Subdividing a Bezier Patch

- Subdivide separately along *u* and *v* parameters
- To subdivide along the *u* parameter, subdivide the curve corresponding to each row of the matrix (used to represent the matrix form of the Bezier patch):

$$[B_0^m(u) \;\ldots\; B_m^m(U)] \begin{bmatrix} \mathbf{b}_{00} & & \mathbf{b}_{0n} \\ & & \\ & & \mathbf{b}_{mn} \end{bmatrix} \begin{bmatrix} B_0^n(w) \\ \vdots \\ B_n^n(w) \end{bmatrix}$$

- To subdivide along the w parameter, subdivide the curve along each column using de Casteljau’s algorithm

Tensor Product B-Spline Patches

- A tensor product B-spline patch is given as:

$$\mathbf{P}(u,w) = \sum_{i=0} \sum_{j=0} \mathbf{b}_{ij} N_{i,k}(u) N_{j,l}(w)$$

where $N_{i,k}(u)$ are the B-spline basis function associated with a knot sequence. Similarly $N_{j,l}(v)$ is another B-spline basis function associated with a different knot sequence

- Evaluated using Cox-DeBoor algorithm
- Represented using matrix form
  - For knot insertion along u, use the curve rep. defined by each row
  - For knot insertion along w, use the curve rep. defined by each column

Rational Bezier Patches

- They are given as:

$$\mathbf{P}(u,w) = \frac{\sum_{i=0}\sum_{j=0} w_{ij}\mathbf{b}_{ij}B_i^m(u)B_j^n(w)}{\sum_{i=0}\sum_{j=0} w_{ij}B_i^m(u)B_j^n(w)}$$

- Rational surfaces are obtained as the projections of tensor product patches (in a higher dimension space), but they are not tensor product patches

- For a tensor product patch, the basis functions $F_{i,j}(u,w)$ can be expressed as products: $F_{i,j}(u,w) = A_i(u)B_j(v)$

Rational Bezier Patches

- The main benefit of rational Bezier patches are:
  - Exact representation of quadric surfaces (sphere, ellipsoid, cone etc.)
  - Exact representation of surfaces of revolution, given as:

$$\mathbf{P}(u,w) = \begin{bmatrix} r(w)\cos(u) \\ r(w)\sin(u) \\ z(w) \end{bmatrix}$$

If $r(w)$ and $z(w)$ are rational functions, than

$\mathbf{P}(u,w)$ can be represented using rational Bezier patches