A Short Introduction to Multiplexed Illumination

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1 Multiplexed Illumination

Capturing images under dim lighting is difficult due to the presence of camera CCD noise. This noise can significantly degrade the quality of the image due to the low signal-to-noise ratio. Schechner et al. [**?**] introduced a technique to significantly reduce the noise in the captured images with multiple low intensity light sources. Using n light sources, we can increase the signal-to-noise ratio by up to $\frac{\sqrt{n}}{2}$ with the same number of images.

1.1 Example

Before the general case is described, we first consider an example of three light sources. An acquired image lit by light source l will be called $a(x, y)$, and the image irradiance is $i(x, y)$. Now suppose that we acquire a set of 3 images, each lit by only one of the light sources. We can write this in matrix form as:

$$\begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

We have used a 1 where the light is on, and 0 where the light is off. In this setup, it is trivial to recover the images as lit by one of the light sources, as the images were acquired under a single light source. However, each of these images are lit by only 1/3 of the available light sources. As these images are being captured with CCD cameras, the signal-noise ratio may be too low for high-quality images. We can improve this ratio by using more of the light sources at a time. If each of the images is captured under 2 of the 3 lights, we can write the system:

$$\begin{bmatrix} a_{1,2} \\ a_{2,3} \\ a_{1,3} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix}$$

Each of the observed images $a(x, y)$ consist of the light from the multiple lights multiplexed together. We want to *demultiplex* these images by reconstructing the images $i(x, y)$ which contain the light from only a single light source. Suppose the we want to reconstruct i_1, the scene lit by light 1. If we add the observed image $a_{1,2}$ to $a_{1,3}$, we get twice i_1 as well as i_2 and i_3. Since $a_{2,3}$ consists of i_2 and i_3, we can subtract this two get twice i_1. Dividing by two gives us the contribution of i_1, as reconstructed from the multiplexed images. Writing this down for each of the 3 lights gives us:

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 & -1 & 1 \\ 1 & 1 & -1 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a_{1,2} \\ a_{2,3} \\ a_{1,3} \end{bmatrix}$$

The advantage of this new method is the improved noise. To see this, assume that each of the images have independent additive noise σ. The noise of the demultiplexed image will be a combination of the noise from the three component images. To get the total noise, we add the variances of the individual noisy images together (the square of the standard deviation):

$$(\pm\frac{1}{2}\sigma_1)^2 + (\pm\frac{1}{2}\sigma_1)^2 + (\pm\frac{1}{2}\sigma_1)^2 = \frac{3}{4}\sigma^2$$

So the noise has been reduced to $\sqrt{\frac{3}{4}}\sigma$, an improvement of about 14% without increasing the number of images that we have to acquire.

2 General Multiplexed Illumination

Now let's consider the general case of n lights and n images. The light are additive quantities and are linearly related by superposition

$$\underbrace{[a_{\zeta_0}(x,y),\ldots,a_{\zeta_n}(x,y)]^T}_{a(x,y)} = W\,\underbrace{[i_0(x,y),\ldots,i_m(x,y)]^T}_{i(x,y)}$$

where $a_{\zeta_k}(x,y)$ is the light observed at pixel (x,y) under the set of lights ζ_k and $i_l(x,y)$ is the energy contributed by light source l at pixel (x,y). The multiplexing matrix W for the light sources $l = 1,\ldots,m$ describes which light sources illuminate the scene. An element $W_{i,j}$ is one if the light j is illuminated in image i, and zero if the light was "off". The sets ζ_k consists of all of the lights in row k which are "on".

In order to recover the images $i_l(x,y)$ as lit under a single light source l, we demultiplex the observed images $a(x,y)$ by inverting the matrix W:

$$i(x,y) = W^{-1}a(x,y)$$

The next question is to determine an effective multiplexing matrix. There are several properties that an optimal multiplexing matrix should have: it should minimize the MSE, and it should be easy to invert. The authors [**?**] suggest using a special matrix form called *Hadamard matrices* which were developed in the X-ray astronomy community [**?**]. Hadamard matrices have 0's or 1's as elements of the matrix, and have the property that $\frac{n+1}{2}$ of the elements are 1 along any row (an algorithm for constructing these matrices is given in [**?**]). This means that for each image, a little over half of the lights are on. [*note: Why is this optimal?*]

From this, we can calculate the benefit of using such Hadamard matrices for multiplexing the illumination. Let the light level from one light be L with a standard deviation of σ due to additive independent noise. Thus the SNR of an image under a single light source is:

$$SNR_{single} = \frac{L}{\sigma}$$

Using multiplexing with Hadamard matrices, the number of lights that are used is $\frac{n+1}{2}$ so the signal is increased to $L\frac{n+1}{2}$. The noise level is a combination of the independent additive noise from n images, which means that the variance of the noise is $\sigma^2 n$ and the standard deviation is $\sigma\sqrt{n}$. The SNR of the multiplexed images is:

$$SNR_{multi} = \frac{L(n+1)}{2\sigma\sqrt{n}}$$

This means that the ratio of the multiplexed SNR to the single image SNR is:

$$\frac{SNR_{multi}}{SNR_{single}} = \frac{\sqrt{n}+\frac{1}{\sqrt{n}}}{2} \approx \frac{\sqrt{n}}{2}$$

The larger the number of images, the more useful this technique becomes. For 5 images, the increase is SNR is only 1.34x, but for 50 images, the increase is 3.61x. It's important to note that multiplexed illumination does not require taking any more images than single light illumination. The only added computation is the demultiplexing step, which is negligible when using Hadamard matrices.