LOOP SURFACES

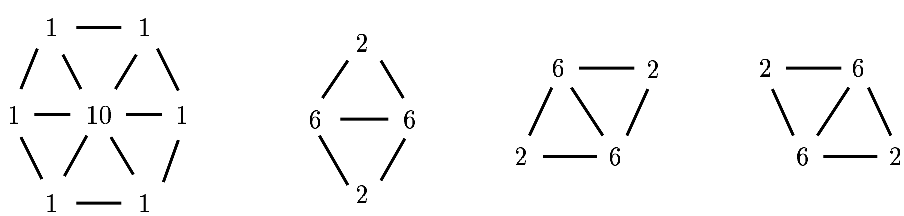
Kenneth I. Joy
Visualization and Graphics Research Group
Department of Computer Science
University of California, Davis

Overview

Loop surfaces are similar to Doo-Sabin or Catmull-Clark surfaces in that they are based upon the subdivision paradigm. However, as the Doo-Sabin and Catmull-Clark methods are based upon quadratic and cubic uniform B-spline surface subdivision, the Loop algorithm is based upon the subdivision of quartic uniform box splines – and therefore a mesh of triangles.

Loop Surfaces

Given a triangular mesh, the Loop refinement scheme generates both vertex points and edge points and utilizes the following subdivision masks



vertex mask   edge masks

The vertex mask generates new control points for each vertex, and the edge masks generate new control points for each edge of the original triangular mesh.

The edge masks compute the new edge points as the average of three values : the two centers of the faces that share the edge and the midpoint of the edge. The vertex mask can be stated as a convex combination of the points $\mathbf{V}$, the original vertex, and $\mathbf{Q}$ the average of the original points that share an edge with $\mathbf{V}$. This convex combination can be seen to be the following: If $\mathbf{V}^1$ is the new vertex point, then

$$\begin{aligned}\mathbf{V}^1 &= \frac{10\mathbf{V} + \mathbf{Q}_1 + \mathbf{Q}_2 + \mathbf{Q}_3 + \mathbf{Q}_4 + \mathbf{Q}_5 + \mathbf{Q}_6}{16} \\ &= \frac{5}{8}\mathbf{V} + \frac{\mathbf{Q}_1 + \mathbf{Q}_2 + \mathbf{Q}_3 + \mathbf{Q}_4 + \mathbf{Q}_5 + \mathbf{Q}_6}{16} \\ &= \frac{5}{8}\mathbf{V} + \frac{6\mathbf{Q}}{16} \\ &= \frac{5}{8}\mathbf{V} + \frac{3}{8}\mathbf{Q}\end{aligned}$$

Specifying the Refinement Procedure

For an arbitrary triangular mesh we can apply the same rules to generate the new edge points and new vertex points for the refined mesh. However, Loop noted that with the above vertex rule, the surface exhibited some points for which a tangent plane was discontinuous. Upon further examination, he noted that a rule of the form

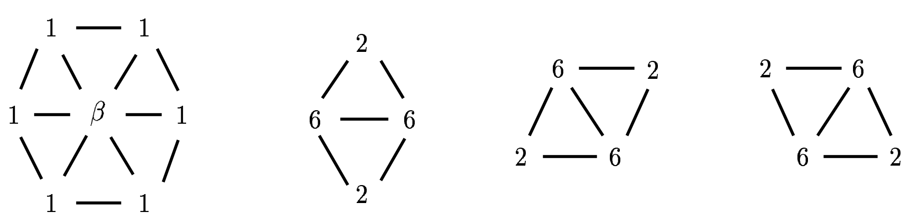
$$\mathbf{V}^1 = \alpha_n \mathbf{V} + (1 - \alpha_n)\mathbf{Q}$$

where

$$\alpha_n = \left(\frac{3}{8} + \frac{1}{4}\cos\frac{2\pi}{n}\right)^2 + \frac{3}{8}$$

gave a surface with a smooth tangent surface. (We note that when $n = \frac{1}{2}$ then $\alpha_n = \frac{5}{8}$, as required).

These surfaces are known as *Loop surfaces* and are generated with the subdivision masks



vertex mask

edge masks

where for a vertex of n edges, β is utilized to find the new vertex control point. That is

$$\alpha_n \mathbf{V} + (1 - \alpha_n)\mathbf{Q} = \frac{\beta\mathbf{V} + n\mathbf{Q}}{\beta + n}$$

or

$$\alpha_n = \frac{\beta}{\beta + n}$$

or

$$\beta = \frac{n\alpha_n}{1 - \alpha_n}$$
