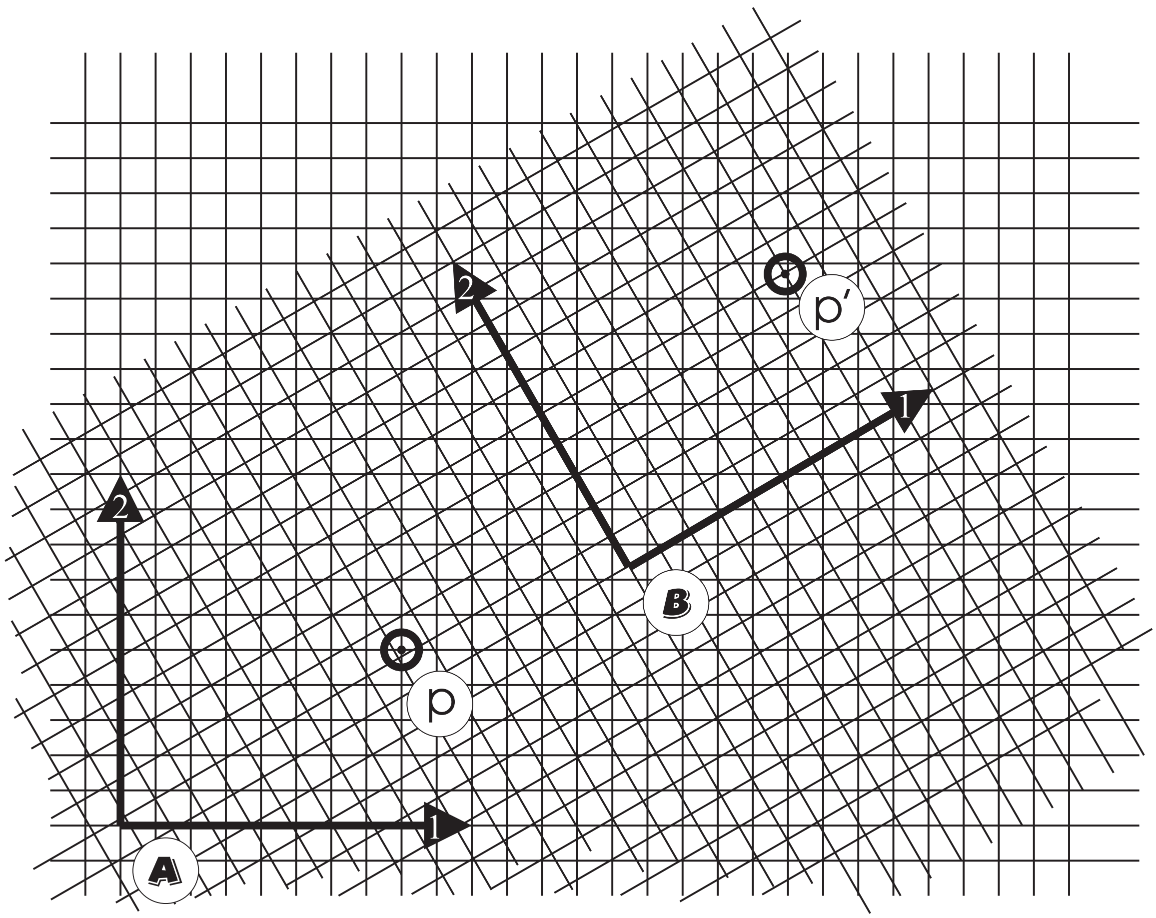
COMP 770, Homework 2, Due Monday Mar. 16



The series of exercises on the following page use this diagram. It shows two coordinate systems, A and B, and two points, p and p'. For your convenience, I have provided a grid aligned with each C.S., which will simplify coordinate measurements. The axis arrows are unit length, so the grid squares have spacing of 0.1 units. Your measurements need not be exact, but take your best guess as to the second decimal place.

1. Measure directly from the diagram the following quantities:

   (a) The placement of B with respect to A (R and T).

   (b) The placement of A with respect to B (R' and T').

   (c) The coordinates of p with respect to A (p_A).

   (d) The coordinates of p with respect to B (p_B).

(e) The coordinates of p' with respect to A (p'_A).

(f) The coordinates of p' with respect to B (p'_B).

2. Compute the following expressions from the quantities you measured in step 1.

   (a) $Rp_A + T$
   (b) $Rp_B + T$
   (c) $Rp'_A + T$
   (d) $Rp'_B + T$
   (e) $R'p_A + T'$
   (f) $R'p_B + T'$
   (g) $R'p'_A + T'$
   (h) $R'p'_B + T'$
   (i) R^{-1}
   (j) $-R^{-1}T$
   (k) $(R')^{-1}$
   (l) $-(R')^{-1}T$

3. Which of the expressions computed in step 2 match any of the quantities measured in step 1? Write these down as equalities. What matches did you expect to find? Don't expect to find any exact matches, due to imprecision in measurements. What do you think this exercise is intended to demonstrate?

4. Consider the quaternion $q = \frac{1}{5}(1 + 2i + 4j + 2k)$. Its inverse is $q^{-1} = \frac{1}{5}(1 - 2i - 4j - 2k)$. To apply the rotation to a point with coordinates $v = (1, 3, 2)$, we form a "pure vector" quaternion $p = (0+1i+3j+2k)$ from v, and then compute $p' = qpq^{-1}$. The result is a "pure vector" quaternion whose coordinates give the rotated point.

   (a) Verify that q is a unit quaternion.

   (b) Compute the quantity qpq^{-1}. Since quaternion multiplication is associative, the order of evaluation is arbitrary.

   (c) Write the rotation matrix R equivalent to q. Verify that applying R to v yields the same coordinates as applying q to p (from part b). The correct formula for column vector convention given a unit quaternion $q = (w + xi + yj + zk)$ is

$$R = \begin{pmatrix} 1 - 2y^2 - 2z^2 & 2xy - 2wz & 2xz + 2wy \\ 2xy + 2wz & 1 - 2x^2 - 2z^2 & 2yz - 2wx \\ 2xz - 2wy & 2yz + 2wx & 1 - 2x^2 - 2y^2 \end{pmatrix}$$

   (d) Write down the axis-angle form for the rotation represented by q.

   (e) **Optional, Extra Credit** using any means necessary, find a roll-pitch-yaw representation for the rotation represented by q.