarrow '+' \mid '-' \mid '*' \mid '/'$

Substitute **values** of identifier tokens.

Program

slope * x + intercept

$expr \Rightarrow expr op expr$

$\Rightarrow expr op id$

$\Rightarrow expr + id$

$\Rightarrow expr op expr + id$

$\Rightarrow expr op id + id$

$\Rightarrow expr * id + id$

$\Rightarrow id * id + id$

slope * x + intercept

Example Derivation

Arithmetic grammar:

$expr \rightarrow id \mid number \mid '-' expr \mid '(' expr ') \mid expr op expr$

$op \rightarrow '+' \mid '-' \mid '*' \mid '/'$

This is a **right-most** derivation.

Program

slope * x + intercept

$expr \Rightarrow expr op expr$

$\Rightarrow expr op id$

$\Rightarrow expr + id$

$\Rightarrow expr op expr + id$

$\Rightarrow expr op id + id$

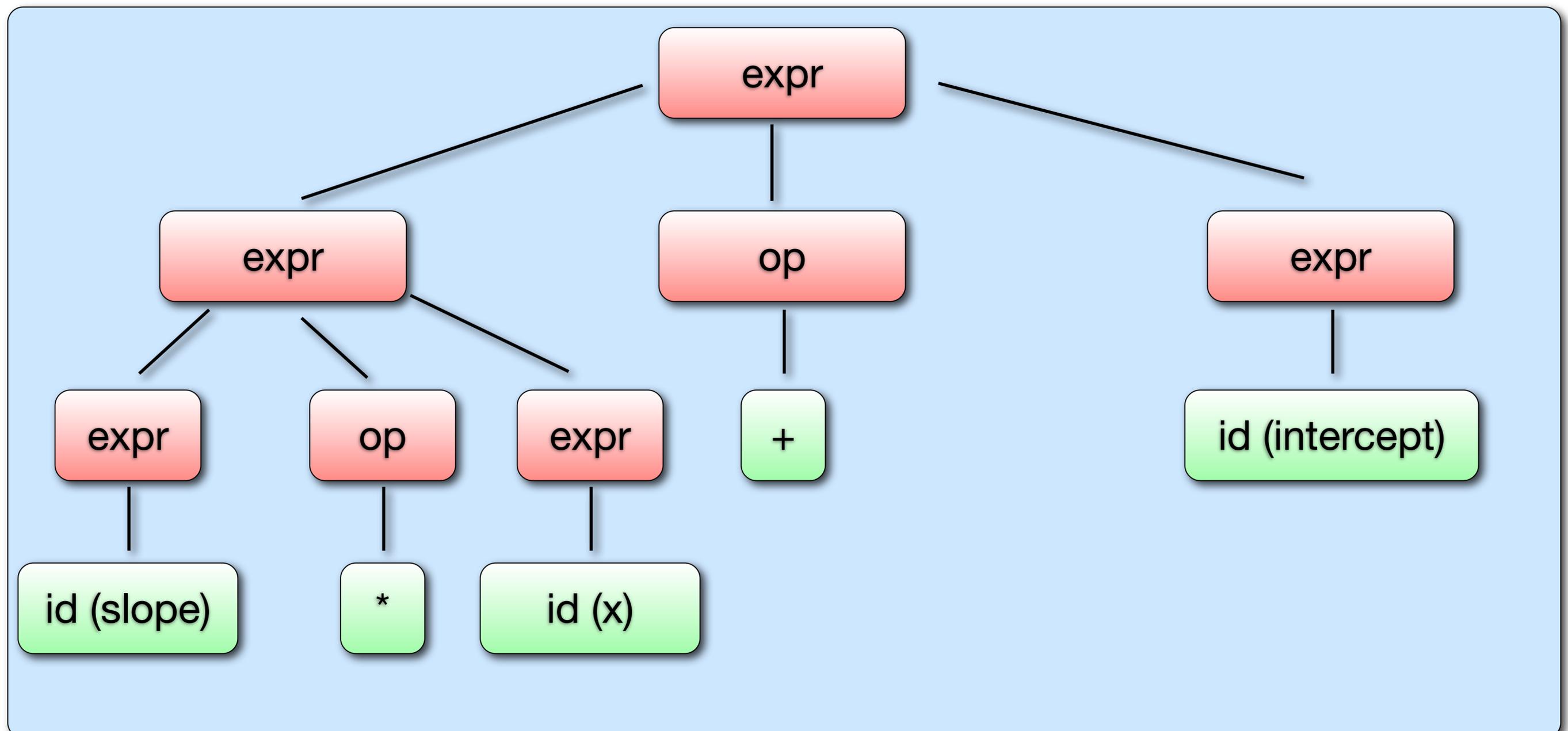
$\Rightarrow expr * id + id$

$\Rightarrow id * id + id$

slope * x + intercept

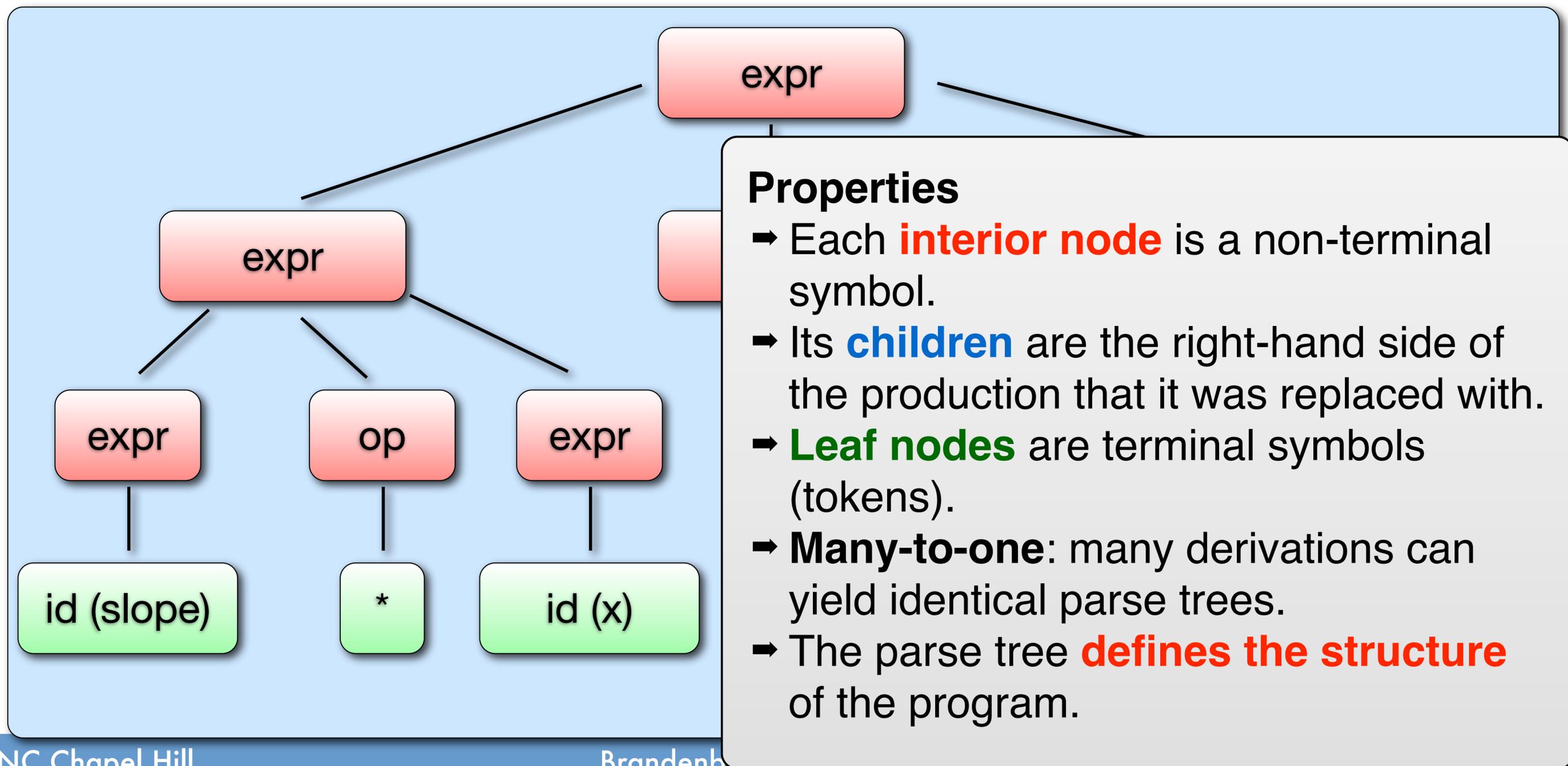
Parse Tree

A parse tree is a **hierarchical representation** of the derivation that does not show the derivation order.



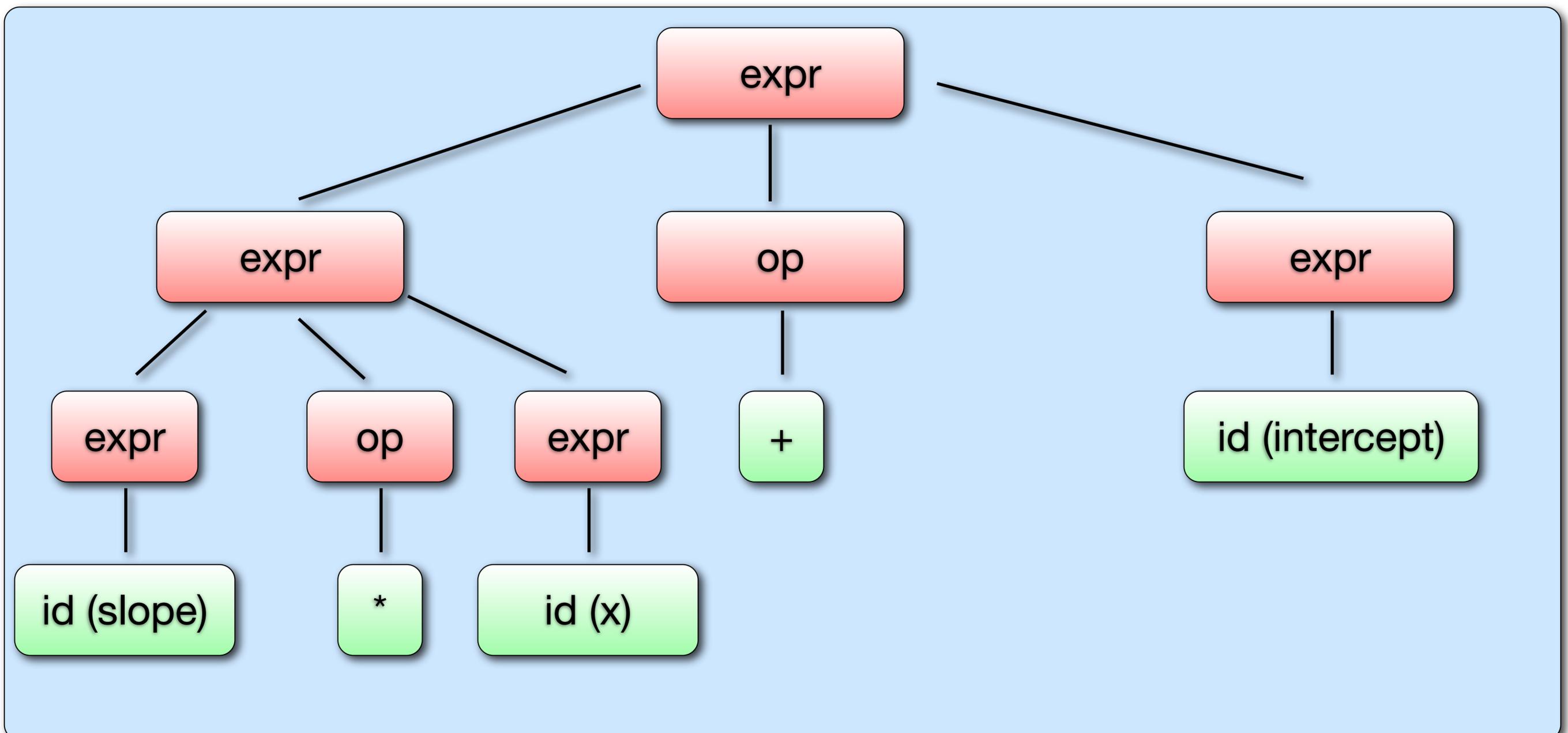
Parse Tree

A parse tree is a **hierarchical representation** of the derivation that does not show the derivation order.



Parse Tree

This parse tree represents the formula
slope * x + intercept.



Parse Tree

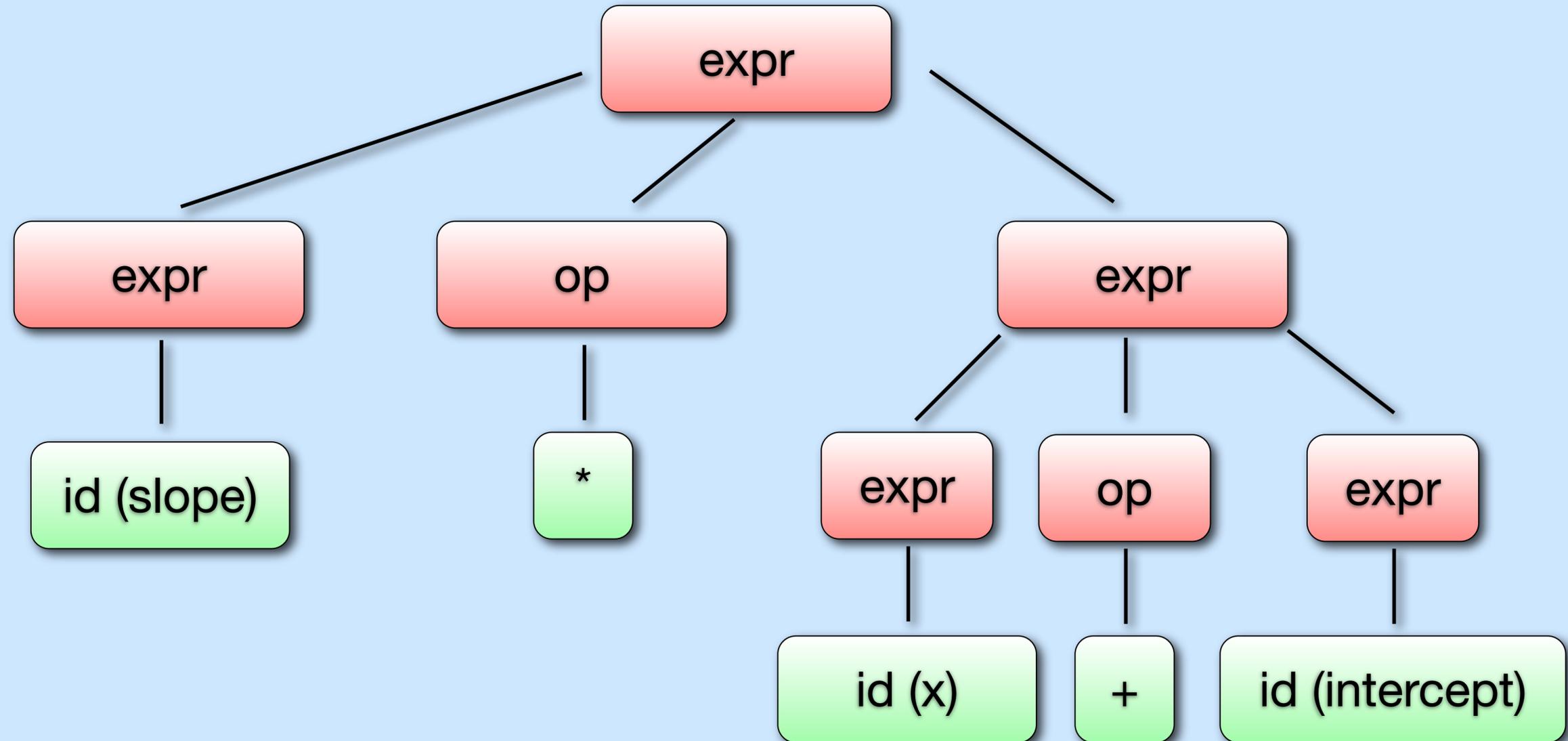
Let's do a **left-most derivation** of
slope * x + intercept.

Arithmetic grammar:

$$\begin{aligned} \text{expr} &\rightarrow \text{id} \mid \text{number} \mid \text{'-'} \text{expr} \mid \text{'(' expr ')'} \mid \text{expr op expr} \\ \text{op} &\rightarrow \text{'+'} \mid \text{'-'} \mid \text{'*'} \mid \text{'/'} \end{aligned}$$

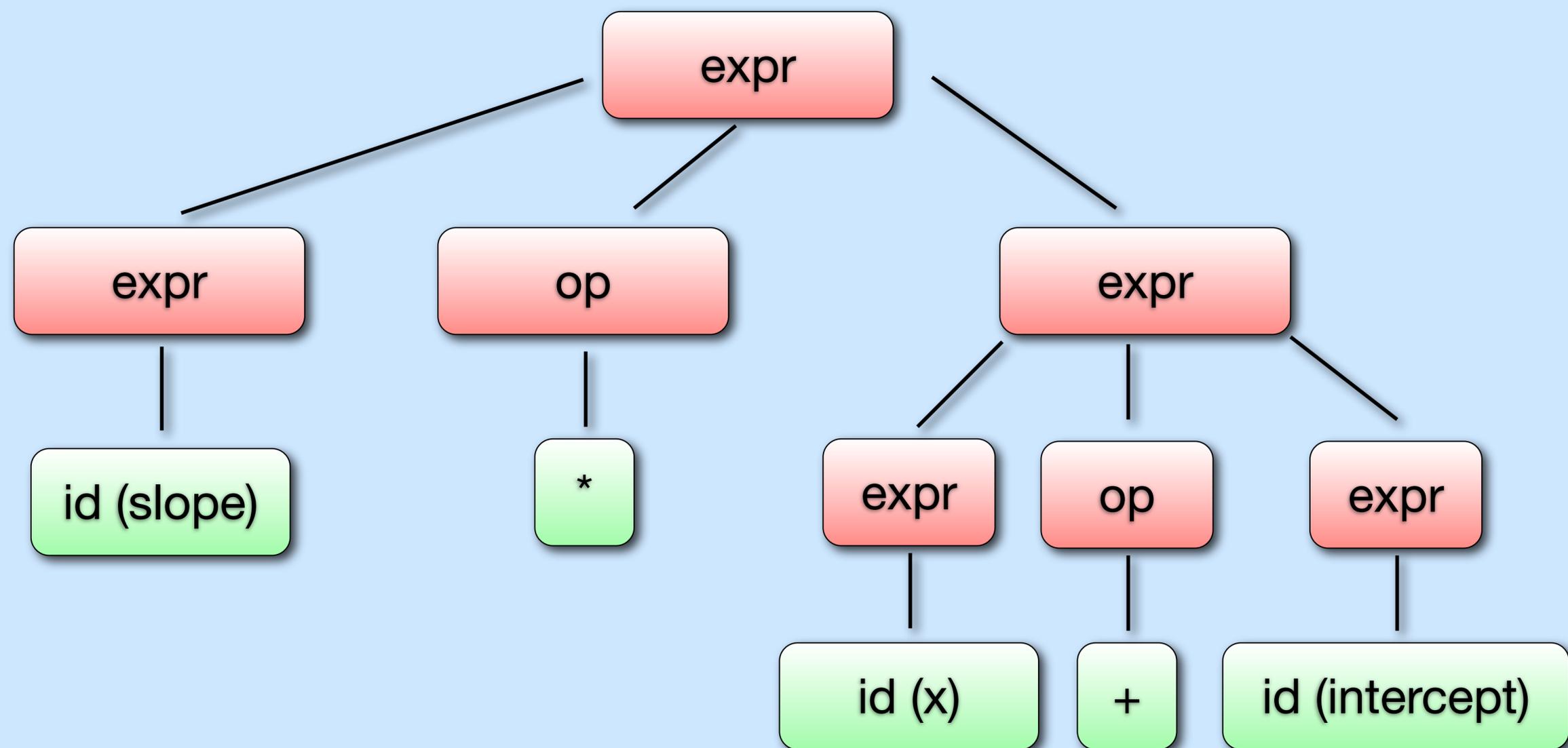
Parse Tree

Let's do a **left-most derivation** of
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Parse Tree (Ambiguous)

This parse tree represents the formula **slope * (x + intercept)**, which is **not equal** to **slope * x + intercept**.

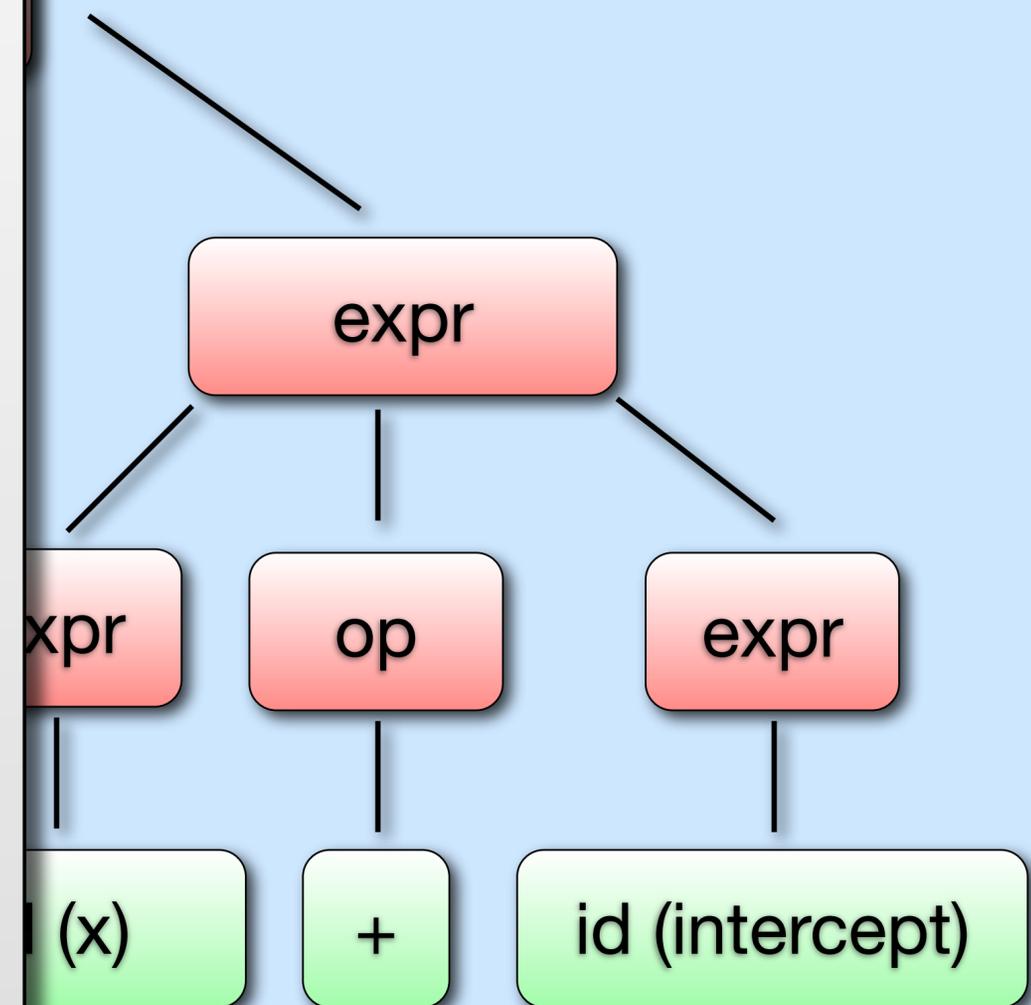


Parse Tree (Ambiguous)

This parse tree represents the formula `slope * (x + intercept)`, which is **not equal** to `slope * x + intercept`.

Ambiguity

- The parse tree **defines the structure** of the program.
- A program should have **only one valid interpretation!**
- Two solutions:
 - Make grammar unambiguous, i.e., ensure that all derivations yield **identical** parse trees.
 - Provide **disambiguating** rules.



Disambiguating the Grammar

- ▶ The problem with our original grammar is that it does **not fully express the grammatical structure** (i.e., associativity and precedence).
- ▶ To create an unambiguous grammar, we need to fully specify the grammar and differentiate between **terms** and **factors**.

Disambiguating the Grammar

- ▶ The problem with our original grammar is that it does **not fully express the grammatical structure** (i.e., associativity and precedence).
- ▶ To create an unambiguous grammar, we need to fully specify the grammar and differentiate between **terms** and **factors**.

$$\text{expr} \rightarrow \text{term} \mid \text{expr add_op term}$$
$$\text{term} \rightarrow \text{factor} \mid \text{term mult_op factor}$$
$$\text{factor} \rightarrow \text{id} \mid \text{number} \mid - \text{factor} \mid (\text{expr})$$
$$\text{add_op} \rightarrow + \mid -$$
$$\text{mult_op} \rightarrow * \mid /$$

Disambiguating the Grammar

- ▶ The problem with our original grammar is that it does **not fully express the grammatical structure** (i.e., associativity and precedence).
- ▶ To create a grammar that specifies the structure and differentiates between **terms** and **factors**.

This gives **precedence** to multiply.

$expr \rightarrow term \mid expr \text{ add_op } term$

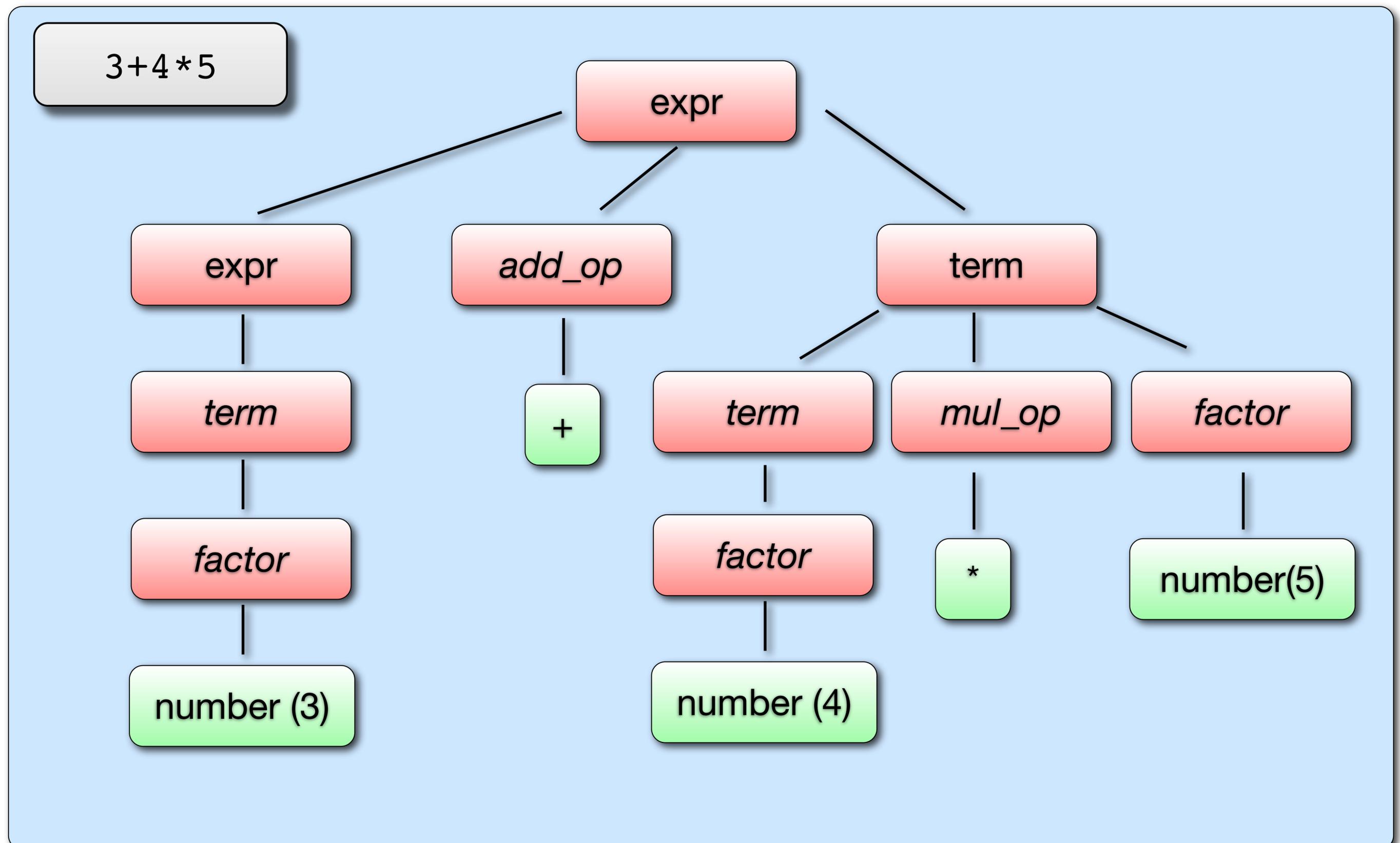
$term \rightarrow factor \mid term \text{ mult_op } factor$

$factor \rightarrow id \mid number \mid - factor \mid (expr)$

$add_op \rightarrow + \mid -$

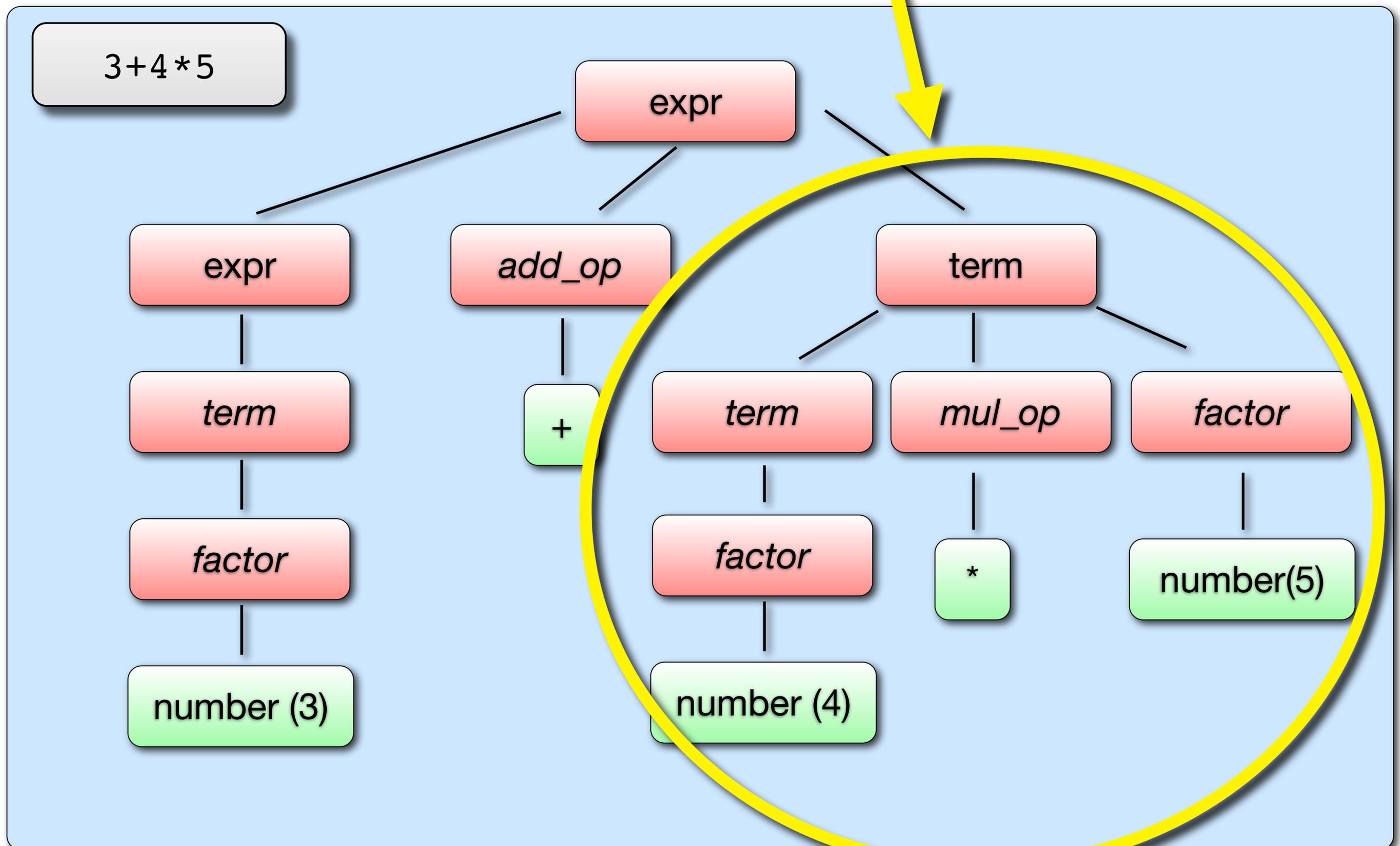
$mult_op \rightarrow * \mid /$

Example Parse Tree



Ex

Multiplication precedes addition.



Another Example

Lets try deriving “3*4+5*6+7”.

$expr \rightarrow term \mid expr \text{ add_op } term$

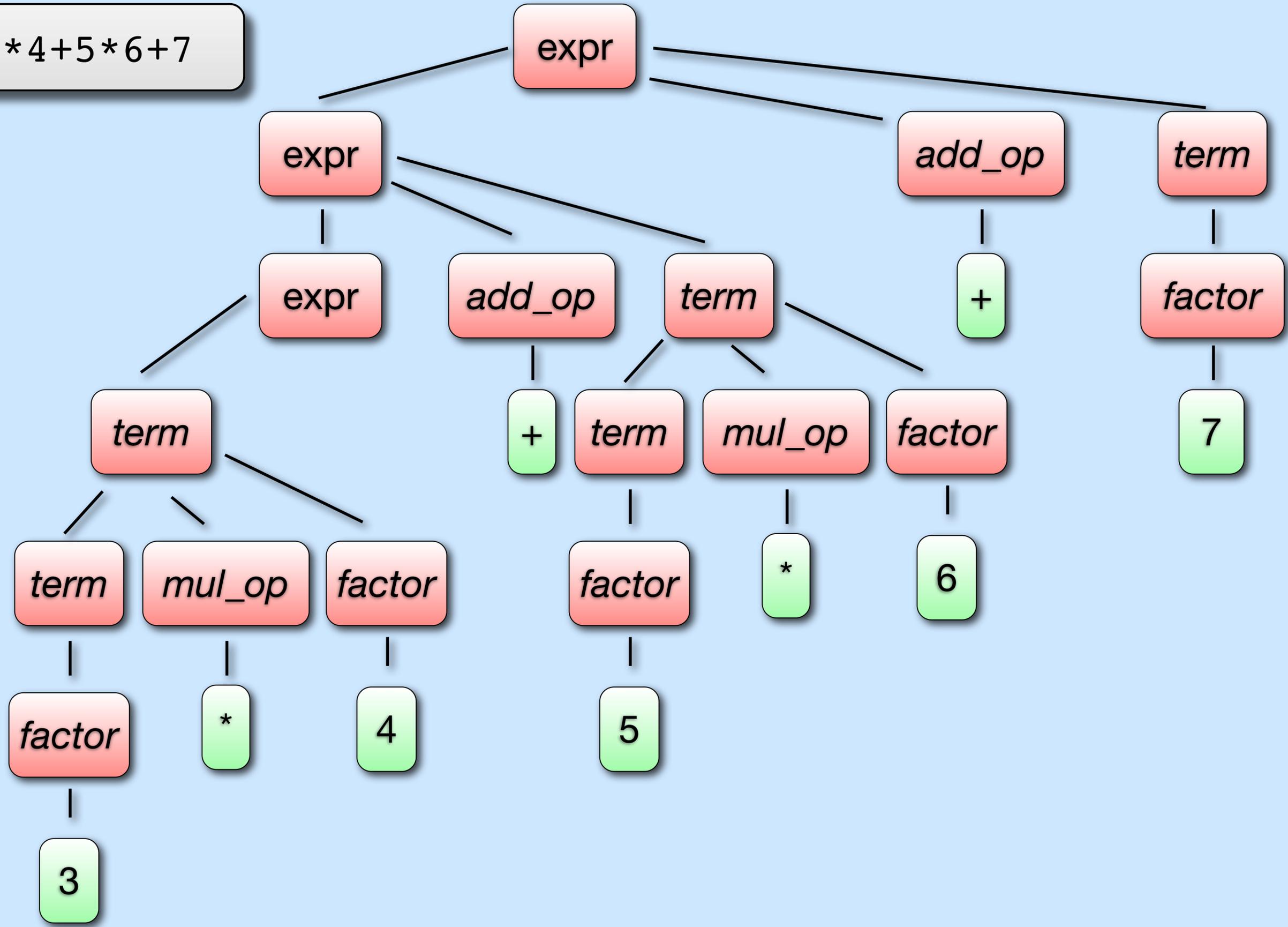
$term \rightarrow factor \mid term \text{ mult_op } factor$

$factor \rightarrow id \mid number \mid - factor \mid (expr)$

$add_op \rightarrow + \mid -$

$mult_op \rightarrow * \mid /$

3 * 4 + 5 * 6 + 7



Parser

The purpose of the parser is to
construct the parse tree
that
corresponds to the input token stream.

(If such a tree exists, i.e., for correct input.)

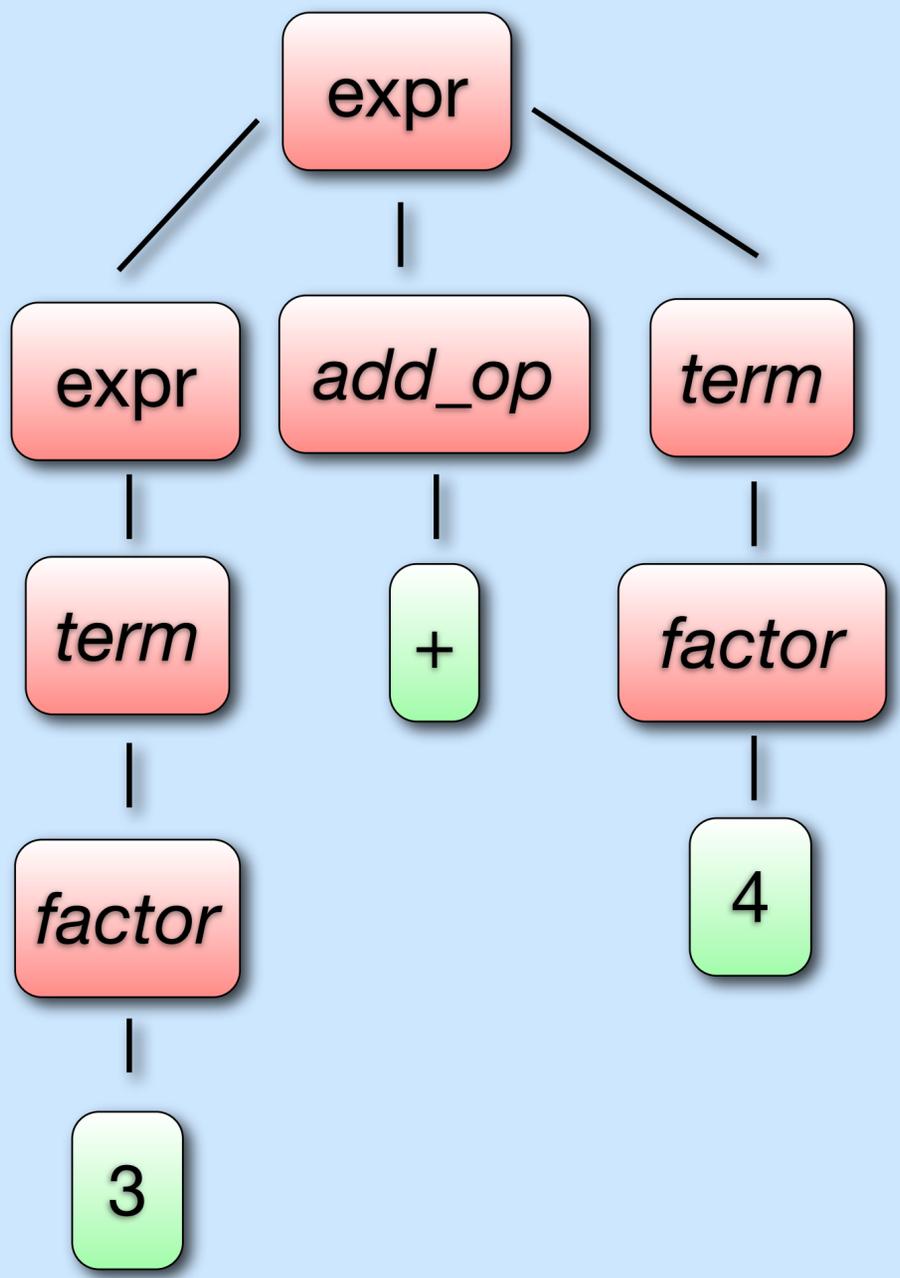
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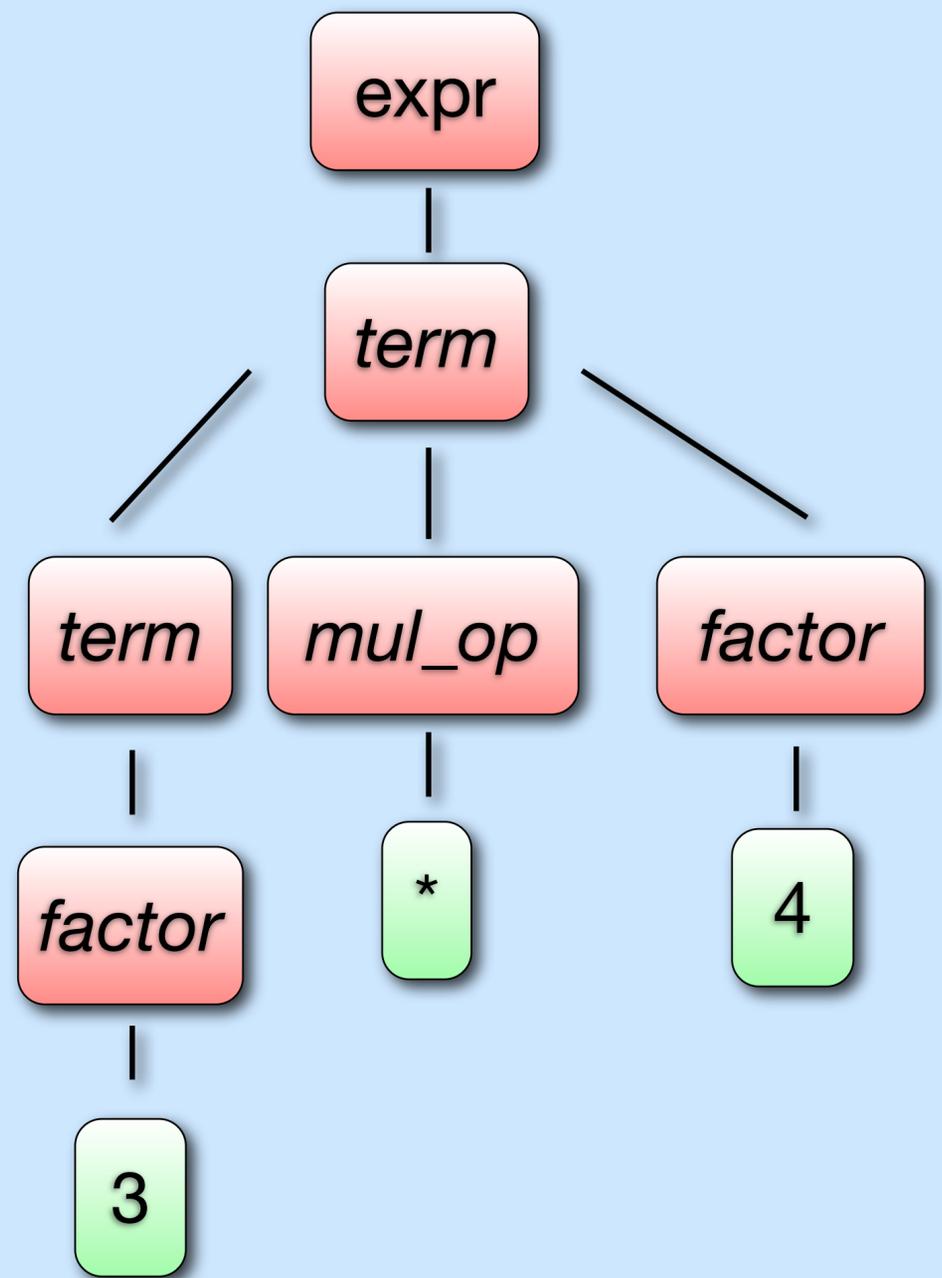
(If such a tree exists, i.e., for correct input.)

This is a **non-trivial problem**:
for example, consider “ $3 * 4$ ” and “ $3 + 4$ ”.

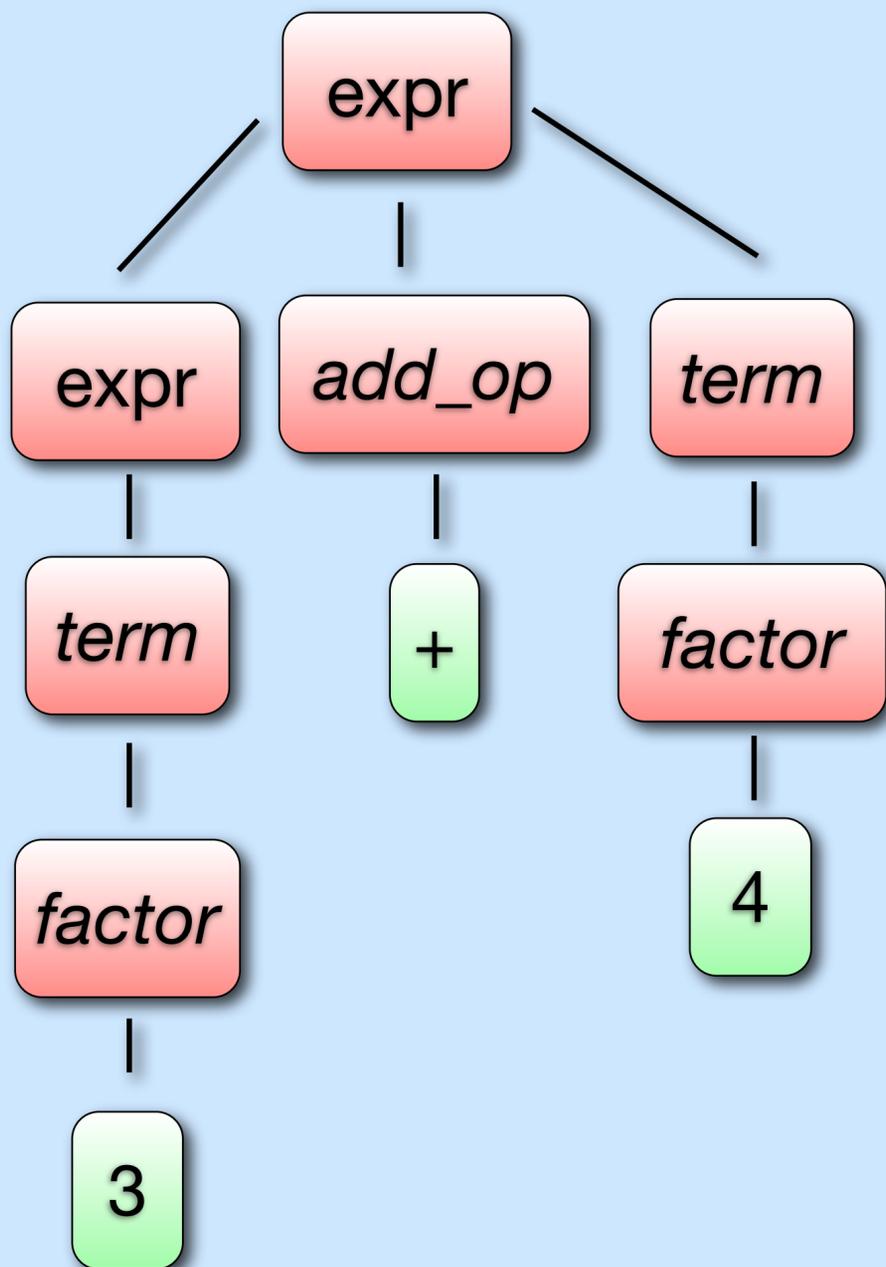
3+4



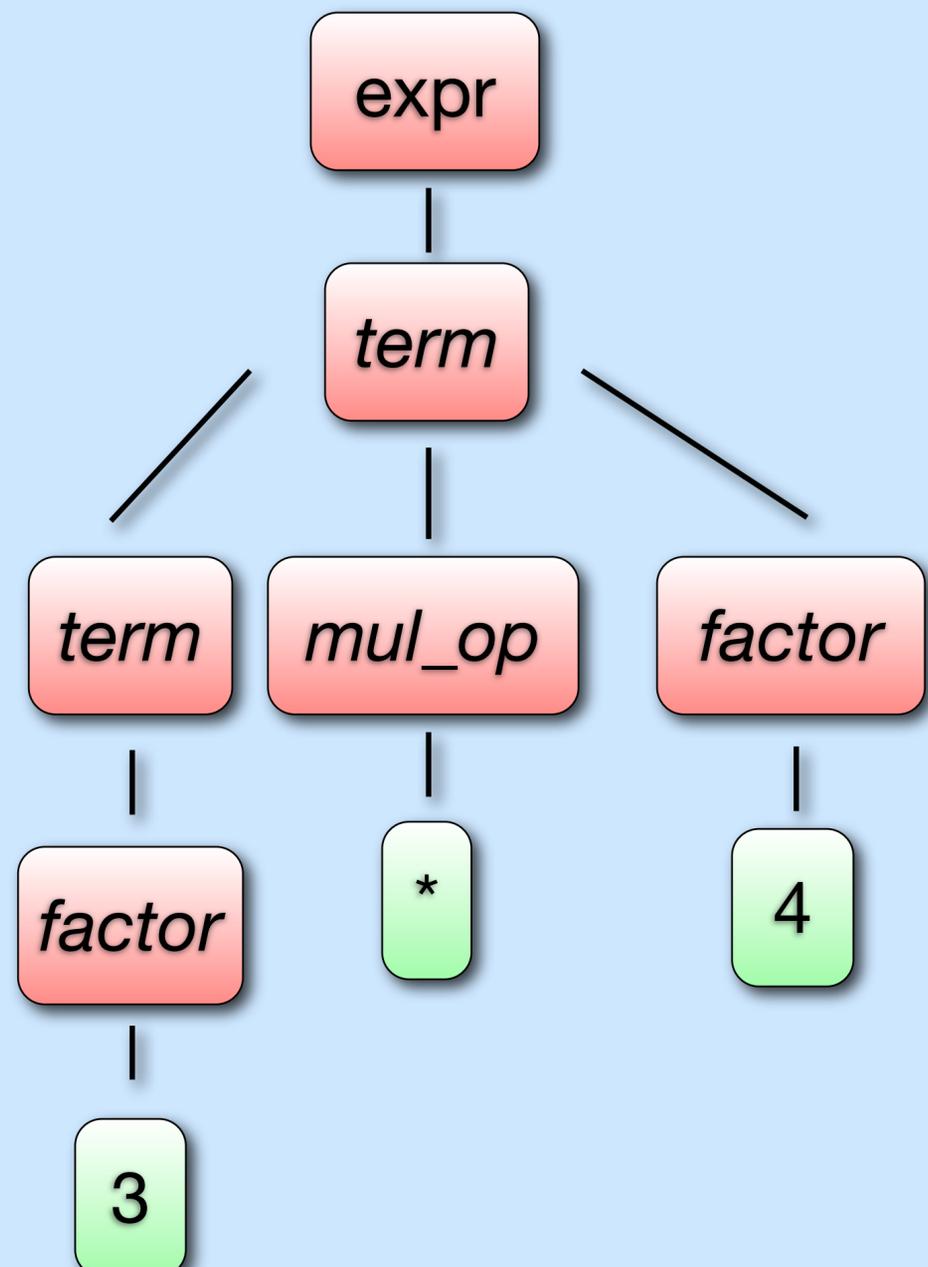
3*4



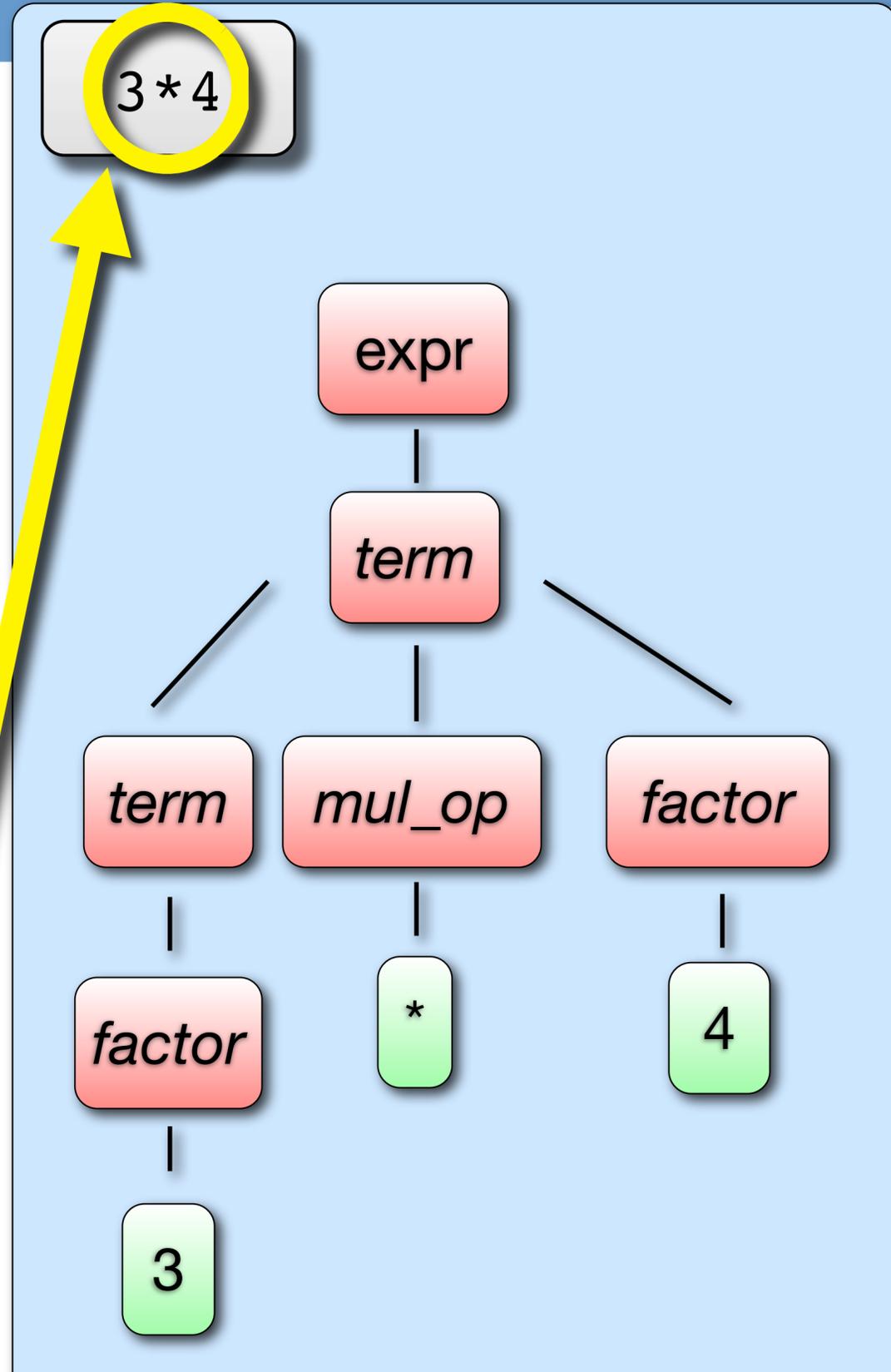
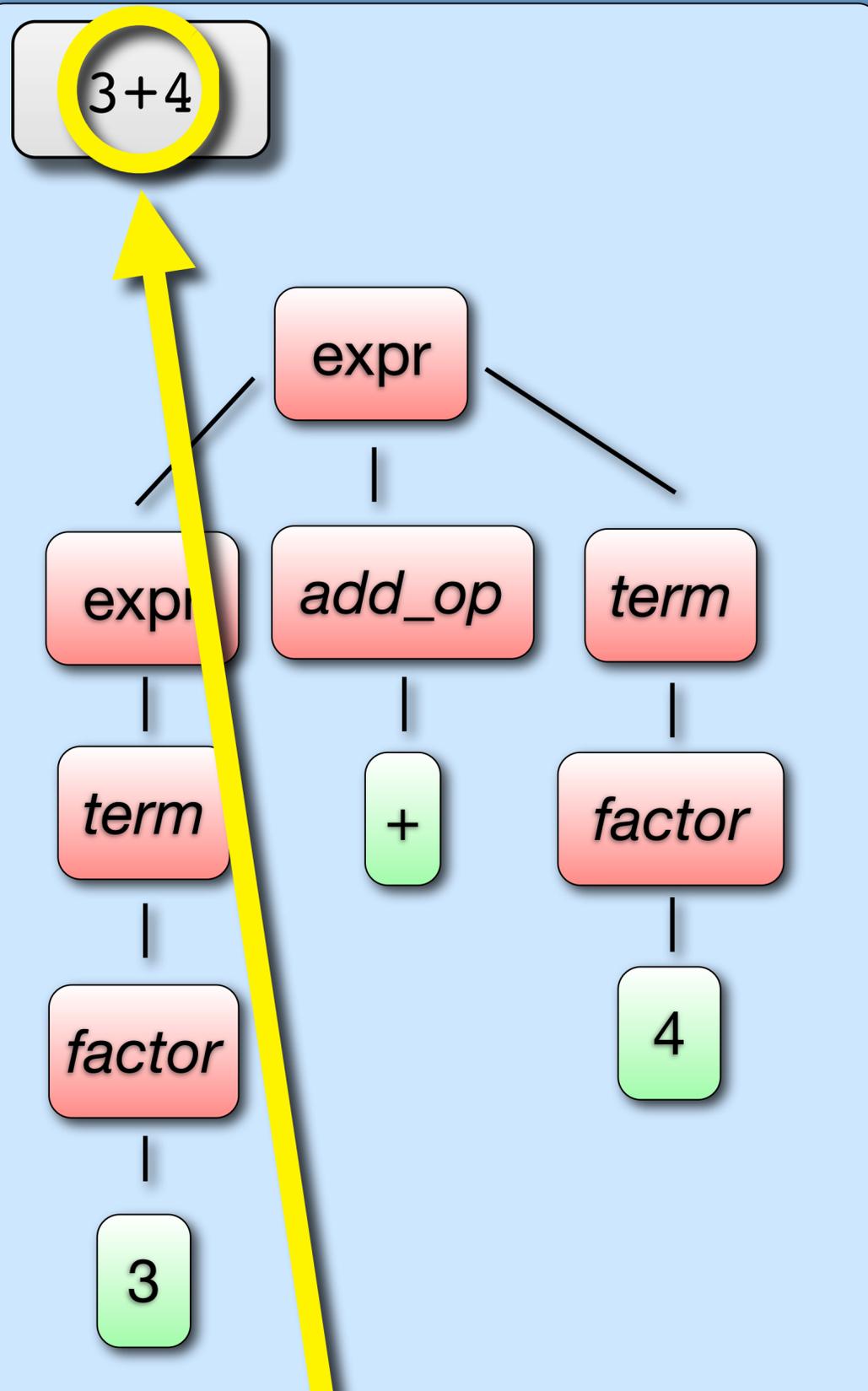
3+4



3*4

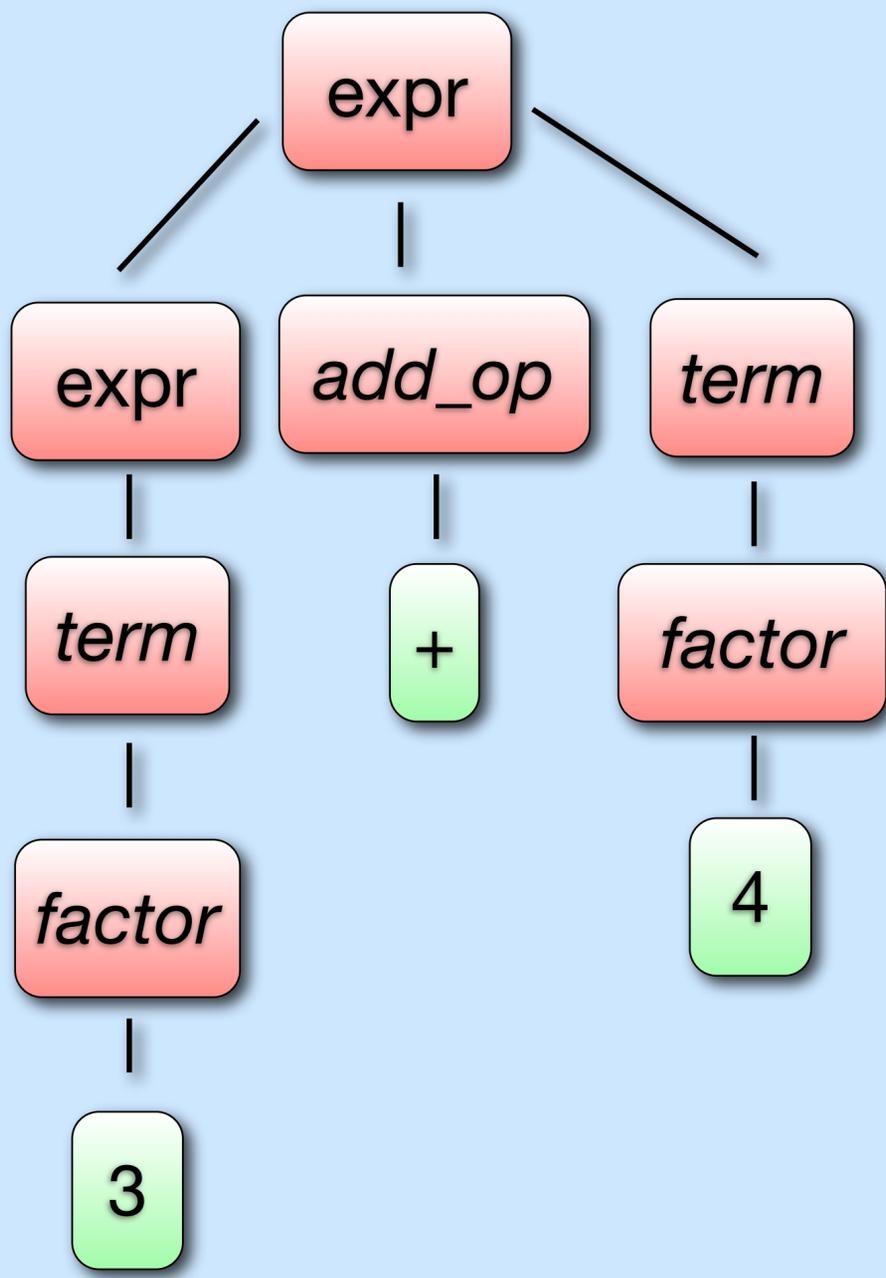


How can a computer derive these trees by examining **one token at a time**?

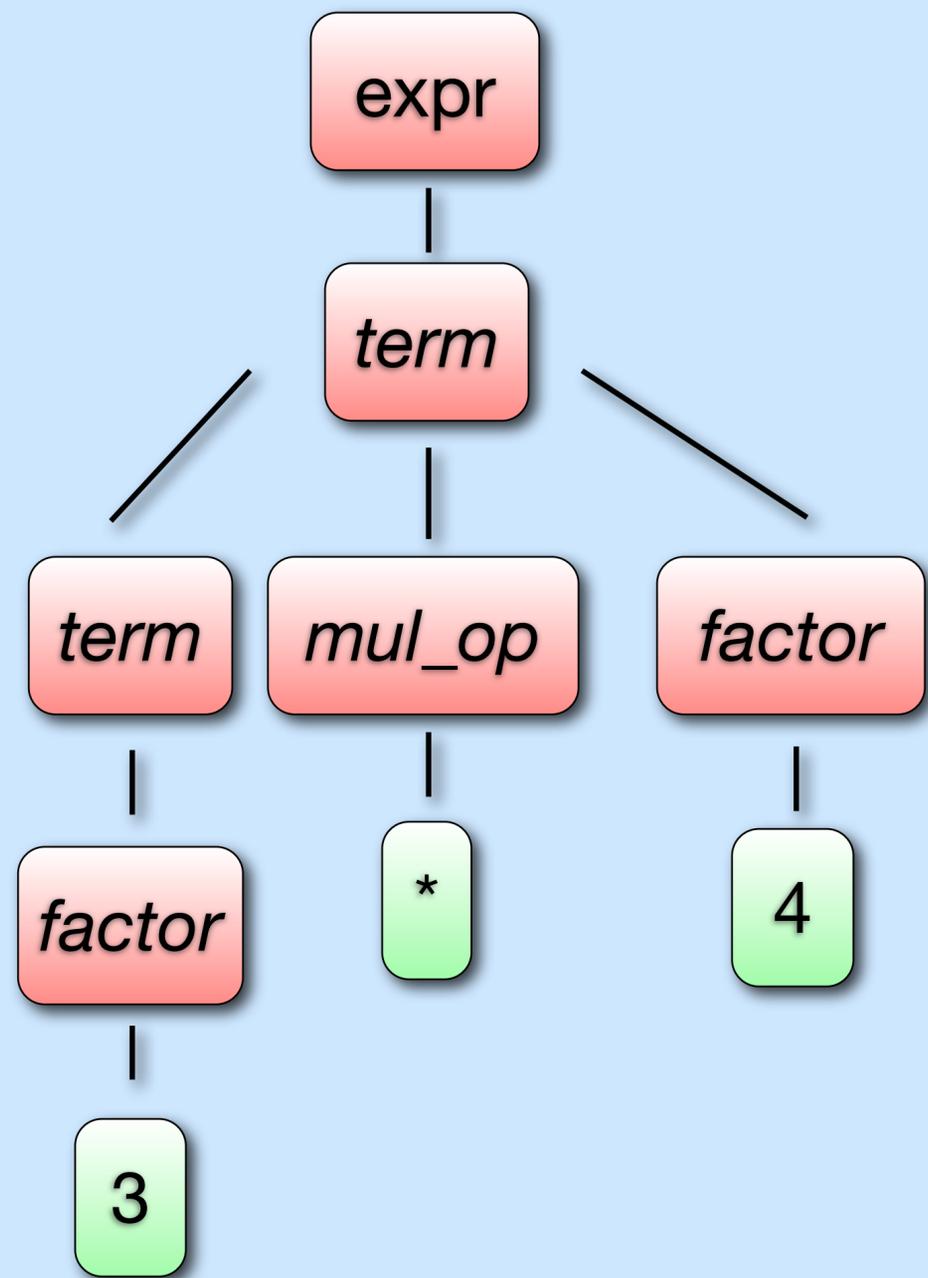


In order to derive these trees, the **first character** that we need to examine is the math operator **in the middle**.

3+4



3*4



Writing **ad-hoc parsers** is difficult, tedious, and error-prone.

Complexity of Parsing

Arbitrary CFGs can be parsed in **$O(n^3)$** time.

- **n** is length of the program (in tokens).
- Earley's algorithm.
- Cocke-Younger-Kasami (CYK) algorithm.
- This is **too inefficient** for most purposes.

Efficient parsing is possible.

- There are (restricted) types of grammars that can be parsed in **linear time**, i.e., **$O(n)$** .
- Two important classes:
 - **LL**: “Left-to-right, Left-most derivation”
 - **LR**: “Left-to-right, Right-most derivation”
- These are **sufficient** to express most programming languages.

Complexity of Parsing

The class of all grammars for which a **left-most** derivation always yields a parse tree.

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Complexity of Parsing

Arbitrary CFGs can be parsed in **$O(n^3)$** time.

→ **n** is length of the program (in tokens).

The class of all grammars for which a **right-most** derivation always yields a parse tree.

Efficient parsing is possible.

→ There are (restricted) types of grammars that can be parsed in **linear time**, i.e., **$O(n)$** .

→ Two important classes:

▶ **LL**: “Left-to-right, Left-most derivation”

▶ **LR**: “Left-to-right, Right-most derivation”

→ These are **sufficient** to express most programming languages.

LL-Parsers vs. LR-Parsers

LL-Parsers

- Find **left-most** derivation.
- Create parse-tree in **top-down** order, **beginning at the root**.
- Can be either **constructed manually** or automatically generated with tools.
- Easy to understand.
- LL grammars sometimes appear “**unnatural**.”
- Also called **predictive** parsers.

LR-Parsers

- Find **right-most** derivation.
- Create parse-tree in **bottom-up** order, **beginning at leaves**.
- Are usually **generated by tools**.
- Operating is less intuitive.
- LR grammars are often “**natural**.”
- Also called **shift-reduce** parsers.
- **Strictly more expressive**: every LL grammar is also an LR grammar, but the converse is not true.

Both are used in practice.
We focus on LL.

LL vs. LR Example

A simple grammar for a list of identifiers.

$id_list \rightarrow id\ id_list_tail$

$id_list_tail \rightarrow ,\ id\ id_list_tail$

$id_list_tail \rightarrow ;$

Input

A, B, C;

LL Example

the
(as of yet empty)
parse tree

$id_list \rightarrow id\ id_list_tail$

$id_list_tail \rightarrow ,\ id\ id_list_tail$

$id_list_tail \rightarrow ;$

A , B , C ;



current token

LL Example

id_list

id_list → *id id_list_tail*

id_list_tail → *, id id_list_tail*

id_list_tail → *;*

A , B , C ;

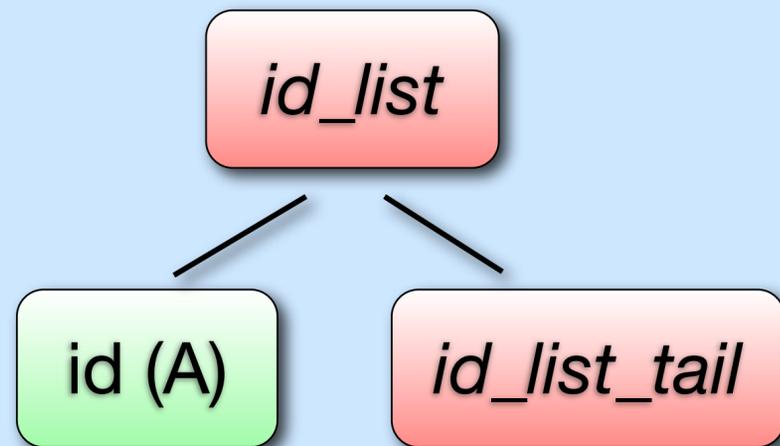


current token

Step 1

Begin with **root** (start symbol).

LL Example



$$id_list \rightarrow id\ id_list_tail$$

$$id_list_tail \rightarrow ,\ id\ id_list_tail$$

$$id_list_tail \rightarrow ;$$

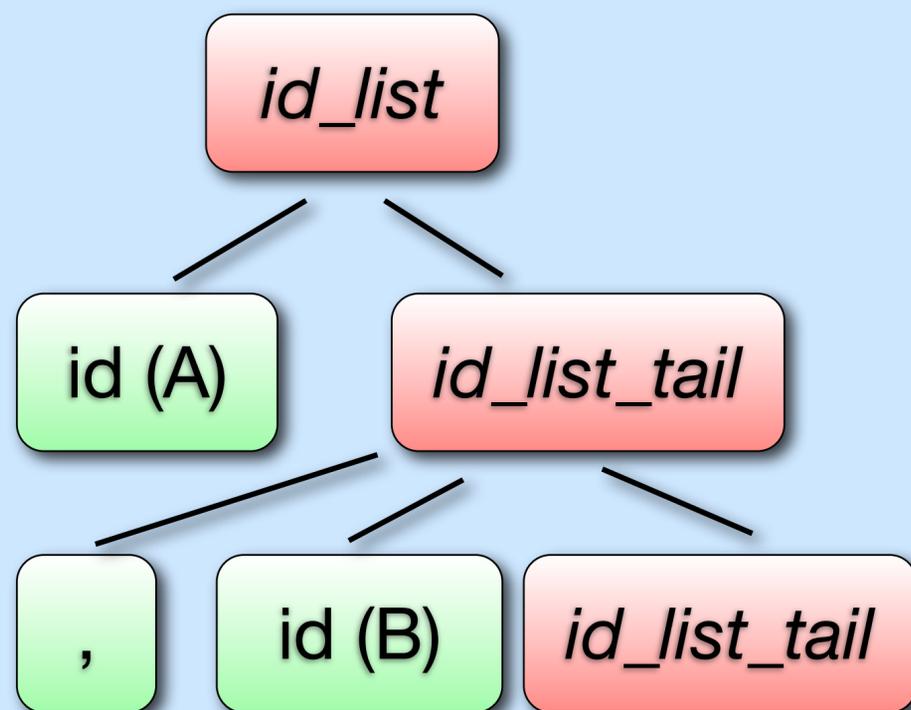
A , B , C ;

current token

Step 2

Apply *id_list* production. This **matches** the first identifier to the **expected id token**.

LL Example



$$id_list \rightarrow id\ id_list_tail$$

$$id_list_tail \rightarrow ,\ id\ id_list_tail$$

$$id_list_tail \rightarrow ;$$

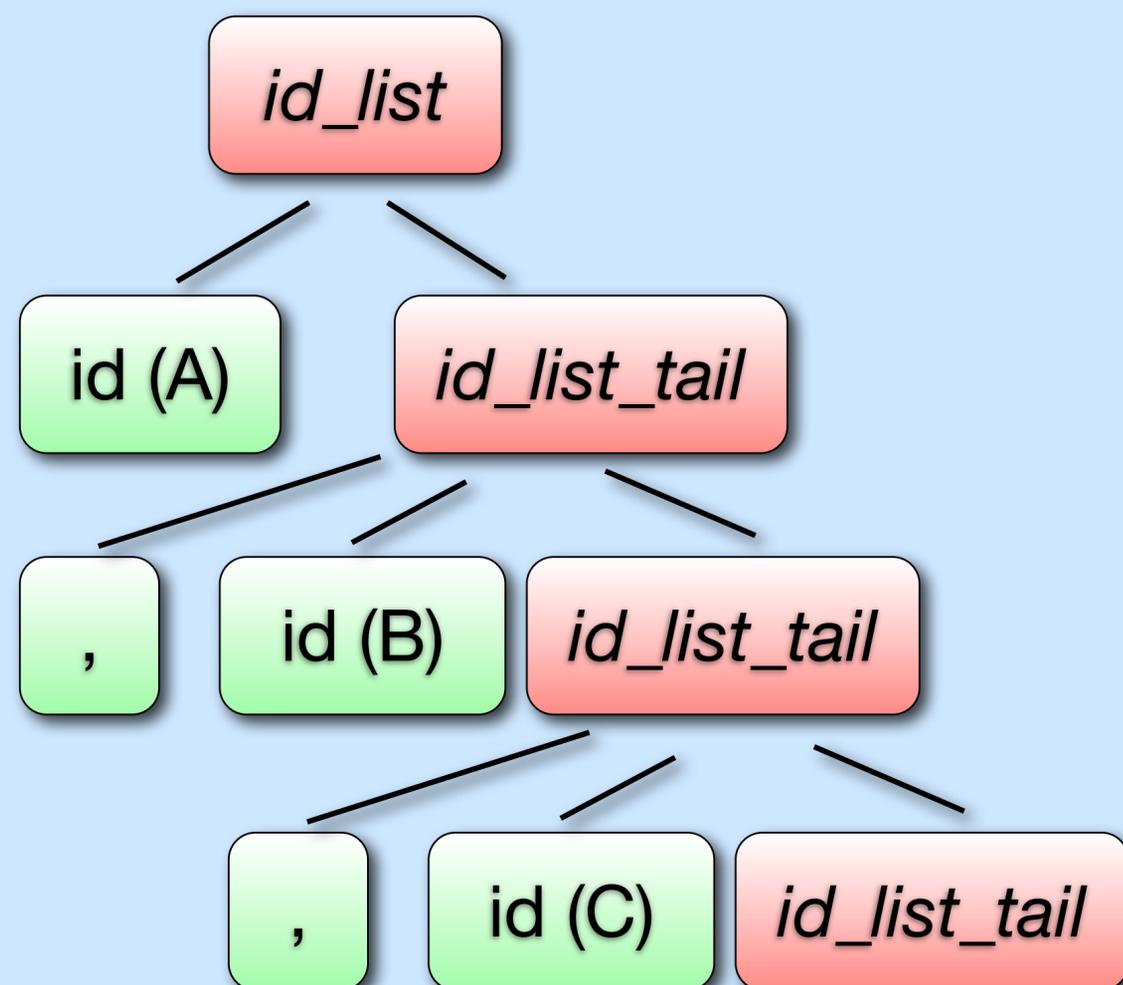
A , B , C ;

current token

Step 3

Apply a production for `id_list_tail`.
 There are two to choose from.
Predict that the first one applies.
 This matches two more tokens.

LL Example



$$id_list \rightarrow id\ id_list_tail$$

$$id_list_tail \rightarrow ,\ id\ id_list_tail$$

$$id_list_tail \rightarrow ;$$

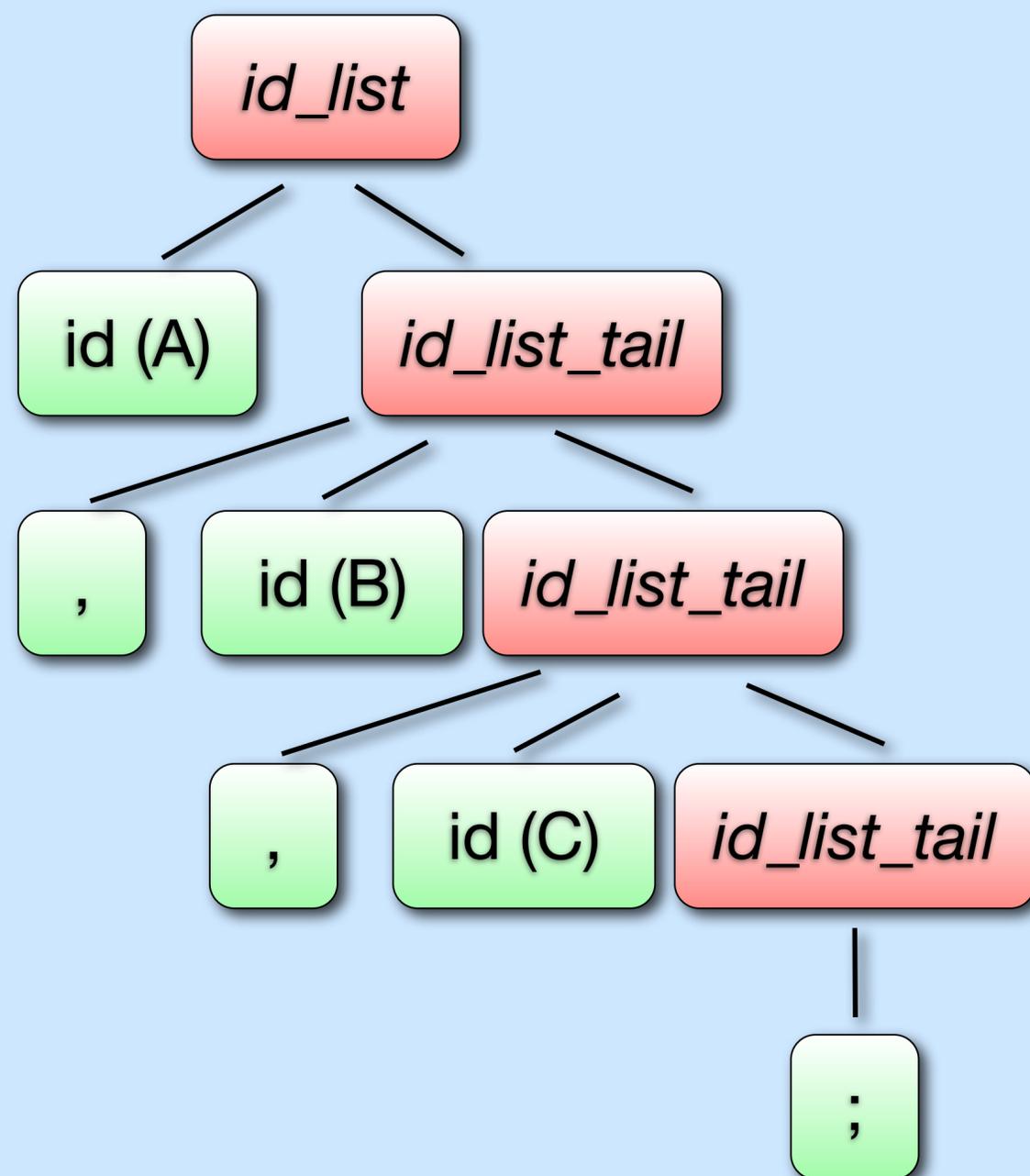
A , B , C ;

current token

Step 4

Substitute the `id_list_tail`, **predicting** the first production again. This matches a comma and c.

LL Example



$$id_list \rightarrow id\ id_list_tail$$

$$id_list_tail \rightarrow ,\ id\ id_list_tail$$

$$id_list_tail \rightarrow ;$$

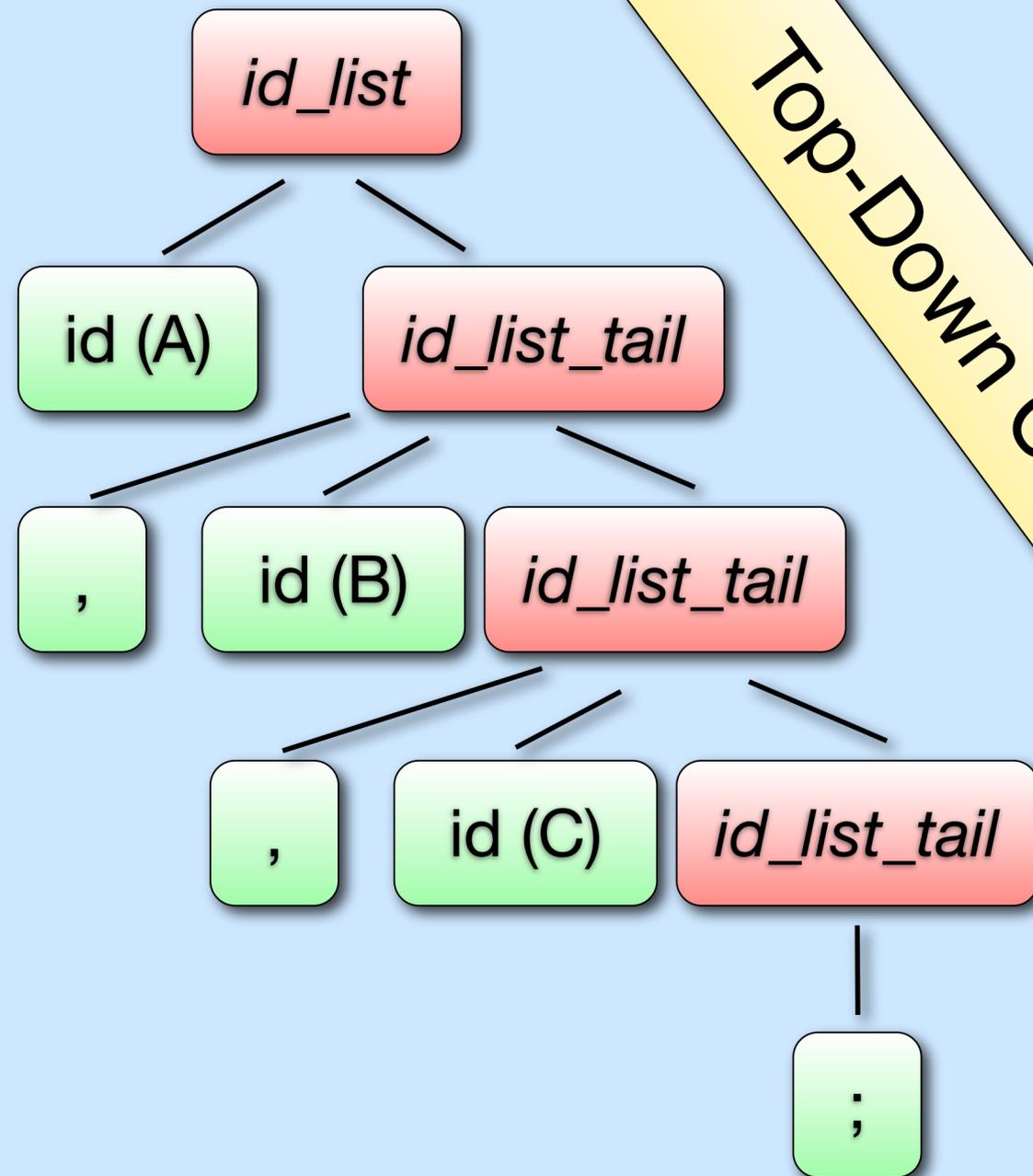
A , B , C ;

current token

Step 5

Substitute the final `id_list_tail`. This time **predict the other production**, which matches the ‘;’.

LL Parse Tree



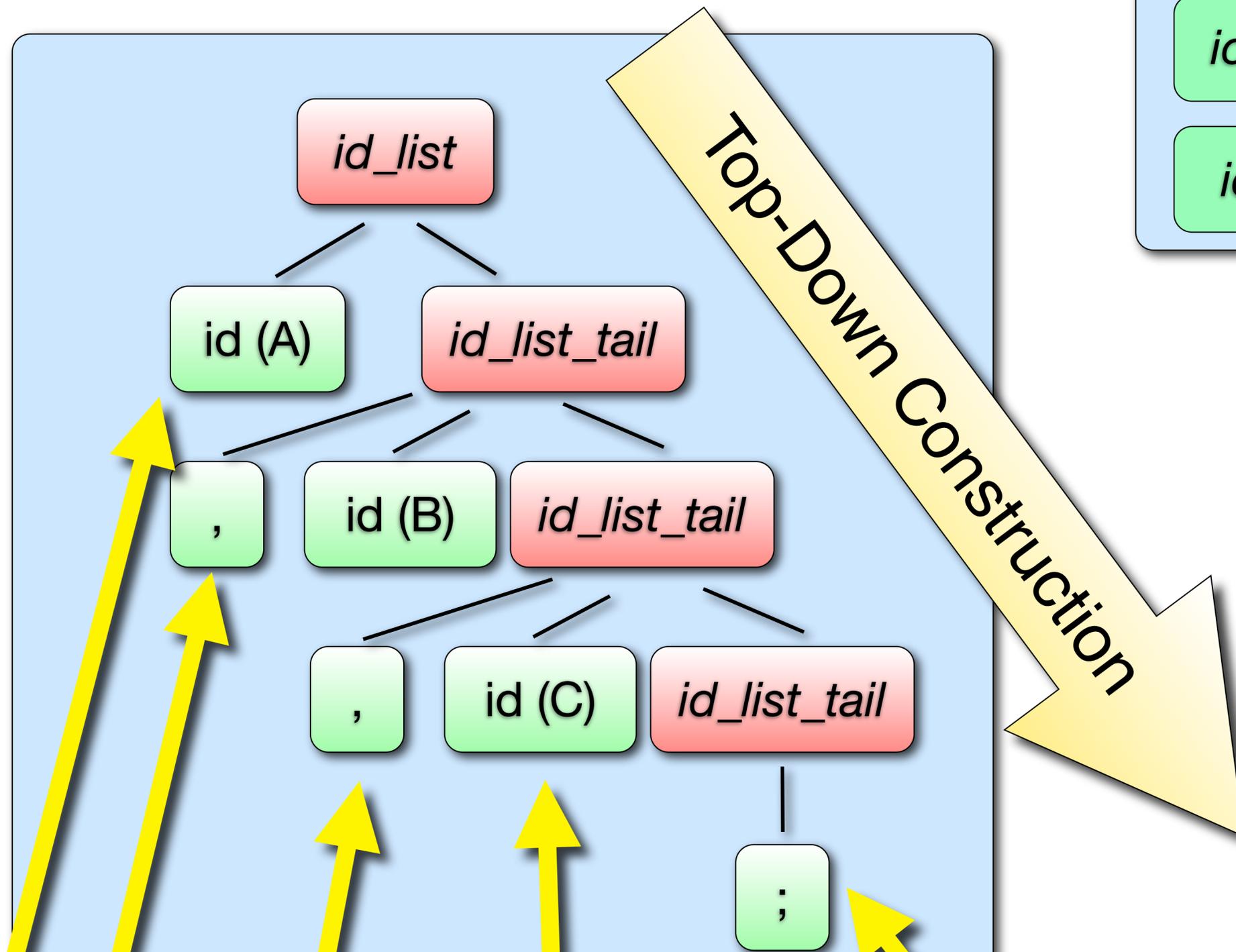
$$id_list \rightarrow id\ id_list_tail$$

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$$id_list_tail \rightarrow ;$$

A , B , C ;

LL Parse Tree



$$id_list \rightarrow id\ id_list_tail$$

$$id_list_tail \rightarrow ,\ id\ id_list_tail$$

$$id_list_tail \rightarrow ;$$

A , B , C ;

Notice that the input **tokens** are placed in the tree from the **left** to **right**.

LR Example

$id_list \rightarrow id\ id_list_tail$

$id_list_tail \rightarrow ,\ id\ id_list_tail$

$id_list_tail \rightarrow ;$

A , B , C ;

forest (a stack)

LR Example

Forest = set of (partial) trees.

$id_list \rightarrow id\ id_list_tail$

$id_list_tail \rightarrow ,\ id\ id_list_tail$

$id_list_tail \rightarrow ;$

A , B , C ;

forest (a stack)

LR Example

Step 1

Shift encountered token into forest.

$id_list \rightarrow id\ id_list_tail$

$id_list_tail \rightarrow ,\ id\ id_list_tail$

$id_list_tail \rightarrow ;$

A , B , C ;

current token

id (A)

forest (a stack)

LR Example

Step 2

Determine that **no right-hand side** of any production **matches the top of the forest**.
Shift next token into forest.

$$id_list \rightarrow id\ id_list_tail$$

$$id_list_tail \rightarrow ,\ id\ id_list_tail$$

$$id_list_tail \rightarrow ;$$

A , B , C ;

current token

id (A)

,

forest (a stack)

LR Example

Steps 3-6

No right hand side **matches** top of forest.
Repeatedly shift next token into forest.

$id_list \rightarrow id\ id_list_tail$

$id_list_tail \rightarrow ,\ id\ id_list_tail$

$id_list_tail \rightarrow ;$

A , B , C ;

current token

id (A)

,

id (B)

,

id (C)

;

forest (a stack)

LR Example

Step 7

Detect that **last production matches the top of the forest.**

Reduce top token to partial tree.

$$id_list \rightarrow id\ id_list_tail$$

$$id_list_tail \rightarrow ,\ id\ id_list_tail$$

$$id_list_tail \rightarrow ;$$

A , B , C ;

current token

id (A)

,

id (B)

,

id (C)

id_list_tail

;

forest (a stack)

LR Example

Step 8

Detect that **second production matches**. **Reduce top of forest.**

$id_list \rightarrow id\ id_list_tail$

$id_list_tail \rightarrow ,\ id\ id_list_tail$

$id_list_tail \rightarrow ;$

A , B , C ;

current token

id (A)

,

id (B)

id_list_tail

id_list_tail

,

id (C)

;

forest (a stack)

LR Example

Step 9

Detect that **second production matches**. **Reduce top of forest.**

$id_list \rightarrow id\ id_list_tail$

$id_list_tail \rightarrow ,\ id\ id_list_tail$

$id_list_tail \rightarrow ;$

id (A)

id_list_tail

id_list_tail

,

id (B)

id_list_tail

,

id (C)

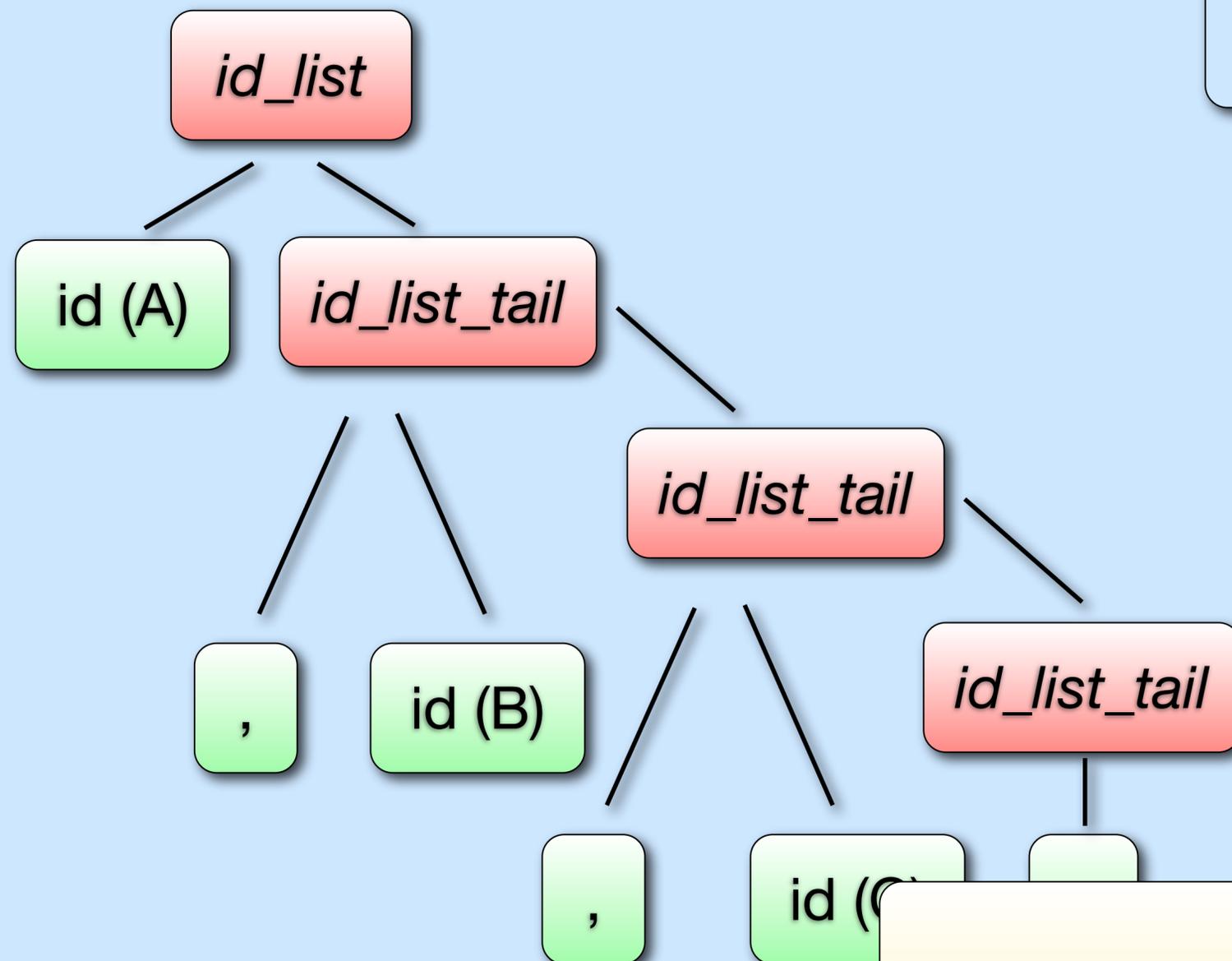
;

A , B , C ;

current token

forest (a stack)

LR Example



forest (a stack)

$id_list \rightarrow id\ id_list_tail$

$id_list_tail \rightarrow ,\ id\ id_list_tail$

$id_list_tail \rightarrow ;$

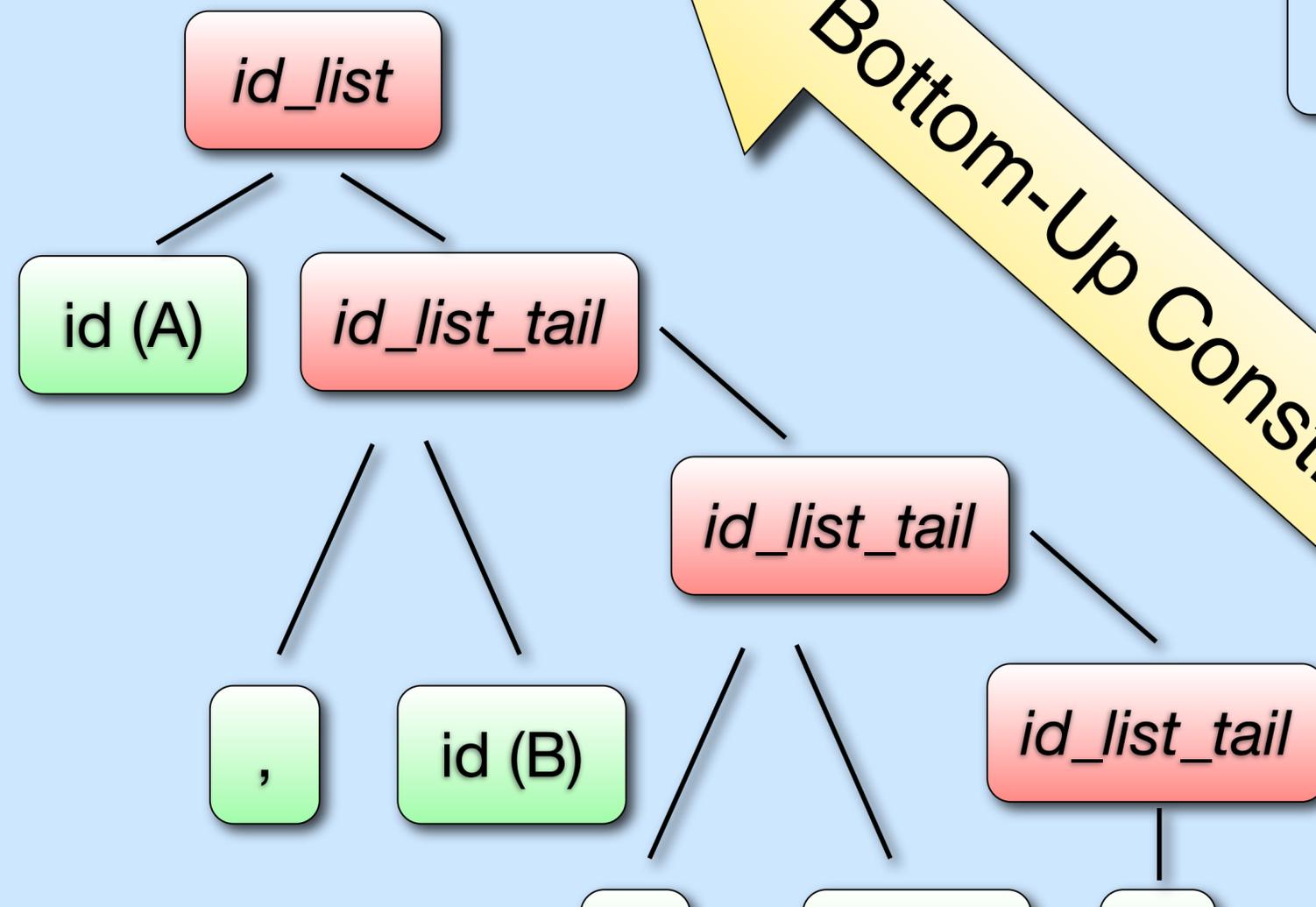
A , B , C ;

current token

Step 10

Detect that **first production matches**. **Reduce top of forest.**

LR Parse Tree



$$id_list \rightarrow id\ id_list_tail$$

$$id_list_tail \rightarrow ,\ id\ id_list_tail$$

$$id_list_tail \rightarrow ;$$

A , B , C ;

The problem with this grammar is that it can require an **arbitrarily large** number of terminals to be **shifted** before **reduction** takes place.

An Equivalent Grammar

better suited to LR parsing

$id_list \rightarrow id_list_prefix ;$

$id_list_prefix \rightarrow id_list_prefix, id$

$id_list_prefix \rightarrow id$

This grammar limits the number of “suspended” non-terminals.

An Equivalent Grammar

better suited to LR parsing

$id_list \rightarrow id_list_prefix ;$

$id_list_prefix \rightarrow id_list_prefix, id$

$id_list_prefix \rightarrow id$

However, this creates a problem for the LL parser.

When the parser discovers an “*id*” it cannot predict the number of *id_list_prefix* productions that it needs to match.

Two Approaches to LL Parser Construction

Recursive Descent.

- A mutually **recursive** set of subroutines.
- One subroutine per non-terminal.
- **Case statements** based on current token to **predict** subsequent productions.

Table-Driven.

- Not recursive; instead has an explicit stack of expected symbols.
- A loop that processes the top of the stack.
- Terminal symbols on stack are simply matched.
- Non-terminal symbols are replaced with productions.
- Choice of production is driven by table.

Recursive Descent Example

“recursive descent”

“climb from root to leaves, calling a subroutine for every level”

Identifier List Grammar.

→ Recall our LL-compatible original version.

$id_list \rightarrow id\ id_list_tail$

$id_list_tail \rightarrow ,\ id\ id_list_tail$

$id_list_tail \rightarrow ;$

Recursive Descent Approach.

- We need **one subroutine for each non-terminal**.
- Each subroutine **adds tokens** into the growing parse tree and/or **calls further subroutines to resolve non-terminals**.

Recursive Descent Example

“recursive descent”

“climb from root to leaves, calling a subroutine for every level”

Identifier List Grammar.

→ Recall our LL-compatible original version.

Possibly itself.

Recursive descent: either directly or indirectly.

id_list_tail → ;

Recursive Descent Approach.

- We need **one subroutine for each non-terminal**.
- Each subroutine **adds tokens** into the growing parse tree and/or **calls further subroutines to resolve non-terminals**.

Recursive Descent Example

Helper routine “**match**”.

- Used to **consume** expected terminals/tokens.
- Given an **expected token type** (e.g., `id`, `“:”`, or `“,”`), checks if next token is of correct type.
- Raises **error** otherwise.

`id_list` → `id id_list_tail`

`id_list_tail` → `“,” id id_list_tail`

`id_list_tail` → `“;”`

```

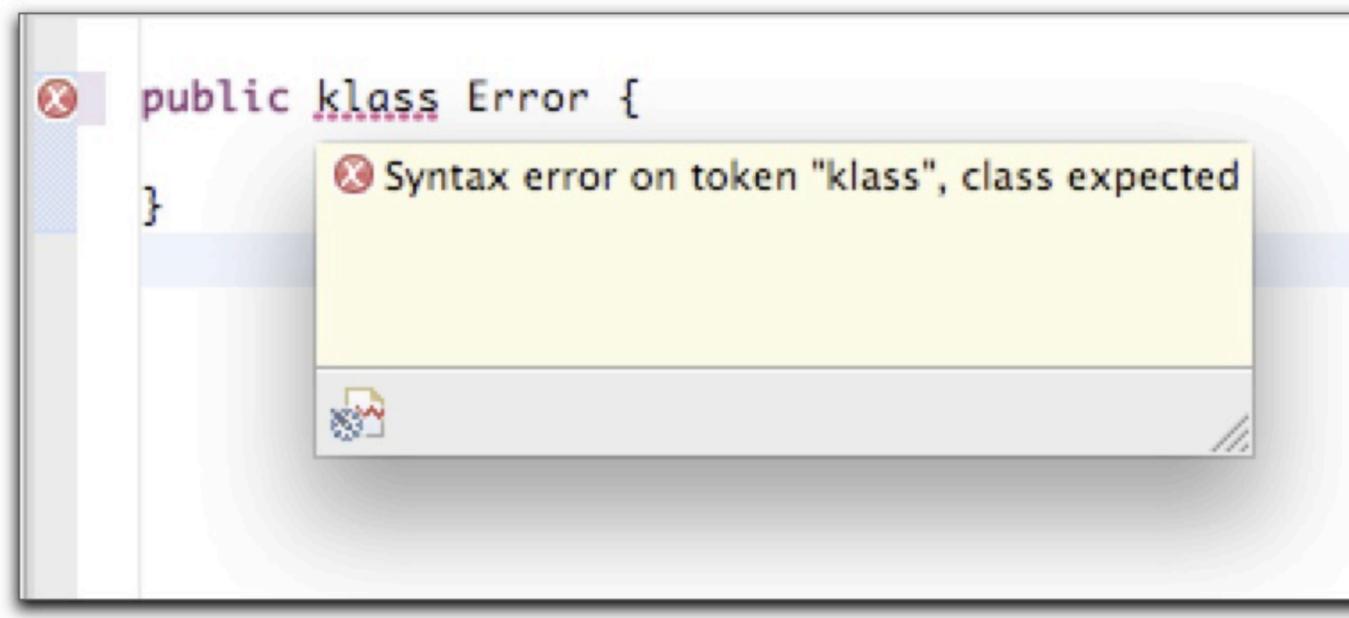
subroutine match(expected_type):
  token = get_next_token()
  if (token.type == expected_type):
    make token a child of left-most non-terminal in tree
  else:
    throw ParseException(“expected ” + expected_type)
  
```

Recursive Descent Example

Helper routine

- Used to **consume** tokens.
- Given an **expected** token or '(', ')', check
- Raises **error**

Example of a match failure:



Eclipse: class token expected, but got id

subroutine

```

token = get_token()
if (token.type == expected_type):
    make token a child of left-most non-terminal in tree
else:
    throw ParseException("expected " + expected_type)
  
```

id_list_tail

id id_list_tail

;

Recursive Descent Example

Parsing *id_list*.

- **Trivial**, there is only one production.
- Simply **match** an *id*, and then **delegate** parsing of the tail to the subroutine for *id_list_tail*.

id_list → *id id_list_tail*

id_list_tail → , *id id_list_tail*

id_list_tail → ;

```
subroutine parse_id_list():  
  match(ID_TOKEN)  
  parse_id_list_tail()
```

Recursive Descent Example

Parsing *id_list*.

→ **Trivial**, there is only one production.

→ Sim
pars
id_l

This **delegation** is the “**descent**” part in recursive descent parsing.

id_list → *id id_list_tail*

list_tail → *, id id_list_tail*

list_tail → *;*

```
subroutine parse_id_list():
  match(ID_TOKEN)
  parse_id_list_tail()
```

Recursive Descent Example

Parsing *id_list_tail*.

- There are **two productions** to choose from.
- This require **predicting** which one is the correct one.
- This requires **looking ahead** and examining the **next token** (without consuming it).

$$id_list \rightarrow id\ id_list_tail$$

$$id_list_tail \rightarrow ,\ id\ id_list_tail$$

$$id_list_tail \rightarrow ;$$

```

subroutine parse_id_list_tail():
  type = peek_at_next_token_type()
  case type of
    COMMA_TOKEN:
      match(COMMA_TOKEN); match(ID_TOKEN); parse_id_list_tail()
    SEMICOLON_TOKEN:
      match(SEMICOLON_TOKEN);
  
```

Recursive Descent Example

Parsing *id_list_tail*.

- There are **two productions** to choose from.
- This requires **predicting** which is the correct one.
- This requires **looking ahead** by examining the **next token** (and consuming it).

$id_list \rightarrow id\ id_list_tail$

$id_list_tail \rightarrow ,\ id\ id_list_tail$

This **delegation** is the “**recursive**” part in recursive descent parsing.

```

subroutine parse_id_list_tail():
  type = peek_at_next_token_type()
  case type of
    COMMA_TOKEN:
      match(COMMA_TOKEN); match(ID_TOKEN); parse_id_list_tail()
    SEMICOLON_TOKEN:
      match(SEMICOLON_TOKEN);
  
```

Recursive Descent

Parsing *id_list_tail*.

- There are **two** productions from.
- This requires **prediction** of the correct one.
- This requires **looking ahead** (examining the **next** token without consuming it).

We need one token “**lookahead.**”

Parsers that require **k** tokens lookahead are called **LL(k)** (or **LR(k)**) parsers.

Thus, this is a **LL(1)** parser.

```

subroutine parse_id_list_tail():
  type = peek_at_next_token_type()
  case type of
    COMMA_TOKEN:
      match(COMMA_TOKEN); match(ID_TOKEN); parse_id_list_tail()
    SEMICOLON_TOKEN:
      match(SEMICOLON_TOKEN);
  
```

LL(k) Parsers

Recall our non-LL compatible grammar.

→ Better for LR-parsing, but **problematic for predictive parsing.**

$id_list \rightarrow id_list_prefix ;$

$id_list_prefix \rightarrow id_list_prefix , id$

$id_list_prefix \rightarrow id$

Cannot be parsed by LL(1) parser.

→ **Cannot predict** which *id_list_production* to choose if next token is of type *id*.

→ However, **a LL(2) parser can parse this grammar.** Just look at the second token ahead and disambiguate based on ‘,’ vs. ‘;’.

LL(k) Parsers

Recall our non-LL compatible grammar.

→ E

using.

Bottom-line:

can enlarge class of supported grammars by using **$k > 1$** lookahead, but at the expense of **reduced performance** / backtracking.

Most production LL parsers use **$k = 1$** .

Can

- **Cannot predict** which *id_list_production* to choose if next token is of type *id*.
- However, **a LL(2) parser can parse this grammar**. Just look at the second token ahead and disambiguate based on ‘,’ vs. ‘;’.

Predict Sets

```
subroutine parse_id_list_tail():  
  type = peek_at_next_token_type()  
  case type of  
    COMMA_TOKEN:  
      match(COMMA_TOKEN); match(ID_TOKEN); parse_id_list_tail()  
    SEMICOLON_TOKEN:  
      match(SEMICOLON_TOKEN);
```

The question is how do we **label the case statements** in general, i.e., for **arbitrary** LL grammars?

First, Follow, and Predict

sets of terminal symbols

FIRST(A):

- The terminals that can be the first token of a valid derivation starting with symbol A .
- Trivially, for each terminal T , **FIRST(T)** = $\{T\}$.

FOLLOW(A):

- The terminals that can follow the symbol A in any valid derivation. (A is usually a non-terminal.)

PREDICT($A \rightarrow \alpha$):

- The terminals that can be the first tokens as a result of the production $A \rightarrow \alpha$. (α is a string of symbols)
- The terminals in this set form the **label in the case statements** to predict $A \rightarrow \alpha$.

First, Follow, and Predict

sets of terminal symbols

Note: For a non-terminal A , the set $\text{FIRST}(A)$ is the union of the predict sets of all productions with A as the head:

if there exist three productions $A \rightarrow \alpha$, $A \rightarrow \beta$, and $A \rightarrow \lambda$, then

$$\text{FIRST}(A) = \text{PREDICT}(A \rightarrow \alpha) \cup \text{PREDICT}(A \rightarrow \beta) \cup \text{PREDICT}(A \rightarrow \lambda)$$

PREDICT($A \rightarrow \alpha$):

- The terminals that can be the first tokens as a result of the production $A \rightarrow \alpha$. (α is a string of symbols)
- The terminals in this set form the **label in the case statements** to predict $A \rightarrow \alpha$.

PREDICT($A \rightarrow \alpha$)

If α is ϵ , i.e., if A is derived to “nothing”:

$$\text{PREDICT}(A \rightarrow \epsilon) = \text{FOLLOW}(A)$$

Otherwise, if α is a string of symbols that starts with X :

$$\text{PREDICT}(A \rightarrow X\dots) = \text{FIRST}(X)$$

Inductive Definition of FIRST(A)

If A is a **terminal** symbol, then:

$$\text{FIRST}(A) = \{A\}$$

If A is a **non-terminal** symbol and there exists a production $A \rightarrow X\dots$, then

$$\text{FIRST}(X) \subseteq \text{FIRST}(A)$$

(X can be terminal or non-terminal)

Notation: X is the first symbol of the production body.

If A is a **terminal** symbol, then:

$$\text{FIRST}(A) = \{A\}$$

If A is a **non-terminal** symbol and there exists a production $A \rightarrow X\dots$, then

$$\text{FIRST}(X) \subseteq \text{FIRST}(A)$$

(X can be terminal or non-terminal)

Inductive Definition of FOLLOW(*A*)

If the substring *AX* exists anywhere in the grammar, then

$$\text{FIRST}(X) \subseteq \text{FOLLOW}(A)$$

If there exists a production $X \rightarrow \dots A$, then

$$\text{FOLLOW}(X) \subseteq \text{FOLLOW}(A)$$

Notation: A is the last symbol of the production body.

If the substring AX exists anywhere in the grammar, then

$$\text{FIRST}(X) \subseteq \text{FOLLOW}(A)$$

If there exists a production $X \rightarrow \dots A$, then

$$\text{FOLLOW}(X) \subseteq \text{FOLLOW}(A)$$

Computing First, Follow, and Predict

Inductive Definition.

- FIRST, FOLLOW, and PREDICT are defined in terms of each other.
- **Exception**: FIRST for **terminals**.
- This the base case for the induction.

Iterative Computation.

- **Start with FIRST for terminals** and set all other sets to be **empty**.
- **Repeatedly apply all definitions** (i.e., include known subsets).
- Terminate when sets do not change anymore.

Predict Set Example

[1]

id_list → *id_list_prefix*;

id_list_prefix → *id_list_prefix*, *id*

[2]

id_list_prefix → *id*

Predict Set Example

$id_list \rightarrow id_list_prefix;$

[1] $id_list_prefix \rightarrow id_list_prefix, id$

[2] $id_list_prefix \rightarrow id$

Base case: $FIRST(id) = \{id\}$

Induction for [2]: $FIRST(id) \subset FIRST(id_list_prefix) = \{id\}$

Induction for [1]: $FIRST(id_list_prefix) \subset FIRST(id_list_prefix)$

Predict sets for (2) and (1) are identical: not LL(1)!

Left Recursion

Leftmost symbol is a recursive non-terminal symbol.

- ➔ This causes a grammar **not to be LL(1)**.
- ➔ Recursive descent would enter **infinite recursion**.
- ➔ It is **desirable for LR grammars**.

```
id_list → id_list_prefix;
```

```
id_list_prefix → id_list_prefix, id
```

```
id_list_prefix → id
```

Leftmost

→ This

→ Recu

→ It is **not suitable for LR grammars.**

“To parse an *id_list_prefix*, call the parser for *id_list_prefix*, which calls the parser for *id_list_prefix*, which...”

symbol.

ion.

id_list → *id_list_prefix*;

id_list_prefix → *id_list_prefix*, *id*

id_list_prefix → *id*

Left-Factoring

Introducing “tail” symbols to avoid left recursion.

→ Split a recursive production in an **unambiguous prefix** and an **optional tail**.

$$expr \rightarrow term \mid expr \text{ add_op } term$$

is equivalent to

prefix

$$expr \rightarrow term \text{ expr_tail}$$

tail

$$expr_tail \rightarrow \epsilon$$

tail

$$expr_tail \rightarrow \text{ add_op } expr$$

Another Predict Set Example

[1]

cond → *if expr then statement*

[2]

cond → *if expr then statement else statement*

Another Predict Set Example

[1]

cond → *if expr then statement*

[2]

cond → *if expr then statement else statement*

PREDICT([1]) = {if}

PREDICT([2]) = {if}

If the next token is an **if**, **which production** is the right one?

Common Prefix Problem

Non-disjoint predict sets.

→ In order to predict which production will be applied, **all predict sets** for a given non-terminal need to be **disjoint!**

If there exist two productions $A \rightarrow \alpha$, $A \rightarrow \beta$ such that there exists a terminal x for which $x \in \text{PREDICT}(A \rightarrow \alpha) \cap \text{PREDICT}(A \rightarrow \beta)$, then an LL(1) parser cannot properly predict which production must be chosen.

Can also be addressed with left-factoring...

Dangling else

Even if left recursion and common prefixes have been removed, a language may not be LL(1).

- In many languages an **else statement** in if-then-else statements **is optional**.
- Ambiguous grammar: **which if to match else to?**

```
if AAA then
  if BBB then
    CCC
else
  DDD
```

Dangling else

```
if AAA then
  if BBB then
    CCC
  else
    DDD
```

- ▶ Can be handled with a tricky LR grammar.
- ▶ There exists **no LL(1)** parser that can parse such statements.
- ▶ Even though a proper LR(1) parser can handle this, it may not handle it in a **method the programmer desires**.
- ▶ Good language design avoids such constructs.

Dangling else

- ▶ To write this code correctly (based on indentation) “begin” and “end” statements must be added.
- ▶ This is LL compatible.

```
if AAA then
  if BBB then
    CCC
else
  DDD
```

```
if AAA then
  begin
    if BBB then
      CCC
    end
else
  DDD
```

Dangling else

statement → ... | *cond* | ...

cond → if *expr* then *block_statement*

cond → if *expr* then *block_statement* else *block_statement*

block_statement → begin *statement** end

A grammar that avoids the “dangling else” problem.