Subdivision Curves & Surfaces

•Work of G. de Rham on Corner Cutting in 40’s and 50’s

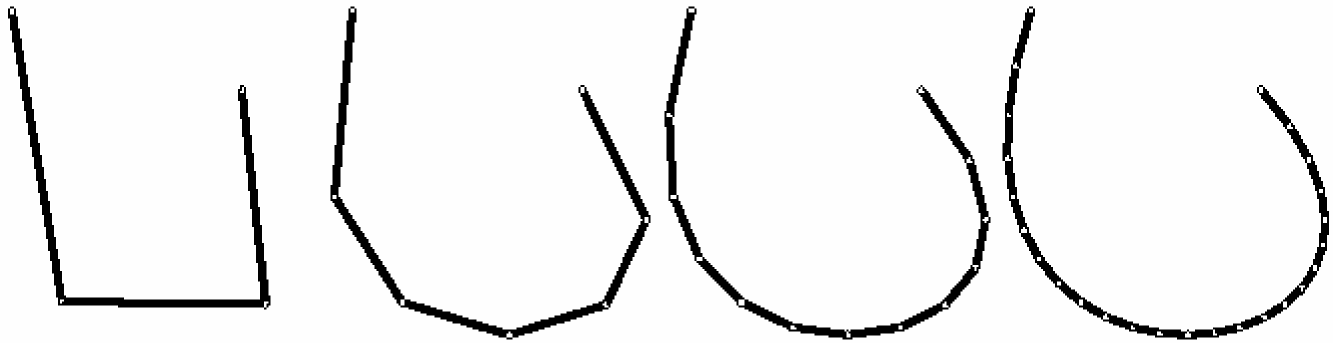
• Work of Chaikin, Catmull/Clark and Doo/Sabin in 70’s

•Work of Loop in mid-80’s

• Work in 90’s: Pixar’s Geri’s Game (Academy Award)

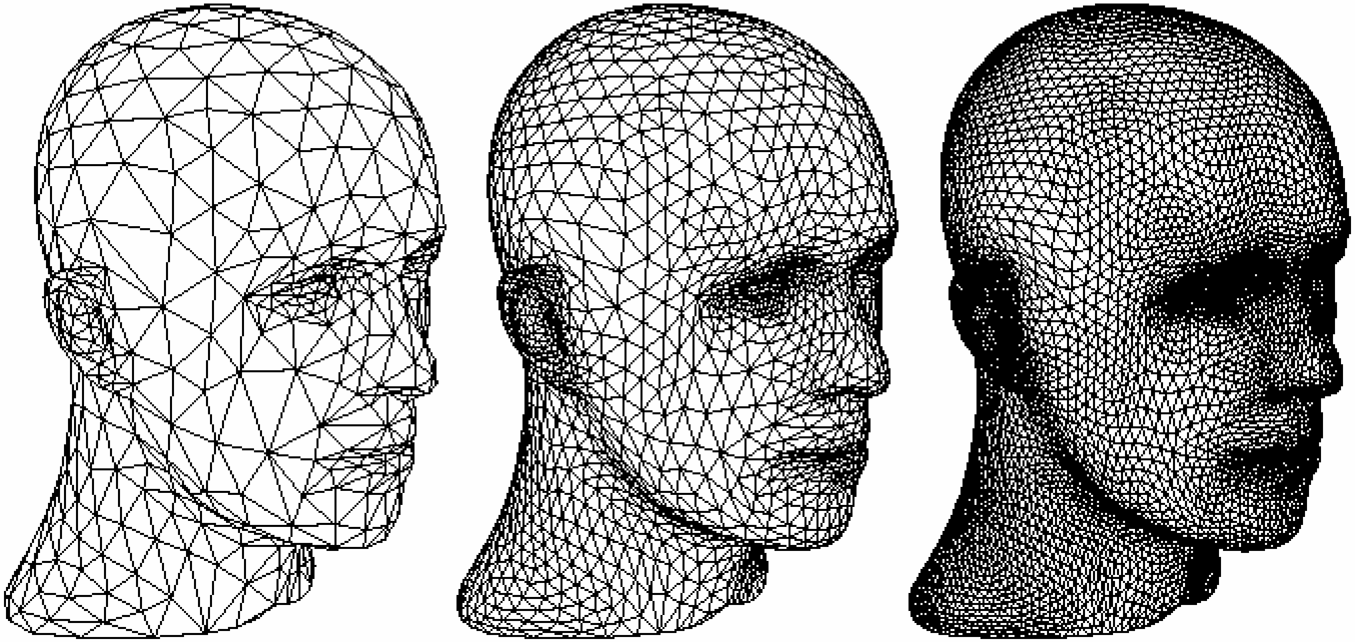
Subdivision

Subdivision defines a smooth curve or surface as the limit of a sequence of successive refinements



Each refined version is obtained by adding a point corresponding to each line segment

Subdivision Surfaces



Example of subdivision for a surface, showing 3 successive levels of refinement. On the left an initial triangular mesh approximating the surface. Each triangle is split into 4 according to a particular subdivision rule (middle). On the right the mesh is subdivided in this fashion once again.

Subdivision Rules

- ***Efficiency:*** the location of new points should be computed with a small number of floating point operations
- ***Compact support:*** the region over which a point influences the shape of the final curve or surface should be small and finite
- ***Local definition:*** the rules used to determine where new points go should not depend on “far away” places
- ***Affine invariance:*** if the original set of points is transformed, e.g., translated, scaled, or rotated, the resulting shape should undergo the same transformation
- ***Simplicity:*** determining the rules themselves should preferably be an offline process and there should only be a small number of rules
- ***Continuity:*** what kind of properties can we prove about the resulting curves and surfaces, for example, are they differentiable?

Possible Advantages

- Handling of arbitrary topology

- Multi-resolution representation: accommodates LOD rendering and adaptive approximation with error bounds

- Uniformity of representations: polygons and patches

- Numerical stability: good properties for finite element solvers

- Implementation simplicity

NURBS vs Subdivision

- Subdivision surfaces do not have a global, closed form mathematical representation
    - NURBS easier to tessellate
    - Easy to compute global properties
    - Easier to implement in hardware
- Intersections and Boolean combinations: not well understood for subdivision surfaces; perhaps more messy…
- Bernstein basis: has some of the best numerical properties in terms of evaluation and intersection computations
- Uniform spline curves are a special case of subdivision curves