1. Given $n+1$ points, $\mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_n$ and a sequence of $n+1$ parameters, $u_0, u_1, \ldots, u_n$, with $u_i < u_{i+1}$ for all i, our goal is compute a C^2 cubic spline curve $\mathbf{F}(u)$, such that

$$\mathbf{F}(u_i) = \mathbf{x}_i,$$

for all i, $0 \leq i \leq n$.

   (a) If we use cubic B-spline curves (and compute the appropriate control points), how will you choose the knot sequence? Is your resulting problem under-constrained or over-constrained?

   (b) If the problem is under-constrained, add sufficient number of tangent conditions at the end points and derive the control points of the resulting B-spline curve.

2. Find the Bézier control points of a closed spline of degree 4 whose control polygon consists of the edges of a square, and whose knot sequence is uniform and consists of simple knots.

3. Given a $m \times n$ tensor product patch, $\mathbf{P}(u,w)$, there are possible ways to evaluate a point:

   - Use the recursive de Casteljau algorithm.
   - Use the tensor product formulation. Compute the coefficients of a u isoparametric line, and then evaluate that curve at v.
   - Use the tensor product formulation. Compute the coefficients of a v isoparametric line, and then evaluate that curve at u.

   Work out the operation count for each of these cases.

4. Given a rational Bézier patch of degree n, $\mathbf{P}(u,w)$, give an algorithm to compute the control points (and associated weight) of its normal or hodograph patch, $\mathbf{N}(u,w)$. Given the parameter values (u_0, w_0), $\mathbf{N}(u_0,w_0)$ denotes the normal at $\mathbf{P}(u_0,w_0)$. $\mathbf{N}(u_0,w_0)$ doesn't represent a normalized vector. I am not expecting a closed form solution in terms of weights and control points of $\mathbf{P}(u,w)$, but show all the steps used to compute a representation of the normal patch in terms of Bernstein polynomials.

5. Degree Elevation of a Triangular Patch: Given a triangular patch of degree n, the degree elevation formula for this patch is given as:

$$\Sigma_{|\mathbf{i}|=n} \mathbf{b}_{\mathbf{i}} B^n_{\mathbf{i}}(\mathbf{u}) = \Sigma_{|\mathbf{i}|=n+1} \mathbf{b}_{\mathbf{i}}^{(1)} B^{n+1}_{\mathbf{i}}(\mathbf{u}),$$

show that the control points $\mathbf{b}_{\mathbf{i}}^{(1)}$ are given as:

$$\mathbf{b}_{\mathbf{i}}^{(1)} = \frac{1}{n+1}[i\mathbf{b}_{\mathbf{i-e1}} + j\mathbf{b}_{\mathbf{i-e2}} + k\mathbf{b}_{\mathbf{i-e3}}]$$

where $\mathbf{i} = (i,j,k)$.