Global Illumination II

Computer Graphics
COMP 770 (236)
Spring 2007

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From last time…

- Rendering equation
- Path tracing
- Photon mapping
- Radiosity

Today's topics

- Monte Carlo integration
  - evaluating integrals by random sampling
- Global illumination in production rendering
  - AKA global illumination hacks

Monte Carlo integration

$$E[f(X)] = \int_{x \in A} f(x)p(x)dx \approx \frac{1}{N}\sum_{i=1}^{N} f(x_i)$$

X	- random variable
p	- probability density function ($X \sim p$)
$E[]$	- expected value
x_i	- a single sample drawn from X

Monte Carlo integration

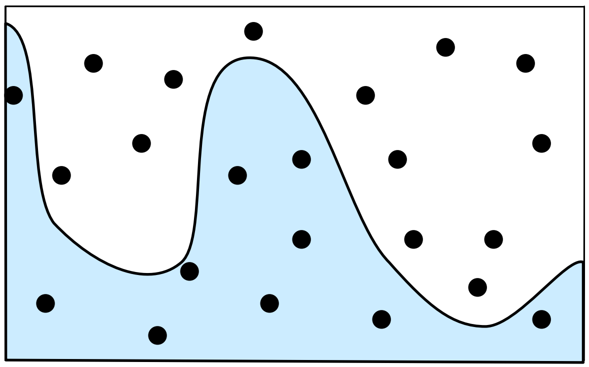
$$\int_{x \in A} f(x)dx \approx \frac{1}{N}\sum_{i=1}^{N}\frac{f(x_i)}{p(x_i)}$$

assumes p is positive wherever f is positive and that f is non-negative

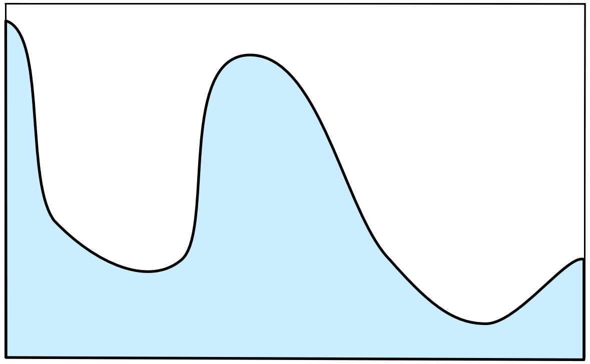
- Most of the time we don't know what f is
  - …but f is easy to evaluate (Like $L(x,\omega)$ in the rendering equation)
  - We may know parts of f. Use a p based on what we know
- Reducing variance
  - stratification
  - importance sampling
    - what happens when p is proportional to f?
- Problem: Given p how do we choose a sample $x \in X \sim p$?

Random sampling

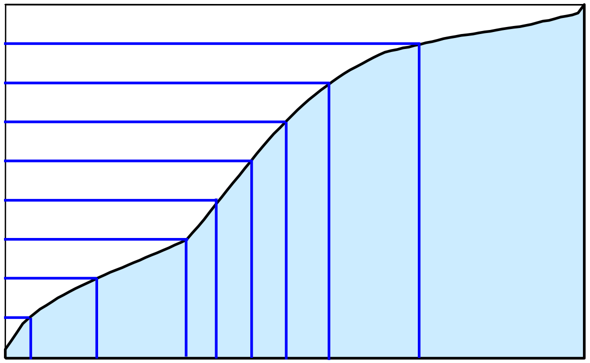
- Rejection sampling
  - uniformly sample domain
  - only keep samples under p



- CDF inversion
  - transform uniform random variable U: $X = P^{-1}(U)$



p



$$P(s) = \int_{-\infty}^{s} p$$

An example

$$p(x) = \frac{5-4x}{3}$$

$$P(x) = \frac{(5-2x)x}{3}$$

$$P^{-1}(u) = \frac{5-\sqrt{25-24u}}{4}$$

$$p(x) = 1$$

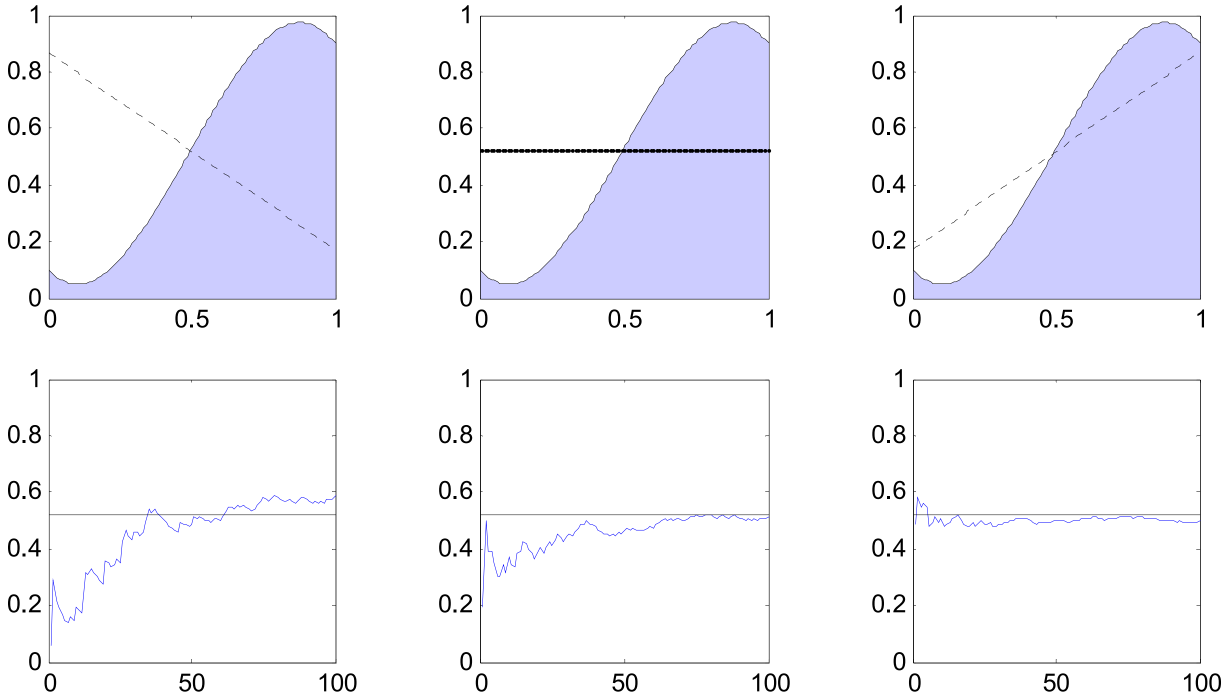
$$P(x) = x$$

$$P^{-1}(u) = u$$

$$p(x) = \frac{1+4x}{3}$$

$$P(x) = \frac{(1+2x)x}{3}$$

$$P^{-1}(u) = \frac{-1+\sqrt{1+24u}}{4}$$



More useful examples

- Uniformly distributed points in a disc

$$(r,\phi) = \left(\sqrt{u_1}, 2\pi u_2\right)$$

- Uniformly distributed points on a hemisphere

$$(\theta,\phi) = \left(\cos^{-1}(1-2u_1), 2\pi u_2\right)$$

- Many others can be found in the Global Illumination Compendium
  - http://www.cs.kuleuven.be/~phil/GI/

Monte Carlo methods

- Advantages
  - sampling is easy
  - guaranteed to converge
- Disadvantages
  - suffers from noise (variance)
  - converges very slowly $\left(1/\sqrt{N}\right)$
  - not always easy to find good sampling distributions

Rendering cheats and hacks

- How is the illumination in this image better than that obtained by the OpenGL local illumination model?

Ever wonder "How'd they do that?"

- Spinosaurus from JP III, by Lucas Digital Ltd.
- Lots of modeling and artist painted textures
- Lots of lights? Ray trace every frame?

Behind the Veneer

- Global illumination methods are still too costly to apply to every frame of a movie production
- What aspects of lighting and rendering are unrealistic?

$$I_{total} = \sum_{i=1}^{lights} k_a I_a + k_d I_d Max((\hat{N}\cdot\hat{L}),0) + k_s I_s Max((\hat{N}\cdot\hat{H}),0)^{n_{shiney}}$$

- Here too

$$I_{\lambda,r} = I_{\lambda,a} k_a + \sum_{i=1}^{lights} I_{\lambda,i}\left((1-k_a-k_s)\rho_\lambda(\bar{l}_i\cdot\bar{n}) + k_s \frac{DGF_\lambda(\theta_i)}{\pi(\bar{v}\cdot\bar{n})}\right)$$

There is light everywhere

- Not only direct light from specific light sources
  - The "Ambient Hack"
- Can't afford to do global illumination to compute the transport of light between every possible pair of points
- Clever Tricks
  - Reflection Mapping
  - Occulsion Mapping

Production-ready global illumination

- http://www.debevec.org/HDRI2004/landis-S2002-course16-prodreadyGI.pdf

landis-S2002-cours
e16-prodreadyGI

Next time

- Geometric modeling