B-reps

A. Boundary representation – a solid is modeled as a set of surfaces forming its boundary.

B. Topology and Geometry
   1. Topology – how the surfaces are connected together.
   2. Geometry – where the surfaces actually are in space.
   3. Topology and Geometry are strongly linked.

C. Manifold surfaces
   1. 2-manifold – the object is locally topologically the same as a disk.
   2. Boundaries are often assumed to be 2-manifolds.
   3. Regularized Boolean operations on manifold solids may not yield manifold results.
   4. Non-manifold geometry (NMG) needed to represent other surfaces.

B-reps (continued)

D. Surfaces must be *orientable*.
E. Boundaries may be linear (planar) polygons or curved patches.
F. Must be able to handle “holes” in faces.
   1. Introduce a “cut” in the face.
   2. Keep track of interior “loops” in faces.

Manifold Solids

A. Euler-Poincare formula:

$$V-E+F-H=2(C-G)$$

V = Vertices
E = Edges
F = Faces
H = Holes in faces (=L-F where L=loops)
C = Components (or shells)
G = Genus (holes through solid)

B. Above formula is necessary but not sufficient.

C. "Tweaking" solids modifies the solid without changing the topology or the above numbers.

D. Euler operators – create and consistently modify the topology of manifold surface solids.
   1. Can be written in form m*x*k*y* – meaning make x features, and kill y features.
   2. More information is needed to modify specific solids – e.g. geometry, which features to modify.

Winged-Edge Data Structure

A. Commonly used form for storing topological information of a B-rep.
B. Faces store 1 edge from each edge cycle (1 cycle for outermost face boundary, other cycles for interior "loops").
C. Vertices store 1 edge that the vertex lies on.
D. Edges store most information:
   1. *From* vertex and *To* vertex.
   2. *Left* face and *Right* face.
   3. *Preceding/Succeeding* edge in clockwise order.
   4. *Preceding/Succeeding* edge in counterclockwise order.
   5. Directions are as viewed from outside the object.
E. Geometric information (e.g. vertex positions and/or plane equations) is kept separately.

Validity of B-reps

A. Ensuring validity of a B-rep can be very difficult – inferences must be made from representation.

B. Topological validity:
   1. “Tame” vs. “wild” structure – usually the nature of design ensures tameness.
   2. Orientability – adjacent faces must have correct orientations
   3. Manifold or not – may need to “shrink” or “expand” in order to deal with non-manifold data.

C. Geometric validity:
   1. Determining position/orientation of vertices/planes, etc.
   2. Must be considered with topological data.
   3. Inaccuracies in geometry and computation can lead to topological inconsistencies.

Boolean Operations on B-reps

A. Intersect the faces of the B-reps, subdivinding each face along the intersection curves.
B. Generate new topological structure for the new faces.
C. Classify the faces as inside/outside of the other solid.
D. Decide which faces to keep to form the boundary of the final solid. Discard the other faces and get the topology of the new solid.
E. CSG → B-rep conversion can be done by converting primitives to B-rep (usually easy) then performing Boolean operations on the B-reps.

Robustness Problems

A. Degeneracies:
   1. Yield non-manifold or other special cases.
   2. Examples: Overlapping faces, vertex lying on face.

B. Numerical errors:
   1. In representation or arising in computation.
   2. Cause conflict between geometric and topological information, or conflict within geometric information.
   3. Example: Plane equations say that an edge should lie within a solid, but topological information says it lies outside.
   4. Example: Intersection of 3 plane equations for faces yields coordinates that are different than those of the vertex shared by those faces.

Curved Surfaces

A. More difficult to deal with than linear surfaces.
B. Intersection curves can be of very high degree – difficult to represent.
C. Often approximations are used.
    1. Represent curved surface as many linear pieces – may need too many for necessary accuracy.
    2. Use spline patches to approximate curved surfaces or spline curves to approximate intersection curves.
    3. Approximations easily yield robustness problems.
D. Local analysis can often be used on linear objects, but usually can’t be used on curved objects.

Other Areas of Solid Modeling Research

A. Other representations for solids (e.g. medial models (skeletons)) – the skeleton of an object, along with thickness, specifies the solid.
B. Constraint-based design – the solid is defined using a group of constraints which must be solved to have give the final outcome. Example constraint: place hole *A* 3cm from hole *B*.
C. Feature-based design/analysis – the solid is looked at in terms of its *features*. Examples are holes, grooves, handles.
D. Functional analysis – incorporating information on the purpose of the design along with the basic geometry.