Bezier Curves

- Interpolating curve
- Polynomial or rational parametrization using Bernstein basis functions
- Use of control points
  - Piecewise segments defining control polygon or characteristic polygon
  - Algebraically: used for linear combination of basis functions

Properties of Basis Functions

- Interpolate the first and last control points, $\mathbf{P}_0$ and $\mathbf{P}_n$.
- The tangent at $\mathbf{P}_0$ is given by $\mathbf{P}_1 - \mathbf{P}_0$ and at $\mathbf{P}_n$ is given by $\mathbf{P}_n - \mathbf{P}_{n-1}$
- Generalize to higher order derivatives: second derivate at $\mathbf{P}_0$ is determined by $\mathbf{P}_0$, $\mathbf{P}_1$ and $\mathbf{P}_2$ and the same for higher order derivatives
- The functions are symmetric w.r.t. u and $(1-u)$. That is if we reverse the sequence of control points to $\mathbf{P}_n \mathbf{P}_{n-1} \mathbf{P}_{n-2} \ldots \mathbf{P}_0$, it defines the same curve.

Bezier Basis Function

Use of Bernstein polynomials:

$$\mathbf{P}(u) = \sum_{i=0}^{n} \mathbf{P}_i B_{i,n}(u) \qquad u \in [0,1]$$

Where

$$B_{i,n}(u) = \binom{n}{i} u^i (1-u)^{n-i}$$

Cubic Bezier Curve: Matrix Representation

Let $\mathbf{B} = [\mathbf{P_0}\ \ \mathbf{P_1}\ \ \mathbf{P_2}\ \ \mathbf{P_3}]$

$\mathbf{F} = [B_1(u)\ \ \mathbf{B}_2(u)\ \ \mathbf{B}_3(u)\ \ \mathbf{B}_4(u)]$ or

$$\mathbf{F} = [u^3\ u^2\ u\ 1] \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & 6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

This is the 4 X 4 Bezier basis transformation matrix.

$$\mathbf{P}(u) = \mathbf{U}\,\mathbf{M_B}\,\mathbf{P}, \text{ where}$$

$$U = [u^3\ \ u^2\ \ u\ \ 1]$$

Properties of Bezier Curves

- Invariance under affine transformation
- Convex hull property
- Variation diminishing
- De Casteljau Evaluation (Geometric computation)
- Symmetry
- Linear precision