DEFINITION OF A B-SPLINE CURVE

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Overview

These notes present the direct definition of the B-Spline curve. This definition is given in two ways: first by an analytical definition using the normalized B-spline blending functions, and then through a geometric definition.

The B-Spline Curve – Analytical Definition

A B-spline curve $\mathbf{P}(t)$, is defined by

$$\mathbf{P}(t) = \sum_{i=0}^{n} \mathbf{P}_i N_{i,k}(t)$$

where

- the $\{\mathbf{P}_i : i = 0, 1, ..., n\}$ are the control points,

- k is the order of the polynomial segments of the B-spline curve. Order k means that the curve is made up of piecewise polynomial segments of degree $k - 1$,

- the $N_{i,k}(t)$ are the "normalized B-spline blending functions". They are described by the order k and by a non-decreasing sequence of real numbers

$$\{t_i : i = 0, ..., n + k\}.$$

normally called the "knot sequence". The $N_{i,k}$ functions are described as follows

$$N_{i,1}(t) = \begin{cases} 1 & \text{if } u \in [t_i, t_{i+1}), \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

and if $k > 1$,

$$N_{i,k}(t) = \frac{t - t_i}{t_{i+k-1} - t_i} N_{i,k-1}(t) + \frac{t_{i+k} - t}{t_{i+k} - t_{i+1}} N_{i+1,k-1}(t) \tag{2}$$

- and $t \in [t_{k-1}, t_{n+1})$.

We note that if, in equation (2), either of the N terms on the right hand side of the equation are zero, or the subscripts are out of the range of the summation limits, then the associated fraction is not evaluated and the term becomes zero. This is to avoid a zero-over-zero evaluation problem. We also direct the readers attention to the "closed-open" interval in the equation (1).

The order k is independent of the number of control points $(n + 1)$. In the B-Spline curve, unlike the Bézier Curve, we have the flexibility of using many control points, and restricting the degree of the polymonial segments.

The B-Spline Curve – Geometric Definition

Given a set of Control Points $\{P_0, P_1, ..., P_n\}$, an order k, and a set of knots $\{t_0, t_1, ..., t_{n+k}\}$, the B-Spline curve of order k is defined to be

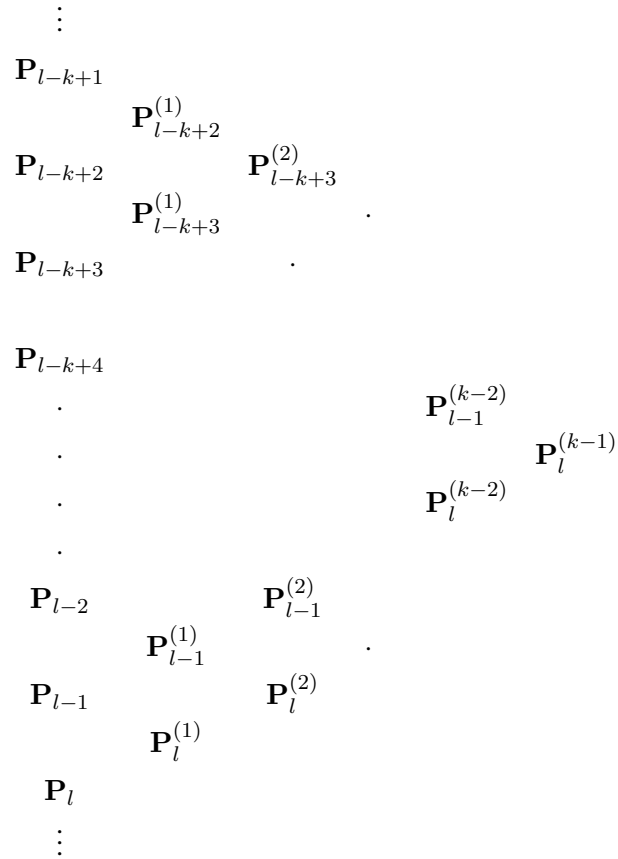$$\mathbf{P}(t) = \mathbf{P}_l^{(k-1)}(t) \text{ if } u \in [t_l, t_{l+1})$$

where

$$\mathbf{P}_i^{(j)}(t) = \begin{cases} (1 - \tau_i^j)\mathbf{P}_{i-1}^{(j-1)}(t) + \tau_i^j \mathbf{P}_i^{(j-1)}(t) & \text{if } j > 0, \\ \mathbf{P}_i & \text{if } j = 0. \end{cases}$$

and

$$\tau_i^j = \frac{t - t_i}{t_{i+k-j} - t_i}$$

It is useful to view the geometric construction as the following pyramid

$$\vdots$$

$$\mathbf{P}_{l-k+1}$$
$$\qquad\qquad \mathbf{P}_{l-k+2}^{(1)}$$
$$\mathbf{P}_{l-k+2} \qquad\qquad\qquad \mathbf{P}_{l-k+3}^{(2)}$$
$$\qquad\qquad \mathbf{P}_{l-k+3}^{(1)} \qquad\qquad\qquad .$$
$$\mathbf{P}_{l-k+3} \qquad\qquad\qquad .$$

$$\mathbf{P}_{l-k+4}$$
$$\qquad . \qquad\qquad\qquad\qquad\qquad \mathbf{P}_{l-1}^{(k-2)}$$
$$\qquad . \qquad\qquad\qquad\qquad\qquad\qquad\qquad \mathbf{P}_{l}^{(k-1)}$$
$$\qquad . \qquad\qquad\qquad\qquad\qquad \mathbf{P}_{l}^{(k-2)}$$
$$\qquad .$$
$$\mathbf{P}_{l-2} \qquad\qquad\qquad \mathbf{P}_{l-1}^{(2)}$$
$$\qquad\qquad \mathbf{P}_{l-1}^{(1)} \qquad\qquad\qquad .$$
$$\mathbf{P}_{l-1} \qquad\qquad\qquad \mathbf{P}_{l}^{(2)}$$
$$\qquad\qquad \mathbf{P}_{l}^{(1)}$$
$$\mathbf{P}_{l}$$

$$\vdots$$

Any $\mathbf{P}$ in this pyramid is calculated as a convex combination of the two $\mathbf{P}$ functions immediately to it's left.

THE UNIFORM B-SPLINE BLENDING FUNCTIONS

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Overview

The normalized B-spline blending functions are defined recursively by

$$N_{i,1}(t) = \begin{cases} 1 & \text{if } u \in [t_i, t_{i+1}) \\ 0 & \text{otherwise} \end{cases} \tag{1}$$

and if $k > 1$,

$$N_{i,k}(t) = \left(\frac{t - t_i}{t_{i+k-1} - t_i} \right) N_{i,k-1}(t) + \left(\frac{t_{i+k} - t}{t_{i+k} - t_{i+1}} \right) N_{i+1,k-1}(t) \tag{2}$$

where $\{t_0, t_1, ..., t_{n+k}\}$ is a non-decreasing sequence of knots, and k is the order of the curve.

These functions are difficult to calculate directly for a general knot sequence. However, if the knot sequence is uniform, it is quite straightforward to calculate these functions – and they have some suprising properties.

Calculating the Blending Functions using a Uniform Knot Sequence

Assume that $\{t_0, t_1, t_2, ..., t_n\}$ is a uniform knot sequence, *i.e.,* $\{0, 1, 2, ..., n\}$. This will simplify the calculation of the blending functions, as $t_i = i$.


Blending Functions for k = 1

if $k = 1$, then by using equation (1), we can write the normalized blending functions as

$$N_{i,1}(t) = \begin{cases} 1 & \text{if } u \in [i, i+1) \\ 0 & \text{otherwise} \end{cases} \tag{3}$$

These are shown together in the following figure, where we have plotted $N_{0,1}$, $N_{1,1}$, $N_{2,1}$, and $N_{3,1}$ respectively, over five of the knots. Note the white circle at the end of the line where the functions value is 1. This represents the affect of the "open-closed" interval found in equation (1).
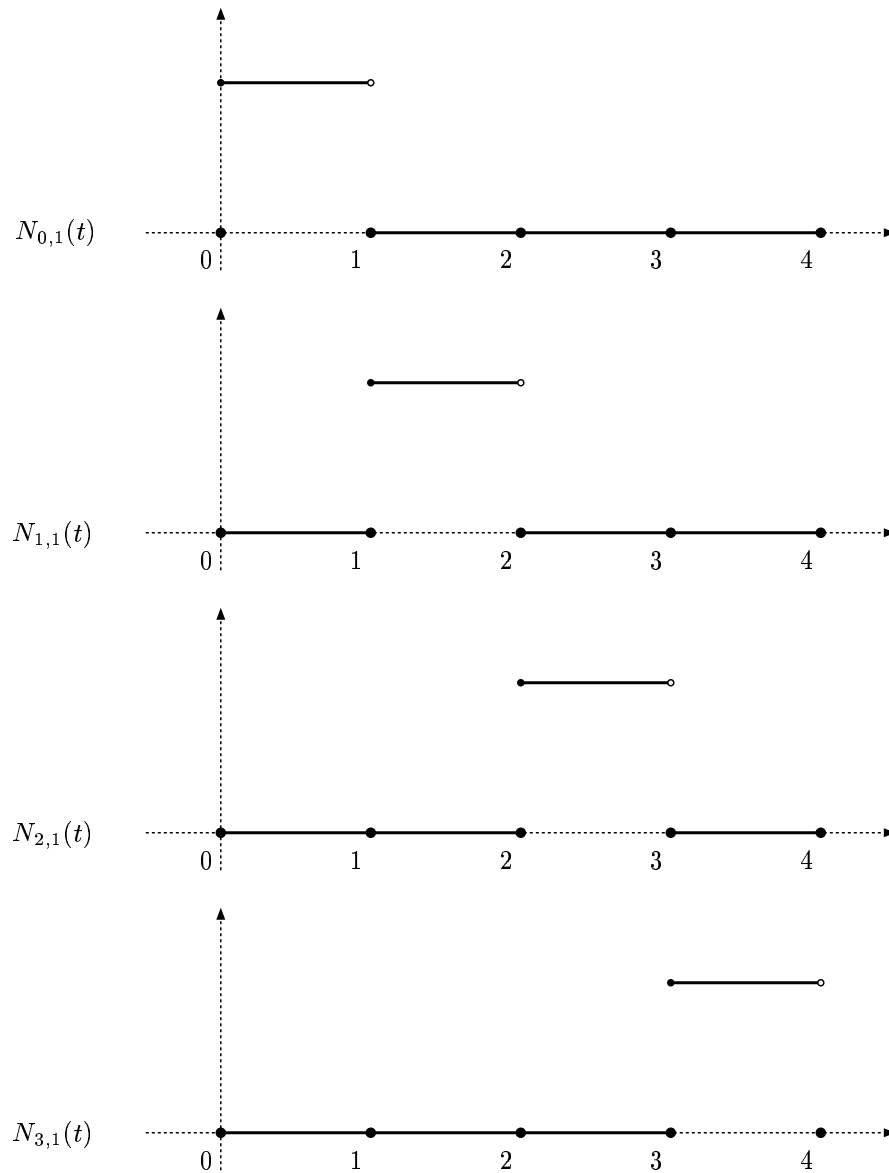
These functions have support (the region where the curve is nonzero) in an interval, with $N_{i,1}$ having support on $[i, i+1)$. They are also clearly shifted versions of each other – e.g., $N_{i+1,1}$ is just $N_{i,1}$ shifted one unit to the right. In fact, we can write $N_{i,1}(t) = N_{0,1}(t-i)$

Blending Functions for k = 2

If $k = 2$ then $N_{0,2}$ can be written as a weighted sum of $N_{0,1}$ and $N_{1,1}$ by equation (2). This gives

$$N_{0,2}(t) = \frac{t - t_0}{t_1 - t_0} N_{0,1}(t) + \frac{t_2 - t}{t_2 - t_1} N_{1,1}(t)$$

$$= t N_{0,1}(t) + (2 - t) N_{1,1}(t)$$

$$= \begin{cases} t & \text{if } 0 \leq t < 1 \\ 2 - t & \text{if } 1 \leq t < 2 \\ 0 & \text{otherwise} \end{cases}$$

This curve is shown in the following figure. The curve is piecewise linear, with support in the interval $[0, 2]$.
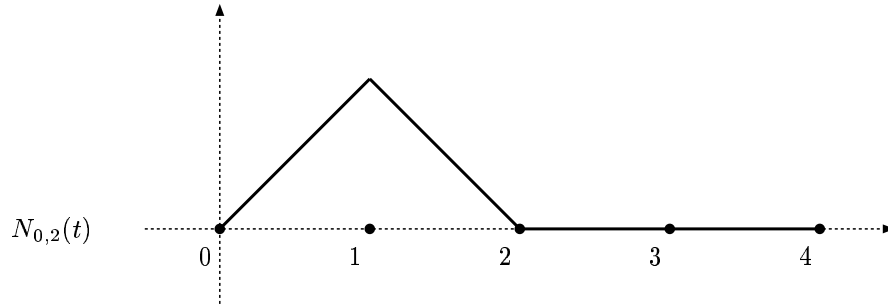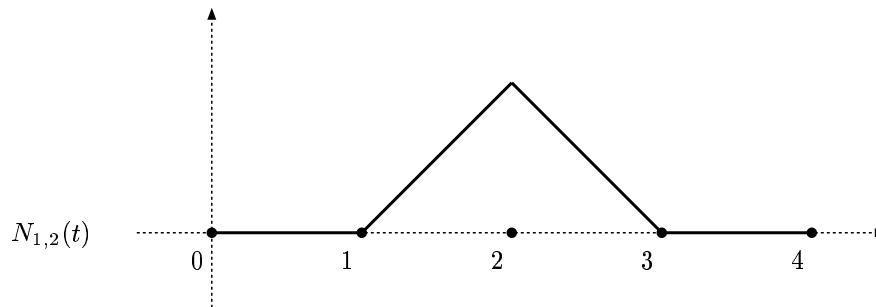

These functions are commonly referred to as "hat" functions and are used as blending functions in many linear interpolation problems.

Similarly, we can calculate $N_{1,2}$ to be

$$N_{1,2}(t) = \frac{t - t_1}{t_2 - t_1} N_{1,1}(t) + \frac{t_3 - t}{t_3 - t_2} N_{2,1}(t)$$

$$= (t - 1) N_{1,1}(t) + (3 - t) N_{2,1}(t)$$

$$= \begin{cases} t - 1 & \text{if } 1 \leq t < 2 \\ 3 - t & \text{if } 2 \leq t < 3 \\ 0 & \text{otherwise} \end{cases}$$

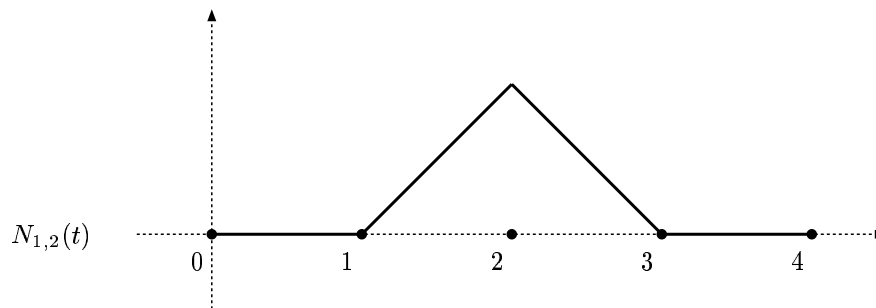This curve is shown in the following figure, and it is easily seen to be a shifted version of $N_{0,2}$.

$$N_{1,2}(t)$$

Finally, we have that

$$N_{1,2}(t) = \begin{cases} t - 2 & \text{if } 2 \leq t < 3 \\ 4 - t & \text{if } 3 \leq t < 4 \\ 0 & \text{otherwise} \end{cases}$$

which is shown in the following illustration.

$$N_{1,2}(t)$$

These nonzero portion of these curves each cover the intervals spanned by three knots – e.g., $N_{1,2}$ spans the interval $[1, 3]$. The curves are piecewise linear, made up of two linear segments joined continuously.

Sinve the curves are shifted versions of each other, we can write

$$N_{i,2}(t) = N_{0,2}(t - i)$$

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Blending Functions for k = 3

For the case $k = 3$, we again use equation (2) to obtain

$$N_{0,3}(t) = \frac{t - t_0}{t_2 - t_0} N_{0,2}(t) + \frac{t_3 - t}{t_3 - t_1} N_{1,2}(t)$$

$$= \frac{t}{2} N_{0,2}(t) + \frac{3 - t}{2} N_{1,2}(t)$$

$$= \begin{cases} \frac{t^2}{2} & \text{if } 0 \le t < 1 \\ \frac{t^2}{2}(2 - t) + \frac{3-t}{2}(t - 1) & \text{if } 1 \le t < 2 \\ \frac{(3-t)^2}{2} & \text{if } 2 \le t < 3 \\ 0 & \text{otherwise} \end{cases}$$

$$= \begin{cases} \frac{t^2}{2} & \text{if } 0 \le t < 1 \\ \frac{-3 + 6t - 2t^2}{2} & \text{if } 1 \le t < 2 \\ \frac{(3-t)^2}{2} & \text{if } 2 \le t < 3 \\ 0 & \text{otherwise} \end{cases}$$

and by nearly identical calculations,

$$N_{1,3}(t) = \frac{t - t_1}{t_3 - t_1} N_{1,2}(t) + \frac{t_4 - t}{t_4 - t_2} N_{2,2}(t)$$

$$= \frac{t - 1}{2} N_{1,2}(t) + \frac{4 - t}{2} N_{2,2}(t)$$

$$= \begin{cases} \frac{(t-1)^2}{2} & \text{if } 1 \le t < 2 \\ \frac{-11 + 10t - 2t^2}{2} & \text{if } 2 \le t < 3 \\ \frac{(4-t)^2}{2} & \text{if } 3 \le t < 4 \\ 0 & \text{otherwise} \end{cases}$$

These curves are shown in the following figure. They are clearly piecewise quadratic curves, each made up of three parabolic segments that are joined at the knot values (we have placed tick marks on the curves to show where they join).

$N_{1,3}(t)$

$N_{1,3}(t)$
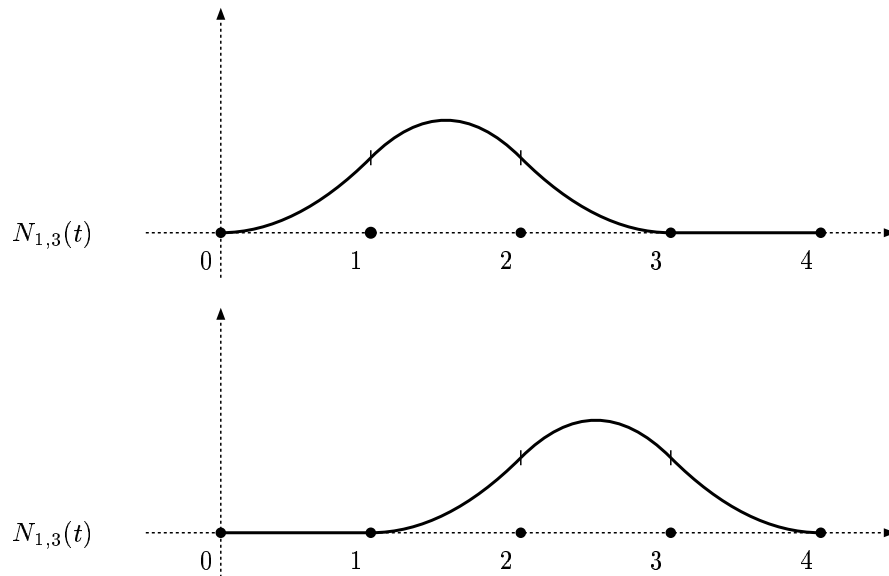
The nonzero portion of these two curves each span the interval between four consecutive knots – e.g., the nonzero portion of $N_{1,3}$ spans the interval $[1,4]$. Again, $N_{1,3}$ can be seen visually to be a shifted version of $N_{0,3}$. (This fact can also be seen analytically by substituting $t+1$ for t in the equation for $N_{1,3}$.) We can write

$$N_{i,3}(t) = N_{0,3}(t-i)$$

Blending Functions of Higher Orders

It is not too difficult to conclude that the $N_{i,4}$ blending functions will be piecewise cubic functions. The support of $N_{i,4}$ will be the interval $[i, i+4]$ and each of the blending functions will be shifted versions of each other, allowing us to write

$$N_{i,4}(t) = N_{0,4}(t-i)$$

In general, the uniform blending functions $N_{i,k}$ will be piecewise $k-1$st degree functions having support in the interval $[i, i+k)$. They will be shifted versions of each other and each can be written in terms of a "basic" function

$$N_{i,k}(t) = N_{0,k}(t-i)$$

7

Summary

In the case of the uniform knot sequence, the blending functions are fairly easy to calculate, are shifted versions of each other, and have support over a simple interval determined by the knots. These characteristics are unique to the uniform blending functions.

For other special characteristics see the notes that describe writing the uniform blending functions as a convolution, and the notes that describe the two-scale relation for uniform B-splines

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THE DEBOOR-COX CALCULATION

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Overview

In 1972, Carl DeBoor and M.G. Cox independently discovered the relationship between the analytic and geometric definitions of B-splines. Starting with the definition of the normalized B-spline blending functions, these two researchers were able to develop the geometric definition of the B-spline. It is this calculation that is discussed in this paper.

Definition of the B-Spline Curve

A B-spline curve $\mathbf{P}(t)$, is defined by

$$\mathbf{P}(t) = \sum_{i=0}^{n} \mathbf{P}_i N_{i,k}(t) \tag{1}$$

where

- the $\{\mathbf{P}_i : i = 0, 1, ..., n\}$ are the control points,

- k is the order of the polynomial segments of the B-spline curve. Order k means that the curve is made up of piecewise polynomial segments of degree $k - 1$,

- the $N_{i,k}(t)$ are the "normalized B-spline blending functions". They are described by the order k and by a non-decreasing sequence of real numbers

$$\{t_i : i = 0, ..., n + k\}.$$

normally called the "knot sequence". The $N_{i,k}$ functions are described as follows

$$N_{i,1}(t) = \begin{cases} 1 & \text{if } t \in [t_i, t_{i+1}), \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

and if $k > 1$,

$$N_{i,k}(t) = \left(\frac{t - t_i}{t_{i+k-1} - t_i} \right) N_{i,k-1}(t) + \left(\frac{t_{i+k} - t}{t_{i+k} - t_{i+1}} \right) N_{i+1,k-1}(t) \tag{3}$$

- the parameter t ranges throughout the interval $[t_{k-1}, t_{n+1})$.

We note that if, in equation (3), either of the N terms on the right hand side of the equation are zero, or the subscripts are out of the range of the summation limits, then the associated fraction is not evaluated and the term becomes zero. This is to avoid a zero-over-zero evaluation problem.

The order k is independent of the number of control points $(n + 1)$. In the B-Spline curve, unlike the Bézier Curve, we have the flexibility of using many control points, and restricting the degree of the polymonial segments.

The DeBoor-Cox Calculation

In the DeBoor-Cox calculation, we substitute the definition of $N_{i,k}(t)$ given in equation (3), into the right-hand side of

$$\mathbf{P}(t) = \sum_{i=0}^{n} \mathbf{P}_i N_{i,k}(t)$$

and simplify. This will give us the definition of $\mathbf{P}(t)$ in terms of $N_{i,k-1}$, which is of lower degree. The general idea is to continue this process until the sum is written with $N_{i,1}$ functions, which we can evaluate easily.

So here we go. If $t \in [t_{k-1}, t_{n+1})$, then by substituting equation (3) into the equation (1), we have

$$\mathbf{P}(t) = \sum_{i=0}^{n} \mathbf{P}_i N_{i,k}(t)$$

$$= \sum_{i=0}^{n} \mathbf{P}_i \left[\left(\frac{t - t_i}{t_{i+k-1} - t_i} \right) N_{i,k-1}(t) + \left(\frac{t_{i+k} - t}{t_{i+k} - t_{i+1}} \right) N_{i+1,k-1}(t) \right]$$

Distributing the sums, we obtain

$$= \sum_{i=0}^{n} \mathbf{P}_i \left(\frac{t - t_i}{t_{i+k-1} - t_i} \right) N_{i,k-1}(t) + \sum_{i=0}^{n} \mathbf{P}_i \left(\frac{t_{i+k} - t}{t_{i+k} - t_{i+1}} \right) N_{i+1,k-1}(t)$$

We now separate out those unique terms of each sum, $N_{0,k-1}$ and $N_{n+1,k-1}$, giving

$$= \mathbf{P}_0 \frac{t - t_0}{t_{k-1} - t_0} N_{0,k-1}(t) + \sum_{i=1}^{n} \mathbf{P}_i \left(\frac{t - t_i}{t_{i+k-1} - t_i} \right) N_{i,k-1}(t)$$

$$+ \sum_{i=0}^{n-1} \mathbf{P}_i \left(\frac{t_{i+k} - t}{t_{i+k} - t_{i+1}} \right) N_{i+1,k-1}(t) + \mathbf{P}_n \left(\frac{t_{n+k} - t}{t_{n+k} - t_{n+1}} \right) N_{n+1,k-1}(t)$$

Now since the support of a B-spline blending function $N_{i,k}(t)$ is the interval $[t_i, t_{i+k}]$, we have that $N_{0,k-1}$ is non-zero only if $t \in [t_0, t_{k-1})$, which is outside the interval $[t_{k-1}, t_{n+1})$ (where $\mathbf{P}(t)$ is defined). Thus, $N_{0,k-1}(t) \equiv 0$. Also $N_{n+1,k-1}$ is non-zero only if $t \in [t_{n+1}, t_{n+k-1})$, which is outside the interval

$[t_{k-1}, t_{n+1})$. Thus, $N_{n+1,k-1}(t) \equiv 0$ and we have

$$\mathbf{P}(t) = \sum_{i=1}^{n} \mathbf{P}_i \left[\frac{t - t_i}{t_{i+k-1} - t_i} \right] N_{i,k-1}(t) + \sum_{i=0}^{n-1} \mathbf{P}_i \left[\frac{t_{i+k} - t}{t_{i+k} - t_{i+1}} \right] N_{i+1,k-1}(t)$$

If we change the summation limits, we get

$$= \sum_{i=1}^{n} \mathbf{P}_i \left[\frac{t - t_i}{t_{i+k-1} - t_i} \right] N_{i,k-1}(t) + \sum_{i=1}^{n} \mathbf{P}_{i-1} \left[\frac{t_{i+k-1} - t}{t_{i+k-1} - t_i} \right] N_{i,k-1}(t)$$

Combining the two sums, and rewriting, we obtain

$$= \sum_{i=1}^{n} \left[\left(\frac{t_{i+k-1} - t}{t_{i+k-1} - t_i} \right) \mathbf{P}_{i-1} + \left(\frac{t - t_i}{t_{i+k-1} - t_i} \right) \mathbf{P}_i \right] N_{i,k-1}(t)$$

If we denote

$$\mathbf{P}_i^{(1)}(t) = \left(\frac{t_{i+k-1} - t}{t_{i+k-1} - t_i} \right) \mathbf{P}_{i-1} + \left(\frac{t - t_i}{t_{i+k-1} - t_i} \right) \mathbf{P}_i$$

then the above result may be written

$$\mathbf{P}(t) = \sum_{i=1}^{n} \mathbf{P}_i^{(1)}(t) N_{i,k-1}(t)$$

Now we have written the summation terms of equation (1) in terms of blending functions of lower degree. Of course, we have transferred some of the complexity to the $\mathbf{P}_i^{(1)}$s, but we retain a similar form with control points $\mathbf{P}_i^{(1)}$s weighted by blending functions, and we can repeat this calculation again.

Once again then, repeating the calculation and manipulating the sums, we obtain

$$\mathbf{P}(t) = \sum_{i=2}^{n} \mathbf{P}_i^{(2)}(t) N_{i,k-2}(t)$$

where

$$\mathbf{P}_i^{(2)}(t) = \left(\frac{t_{i+k-2} - t}{t_{i+k-2} - t_i} \right) \mathbf{P}_{i-1}^{(1)}(t) + \left(\frac{t - t_i}{t_{i+k-2} - t_i} \right) \mathbf{P}_i^{(1)}(t)$$

4

(Note the appearance of the 2s on the right-hand side of the equation.)

If we continue with this process again, we will manipulate the sum so that the blending functions have order $k - 3$. Then again with give us $k - 4$, and eventually we will obtain blending functions of order 1. We are lead to the following result: If we define

$$\mathbf{P}_i^{(j)}(t) = \begin{cases} (1 - \tau_i^j)\mathbf{P}_{i-1}^{(j-1)}(t) + \tau_i^j \mathbf{P}_i^{(j-1)}(t) & \text{if } j > 0 \\ \mathbf{P}_i & \text{if } j = 0 \end{cases} \tag{4}$$

where

$$\tau_i^j = \frac{t - t_i}{t_{i+k-j} - t_i}$$

Then, if t is in the interval $[t_l, t_{l+1})$, we have

$$\mathbf{P}(t) = \mathbf{P}_l^{(k-1)}(t)$$

This can be shown by continuing the DeBoor-Cox calculation $k - 1$ times. When complete, we arrive at the formula

$$\mathbf{P}(t) = \sum_{i=k-1}^{n} \mathbf{P}_i^{(k-1)}(t) N_{i,1}(t)$$

where $\mathbf{P}_i^{(j)}(t)$ is given in equation (4). (Note the algebraic simplification that the τ's provide.) If $t \in [t_l, t_{l+1}]$, then then the only nonzero term of the sum is the lth term, which is one, and the sum must equal $\mathbf{P}_l^{(k-1)}(t)$.

This enables us to define the geometrical definition of the B-spline curve.

Geometric Definition of the B-Spline Curve

Given a set of Control Points

$$\mathbf{P}_0, \mathbf{P}_1, ..., \mathbf{P}_n$$

and a set of knots

$$t_0, t_1, ..., t_{n+k}$$

The B-Spline curve of order k is defined to be

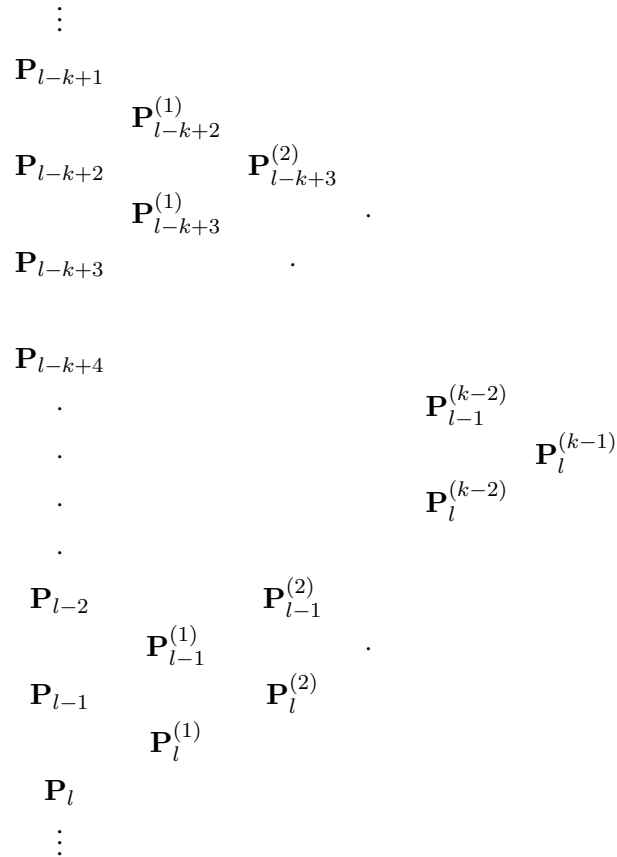$$\mathbf{P}(t) = \mathbf{P}_l^{(k-1)}(t) \text{if } t \in [t_l, t_{l+1})$$

where

$$\mathbf{P}_i^{(j)}(t) = \begin{cases} (1 - \tau_i^j)\mathbf{P}_{i-1}^{(j-1)}(t) + \tau_i^j\mathbf{P}_i^{(j-1)}(t) & \text{if } j > 0 \\ \mathbf{P}_i & \text{if } j = 0 \end{cases}$$

and

$$\tau_i^j = \frac{t - t_i}{t_{i+k-j} - t_i}$$

It is useful to view the geometric construction as the following pyramid

$$\vdots$$

$$\mathbf{P}_{l-k+1}$$
$$\mathbf{P}_{l-k+2}^{(1)}$$
$$\mathbf{P}_{l-k+2} \qquad \mathbf{P}_{l-k+3}^{(2)}$$
$$\mathbf{P}_{l-k+3}^{(1)}$$
$$\mathbf{P}_{l-k+3} \qquad \qquad .$$
$$.$$
$$\mathbf{P}_{l-k+4}$$
$$. \qquad\qquad\qquad\qquad \mathbf{P}_{l-1}^{(k-2)}$$
$$. \qquad\qquad\qquad\qquad\qquad \mathbf{P}_{l}^{(k-1)}$$
$$. \qquad\qquad\qquad\qquad \mathbf{P}_{l}^{(k-2)}$$
$$.$$
$$\mathbf{P}_{l-2} \qquad\qquad \mathbf{P}_{l-1}^{(2)}$$
$$\mathbf{P}_{l-1}^{(1)} \qquad\qquad .$$
$$\mathbf{P}_{l-1} \qquad\qquad \mathbf{P}_{l}^{(2)}$$
$$\mathbf{P}_{l}^{(1)}$$
$$\mathbf{P}_{l}$$

$$\vdots$$

Any $\mathbf{P}_i^{(j)}$ in this pyramid is calculated as a convex combination of the two $\mathbf{P}$ functions immediately to it's left.

Summary

The DeBoor-Cox calculation is a fundamental result in the geometric modeling field. It was used to exhibit the relation between the analytic definition of the B-Spline curve and the geometric definition of the curve. The geometric definition of the curve, because of its computational stability has become the primary technique by which points on these curves are calculated.

7

THE SUPPORT OF A
NORMALIZED B-SPLINE BLENDING FUNCTION

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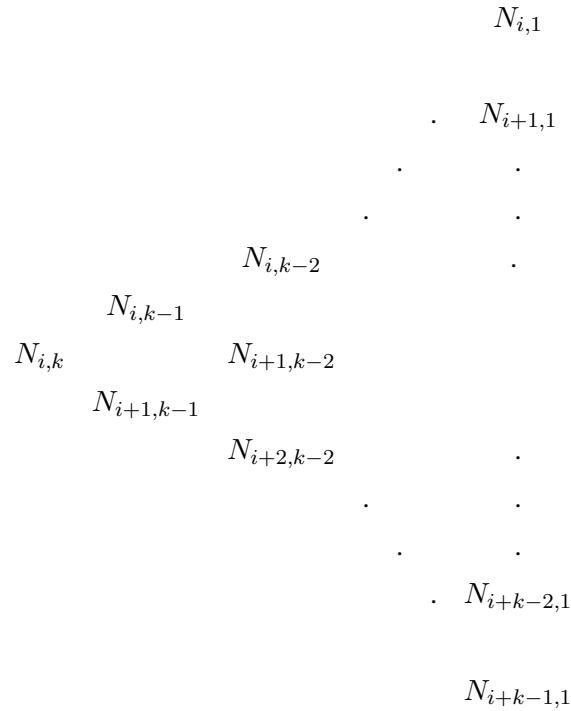University of California, Davis

Overview

A B-spline blending function $N_{i,k}(t)$ has compact support. This means that the function is zero outside of some interval. In these notes, We find this interval explicitly in terms of the knot sequence.

The Support of the Function

Given an order k, and a knot sequence $\{t_0, t_1, t_2, ..., t_{n+k}\}$, the normalized B-spline blending function $N_{i,k}(t)$ is positive if and only if $t \in [t_i, t_{i+k})$.

We can show that this is true by considering the following pyramid structure.

$$N_{i,1}$$

$$. \quad N_{i+1,1}$$

$$. \qquad .$$

$$. \qquad .$$

$$N_{i,k-2} \qquad .$$

$$N_{i,k-1}$$

$$N_{i,k} \qquad\qquad N_{i+1,k-2}$$

$$N_{i+1,k-1}$$

$$N_{i+2,k-2} \qquad .$$

$$. \qquad .$$

$$. \qquad .$$

$$. \quad N_{i+k-2,1}$$

$$N_{i+k-1,1}$$

The definition of the normalized blending function $N_{i,k}$ as a weighted sum of $N_{i,k-1}(t)$ and $N_{i+1,k-1}(t)$. Thus for any of the N functions in the pyramid, it is a weighted sum of the two items immediately to its right. If we follow the pyramid to its right edge, we see that the only blending functions $N_{j,1}$ that contribute to $N_{i,k}$ are those with $i \leq j \leq i + k - 1$, and these function are collectively nonzero when $t \in [t_i, t_{i+k})$.

Summary

A B-spline blending function has compact support. The support of this function depends on the knot sequence and always covers an interval of containing several knots – containing $k + 1$ knots if the curve is or order k.