Subdivision Curves & Surfaces

• Work of G. de Rham on Corner Cutting in 40’s and 50’s

• Work of Chaikin, Catmull/Clark and Doo/Sabin in 70’s

• Work of Loop in mid-80’s

• Work in 90’s: Pixar’s Geri’s Game (Academy Award)
Subdivision defines a smooth curve or surface as the limit of a sequence of successive refinements.

Each refined version is obtained by adding a point corresponding to each line segment.
Subdivision Surfaces

Example of subdivision for a surface, showing 3 successive levels of refinement. On the left an initial triangular mesh approximating the surface. Each triangle is split into 4 according to a particular subdivision rule (middle). On the right the mesh is subdivided in this fashion once again.
Subdivision Rules

• **Efficiency:** the location of new points should be computed with a small number of floating point operations

• **Compact support:** the region over which a point influences the shape of the final curve or surface should be small and finite

• **Local definition:** the rules used to determine where new points go should not depend on “far away” places

• **Affine invariance:** if the original set of points is transformed, e.g., translated, scaled, or rotated, the resulting shape should undergo the same transformation

• **Simplicity:** determining the rules themselves should preferably be an offline process and there should only be a small number of rules

• **Continuity:** what kind of properties can we prove about the resulting curves and surfaces, for example, are they differentiable?
Possible Advantages

• Handling of arbitrary topology

• Multi-resolution representation: accommodates LOD rendering and adaptive approximation with error bounds

• Uniformity of representations: polygons and patches

• Numerical stability: good properties for finite element solvers

• Implementation simplicity
NURBS vs Subdivision

- Subdivision surfaces do not have a global, closed form mathematical representation
  - NURBS easier to tessellate
  - Easy to compute global properties
  - Easier to implement in hardware
- Intersections and Boolean combinations: not well understood for subdivision surfaces; perhaps more messy…
- Bernstein basis: has some of the best numerical properties in terms of evaluation and intersection computations
- Uniform spline curves are a special case of subdivision curves