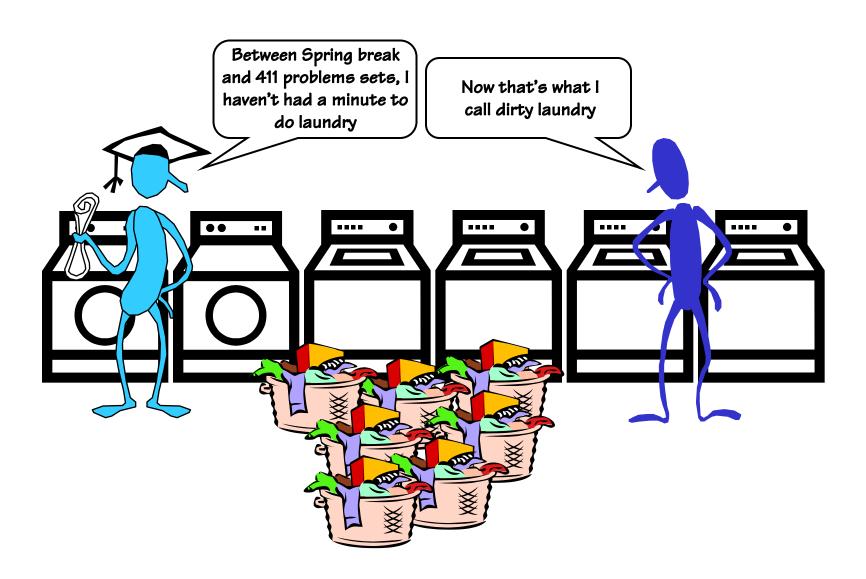
# Pipelining



### Forget 411... Let's Solve a "Relevant Problem"

**INPUT:** dirty laundry



**OUTPUT:** 4 more weeks





Device: Washer

Function: Fill, Agitate, Spin

Washer<sub>PD</sub> = 30 mins



Device: Dryer

Function: Heat, Spin

 $Dryer_{PD} = 60 \text{ mins}$ 

### One Load at a Time

Everyone knows that the real reason that UNC students put off doing laundry so long is \*not\* because they procrastinate, are lazy, or even have better things to do.

The fact is, doing laundry one load at a time is not smart.

(Sorry Mom, but you were wrong about this one!)

Step 1:

Step 2:

Total = Washer<sub>PD</sub> + Dryer<sub>PD</sub>
= \_\_\_\_ mins

### One Load at a Time

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(Sorry Mom, but you were wrong about this one!)

Step 1:

Step 2:

Total = Washer<sub>PD</sub> + Dryer<sub>PD</sub> = 90 mins

# Doing N Loads of Laundry

Here's how they do laundry at Duke, the "combinational" way.

(Actually, this is just an urban legend. No one at Duke actually does laundry. The butler's all arrive on Wednesday morning, pick up the dirty laundry and return it all pressed and starched by dinner)





Step 2:



Step 3:



Step 4:



•••



Total = N\*(Washer<sub>PD</sub> + Dryer<sub>PD</sub>)
= mins

# Doing N Loads of Laundry

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Step 2:



Step 3:



Step 4:



•••



Total = 
$$N*(Washer_{PD} + Dryer_{PD})$$
  
=  $N*90$  mins

## Doing N Loads... the UNC way

UNC students "pipeline" the laundry process.

Step 1:



Step 2:



That's why we wait!

Step 3:



•••

## Doing N Loads... the UNC way

UNC students "pipeline" the laundry process.

Step 1:



Step 2:



That's why we wait!

Step 3:



• • •

# Doing N Loads... the UNC way

UNC students "pipeline" the laundry process.

Step 1:



Step 2:



That's why we wait!

Step 3:



Actually, it's more like N\*60 + 30 if we account for the startup transient correctly. When doing pipeline analysis, we're mostly interested in the "steady state" where we assume we have an infinite supply of inputs.

Total =  $N * Max(Washer_{PD}, Dryer_{PD})$ 

N\*60 mins

#### Latency:

The delay from when an input is established until the output associated with that input becomes valid.

```
(Duke Laundry = _____ mins)
( UNC Laundry = ____ mins)
```

### Throughput:

```
(Duke Laundry = _____ outputs/min)
(UNC Laundry = ____ outputs/min)
```

#### Latency:

The delay from when an input is established until the output associated with that input becomes valid.

### Throughput:

```
(Duke Laundry = _____ outputs/min)
(UNC Laundry = ____ outputs/min)
```

#### Latency:

The delay from when an input is established until the output associated with that input becomes valid.

(Duke Laundry = 
$$\frac{90}{120}$$
 mins)  
(UNC Laundry =  $\frac{120}{120}$  mins)

### Throughput:

```
(Duke Laundry = _____ outputs/min)
(UNC Laundry = ____ outputs/min)
```

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The delay from when an input is established until the output associated with that input becomes valid.

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$$\frac{90}{120}$$
 mins)  
(UNC Laundry =  $\frac{120}{120}$  mins)

Assuming that the wash is started as soon as possible and waits (wet) in the washer until dryer is available.

### Throughput:

#### Latency:

The delay from when an input is established until the output associated with that input becomes valid.

(Duke Laundry = 
$$\frac{90}{120}$$
 mins)  
(UNC Laundry =  $\frac{120}{120}$  mins)

Assuming that the wash is started as soon as possible and waits (wet) in the washer until dryer is available.

### Throughput:

(Duke Laundry = 
$$\frac{1/90}{}$$
 outputs/min)  
(UNC Laundry =  $\frac{1}{}$  outputs/min)

#### Latency:

The delay from when an input is established until the output associated with that input becomes valid.

(Duke Laundry = 
$$\frac{90}{120}$$
 mins)  
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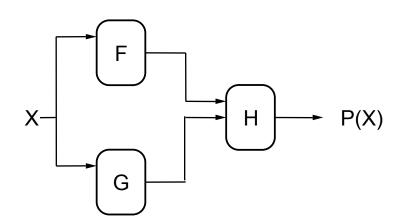
### Throughput:

The rate of which inputs or outputs are processed.

(Duke Laundry = 
$$\frac{1/90}{}$$
 outputs/min)  
(UNC Laundry =  $\frac{1/60}{}$  outputs/min)

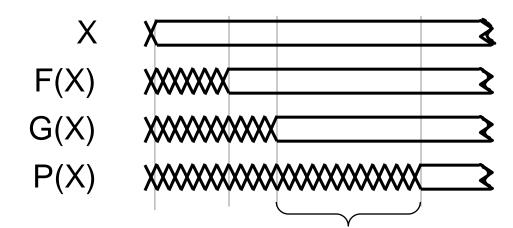
Even though we increase latency, it takes less time per load

## Okay, Back to Circuits...



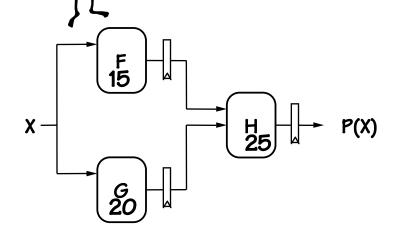
For combinational logic:

latency =  $t_{PD}$ , throughput =  $1/t_{PD}$ . We can't get the answer faster, but are we making effective use of our hardware at all times?



F & G are "idle", just holding their outputs stable while H performs its computation

use registers to hold H's input stable!

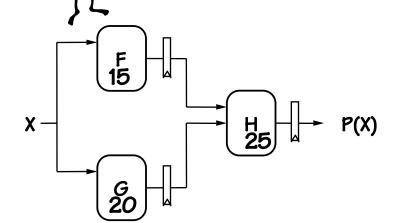


Now F & G can be working on input  $X_{i+1}$ while H is performing its computation on X<sub>i</sub>. We've created a 2-stage pipeline: if we have a valid input X during clock cycle j, P(X) is valid during clock j+2.

Suppose F, G, H have propagation delays of 15, 20, 25 ns and we are using ideal zero-delay registers ( $t_s = 0$ ,  $t_{pd} = 0$ ):

unpipelined 2-stage pipeline latency throughput 1/45

use registers to hold H's input stable!



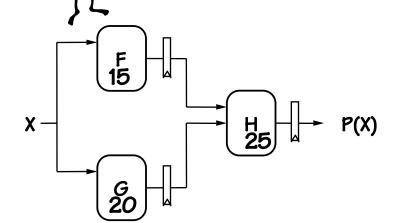
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unpipelined 2-stage pipeline

latency	throughput
45	1/45
50	
worse	

use registers to hold H's input stable!



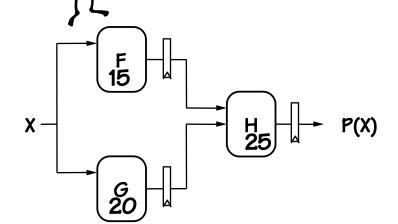
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unpipelined 2-stage pipeline

latency	throughput
45	1/45
50	1/25
worse	_ better \
	* Of

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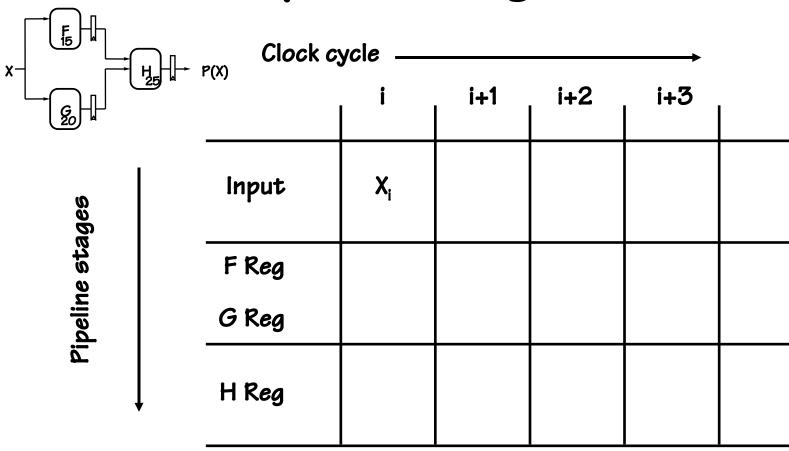
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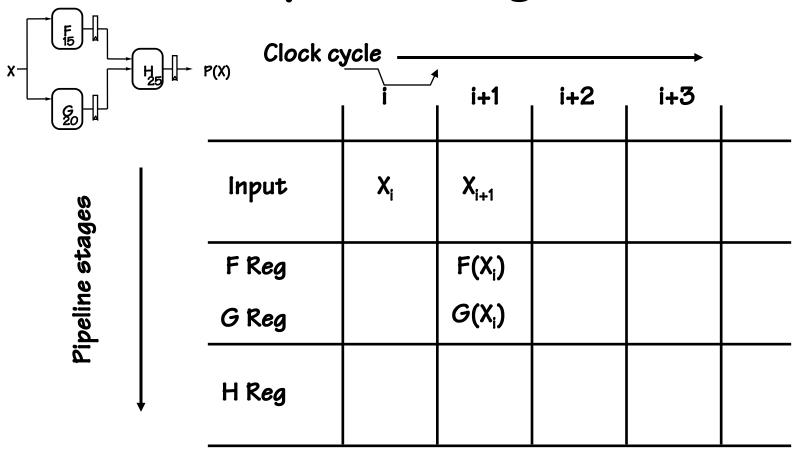
are using ideal zero-delay registers ( $t_s = 0$ ,  $t_{pd} = 0$ ):

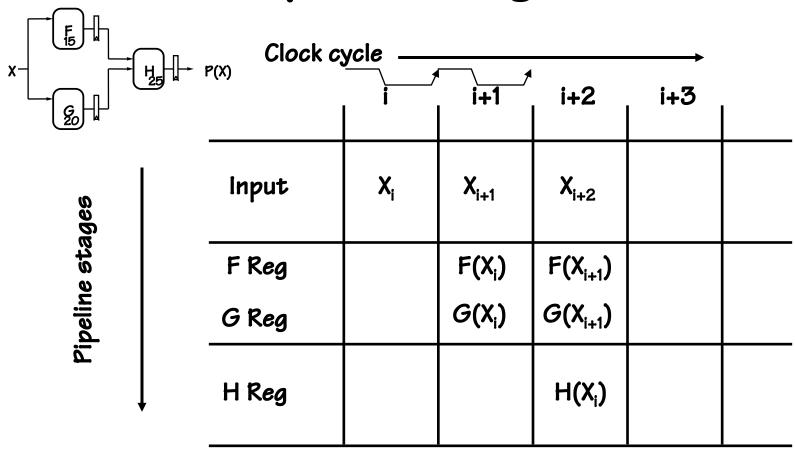
unpipelined 2-stage pipeline

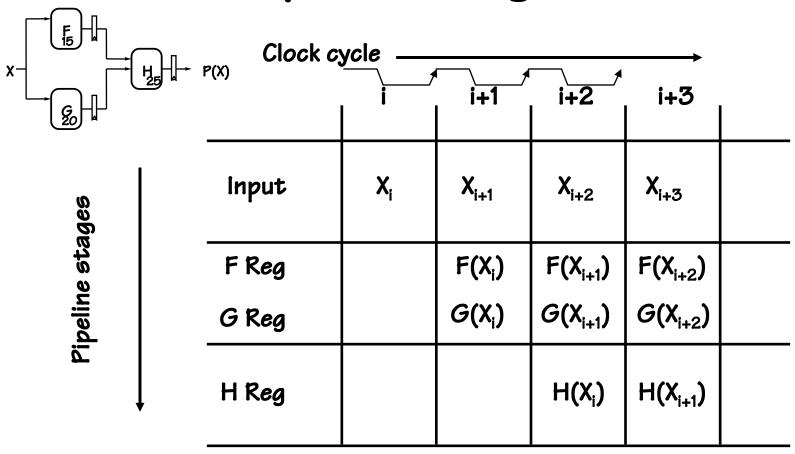
<u>latency</u>	throughput
45	1/45
50	1/25
worse	better

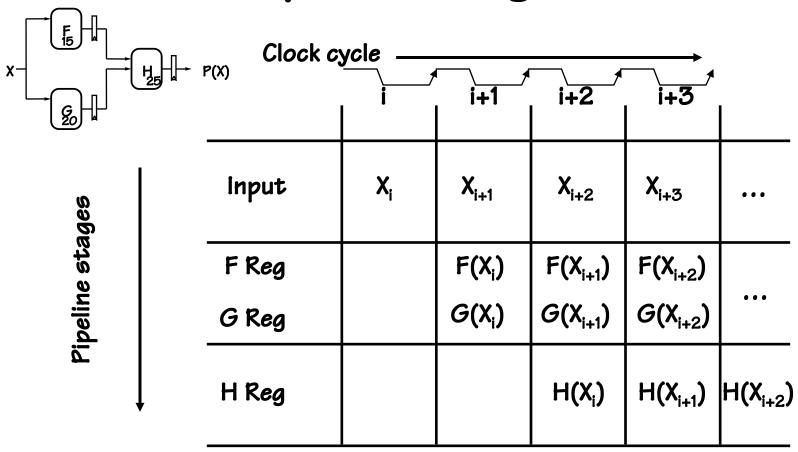
Pipelining uses registers to improve the throughput of combinational circuits

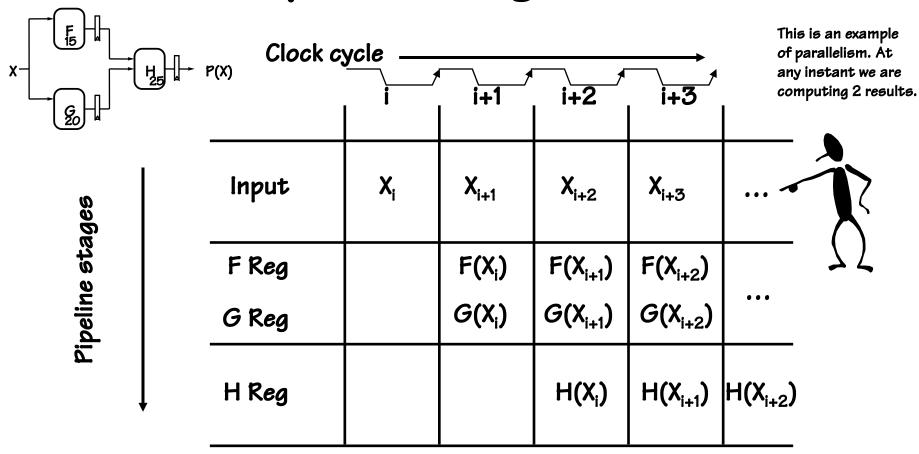












## Pipeline Conventions

#### **DEFINITION:**

a K-Stage Pipeline ("K-pipeline") is an acyclic circuit having exactly K registers on every path from an input to an output.

a COMBINATIONAL CIRCUIT is thus a O-stage pipeline.

#### **CONVENTION:**

Every pipeline stage, hence every K-Stage pipeline, has a register on its OUTPUT (not on its input).

#### **ALWAYS:**

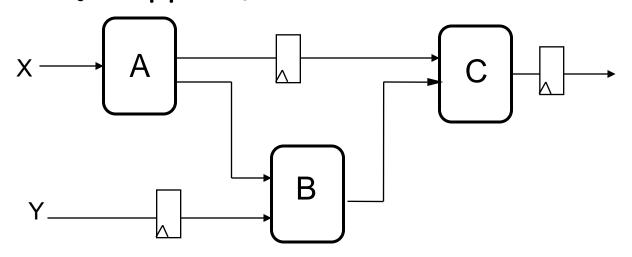
The CLOCK common to all registers \*must\* have a period sufficient to allow for the propagation delays of all combinational paths PLUS (input) register  $t_{PD}$  PLUS (output) register  $t_{SETUP}$ .

The LATENCY of a K-pipeline is K times the period of the clock common to all registers.

The THROUGHPUT of a K-pipeline is the frequency of the clock.

# Ill-Formed Pipelines

Consider a BAD job of pipelining:

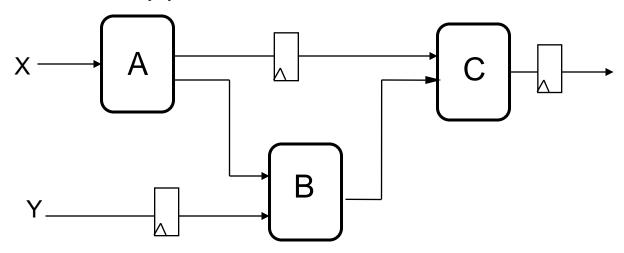


For what value of K is the following circuit a K-Pipeline? ANS: \_\_\_\_\_

Comp 411 L16 - Pipelining 29

# Ill-Formed Pipelines

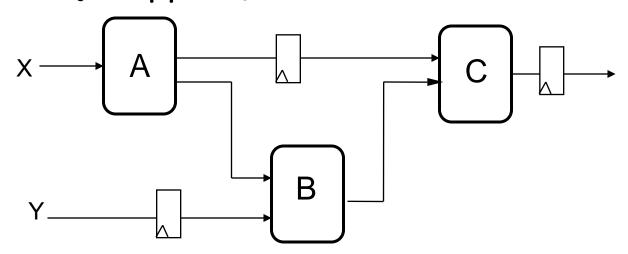
Consider a BAD job of pipelining:



For what value of K is the following circuit a K-Pipeline? ANS: \_ none

## Ill-Formed Pipelines

Consider a BAD job of pipelining:



none For what value of K is the following circuit a K-Pipeline? ANS: \_

#### Problem:

Successive inputs get mixed: e.g.,  $B(A(X_{i+1}), Y_i)$ . This happened because some paths from inputs to outputs had 2 registers, and some had only 1!

Can this happen on a well-formed K pipeline?

#### Step 1:

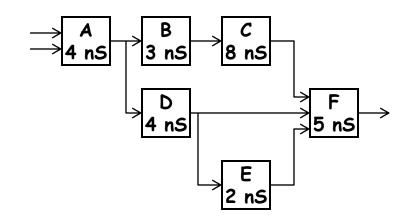
Draw a line that crosses every output in the circuit, and select one endpoint as an origin.

#### Step 2:

Continue to draw new lines from the origin across various circuit connections such that these new lines partition the inputs from the outputs.

Adding a pipeline register at every point where a separating line crosses a connection will always generate a valid pipeline.

#### STRATEGY:



#### Step 1:

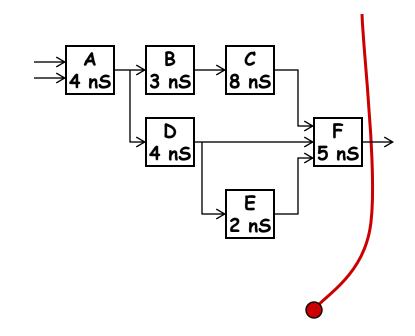
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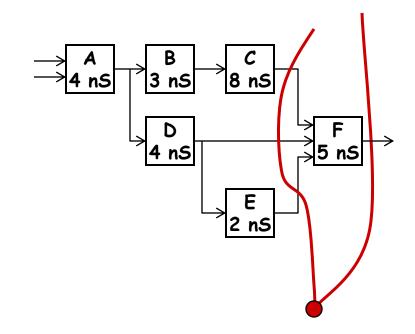
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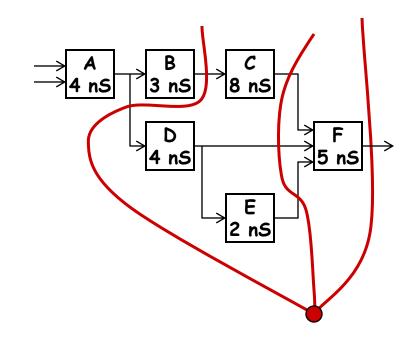
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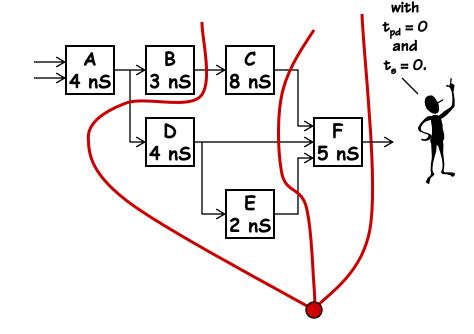
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#### STRATEGY:

Focus your attention on placing pipelining registers around the In these slowest circuit elements examples (BOTTLENECKS). we assume "idealized"



pipeline registers,

# A Pipelining Methodology

#### Step 1:

Draw a line that crosses every output in the circuit, and select one endpoint as an origin.

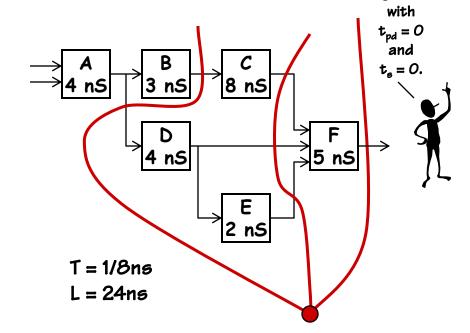
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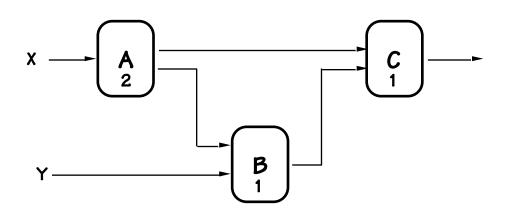
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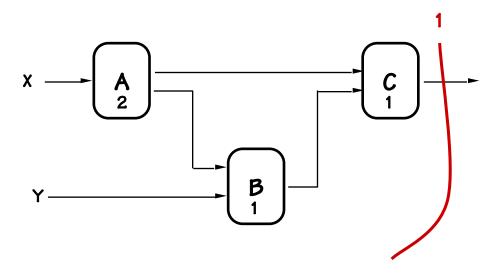


pipeline registers,



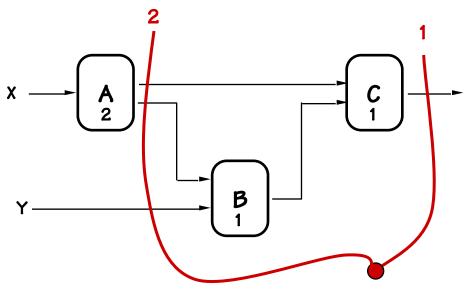
	LATENCY	THROUGHPUT
O-pipe:	4	1/4
1-pipe:		
2-pipe:		
3-pipe:		

- 1-pipeline improves neither L or T.
- T improved by breaking long combinational paths, allowing faster clock.
- Too many stages cost L, don't improve T.
- Back-to-back registers are often required to keep pipeline well-formed.



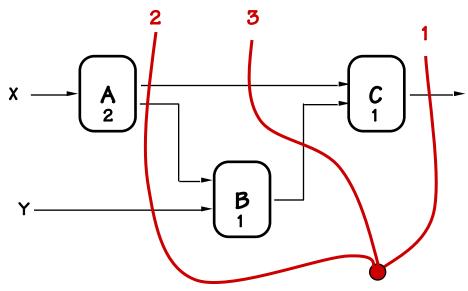
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	LATENCY	THROUGHPUT
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	LATENCY	THROUGHPUT
O-pipe:	4	1/4
1-pipe:	4	1/4
2-pipe:	4	1/2
3-pipe:	6	1/2

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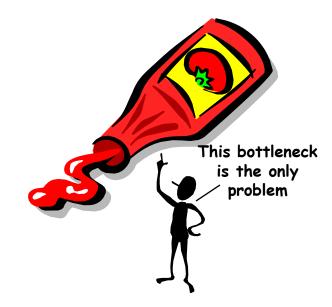
# Pipelining Summary

### Advantages:

- Higher throughput than combinational system
- Different parts of the logic work on different parts of the problem...

### Disadvantages:

- Generally, increases latency
- Only as good as the \*weakest\* link
   (often called the pipeline's BOTTLENECK)



Isn't there a way around this "weak link" problem?

They work around the bottleneck. First, they find a place with twice as many dryers as washers.

Throughput = \_\_\_\_ loads/min

mins/load Latency = \_\_\_\_\_

Step 1:

They work around the bottleneck.

First, they find a place with

twice as many dryers as

washers.

Throughput = \_\_\_\_ loads/min







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mins/load Latency =

Step 1:



Step 2:



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First, they find a place with

twice as many dryers as

washers.

Throughput = \_\_\_\_ loads/min





Step 2:



Step 3:



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First, they find a place with

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Throughput = \_\_\_\_ loads/min





Step 2:



Step 3:



They work around the bottleneck.

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Throughput = \_\_\_\_ loads/min





Step 2:



Step 3:



Step 4:



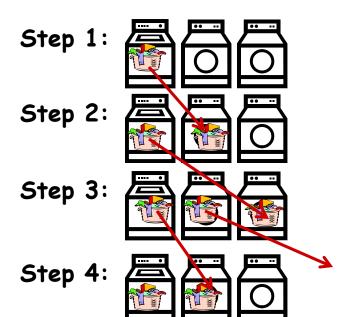
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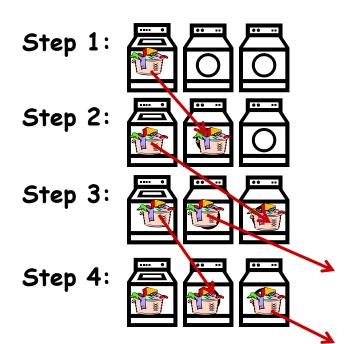
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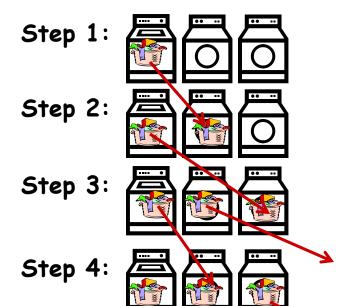
Throughput = \_\_\_\_ loads/min



They work around the bottleneck. First, they find a place with twice as many dryers as washers.

Throughput = \_\_\_\_ loads/min

mins/load Latency =



They work around the bottleneck. First, they find a place with twice as many dryers as washers.

1/30 Throughput = loads/min

mins/load Latency =

Step 1:



Step 2:



Step 3:



Step 4:



They work around the bottleneck.

First, they find a place with

twice as many dryers as

washers.

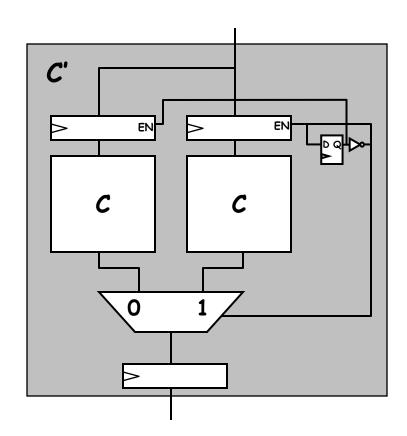
Throughput = 
$$\frac{1/30}{1}$$
 loads/min

Latency = 
$$\frac{90}{\text{mins/load}}$$

One way to overcome a pipeline bottleneck is to replicate the critical element as many times as needed and alternate inputs between the various copies.

N-way interleaving is equivalent to how many pipeline Stages? \_\_\_\_



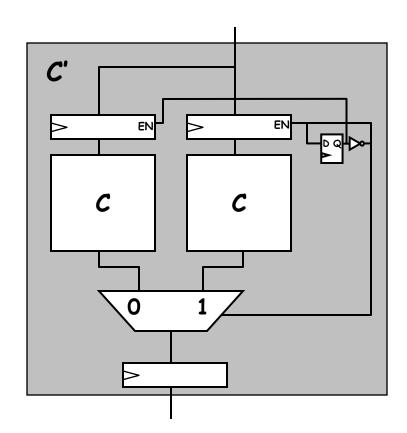


Latency = 2 clocks

One way to overcome a pipeline bottleneck is to replicate the critical element as many times as needed and alternate inputs between the various copies.

N-way interleaving is equivalent to how many pipeline Stages? N





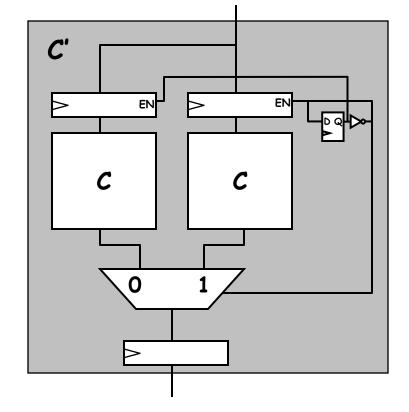
Latency = 2 clocks

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N-way interleaving is equivalent to how many pipeline Stages? N

> But with a reaister constrained to be on the inputs

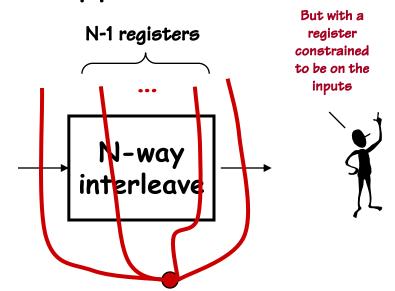


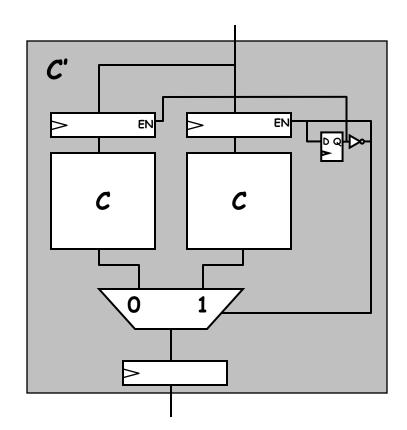


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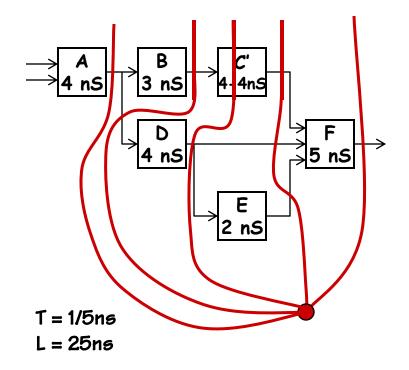


Latency = 2 clocks

# Combining Techniques

We can combine interleaving and pipelining. Here, C' interleaves two C elements with a propagation delay of 8 nS. The resulting C' circuit has a throughput of 4 nS, and latency of 8 nS. This can be considered as an extra pipelining stage that passes through the middle of the C' module. One of our separation lines must pass through this pipeline stage.

By combining interleaving with pipelining we move the bottleneck from the C element to the F element.





We can combine interleaving and pipelining with parallelism.

Throughput = load/min







We can combine interleaving and pipelining with parallelism.

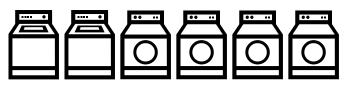


Throughput =

load/min

















We can combine interleaving and pipelining with parallelism.

Throughput = load/min

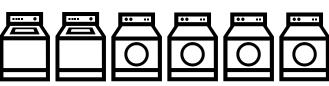


Step 2:









We can combine interleaving and pipelining with parallelism.

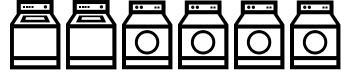
Throughput = load/min



Step 2:









We can combine interleaving and pipelining with parallelism.

Throughput = load/min

Step 1



Step 2:



Step 3



Step 4:



We can combine interleaving and pipelining with parallelism.

Throughput = load/min



Step 2:



Step 3:



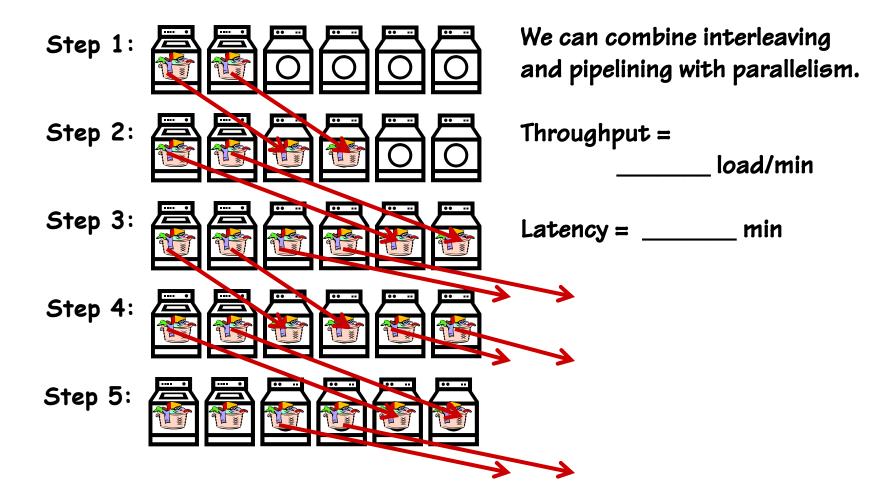


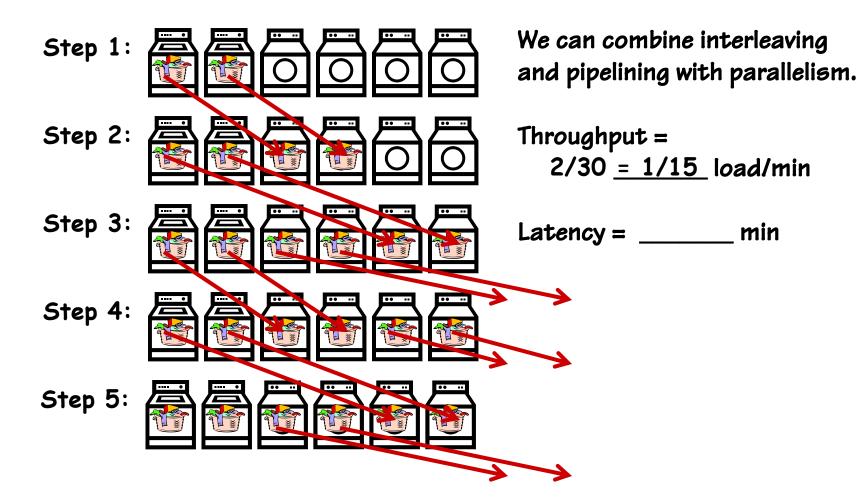
Step 5:

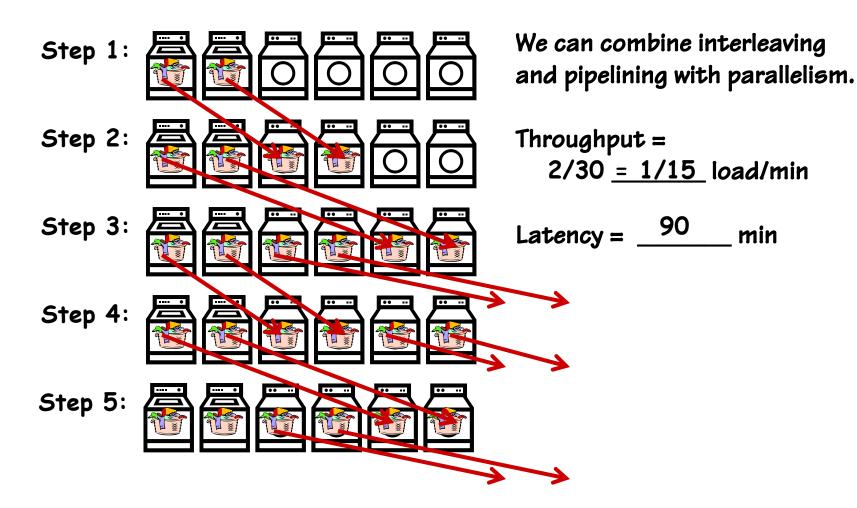


We can combine interleaving and pipelining with parallelism.

Throughput = load/min







#### **Synchronous**

ALL computation "events" occur at active edges of a periodic clock: time is divided into fixed-size discrete intervals.



#### Globally Timed

Timing dictated by centralized FSM according to a fixed schedule.

### Asynchronous

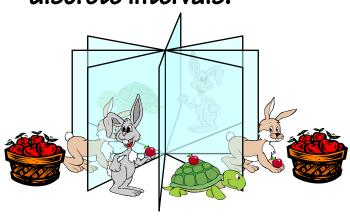
Events - e.g. the loading of a register -- can happen at at arbitrary times.



### Locally Timed

#### **Synchronous**

ALL computation "events" occur at active edges of a periodic clock: time is divided into fixed-size discrete intervals.



Asynchronous

Events – e.g. the loading of a register -- can happen at at arbitrary times.



Timing dictated by

to a fixed schedule.

Laid Back

### Locally Timed

Globally Timed

centralized FSM according

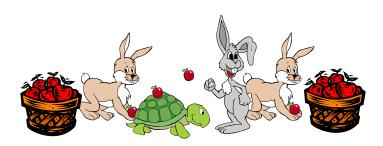
#### **Synchronous**

ALL computation "events" occur at active edges of a periodic clock: time is divided into fixed-size discrete intervals.



#### Globally Timed

Timing dictated by centralized FSM according to a fixed schedule.



#### Asynchronous

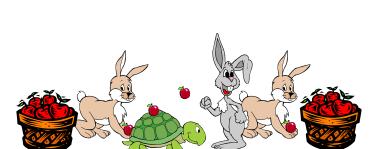
Events – e.g. the loading of a register -- can happen at at arbitrary times.



### Locally Timed

### **Synchronous**

ALL computation "events" occur at active edges of a periodic clock: time is divided into fixed-size discrete intervals.



#### Asynchronous

Events - e.g. the loading of a register -- can happen at at arbitrary times.



Laid Back

#### Globally Timed

Timing dictated by centralized FSM according to a fixed schedule.

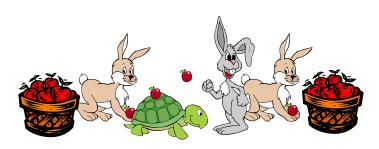


Locally Timed

### **Synchronous**

ALL computation "events" occur at active edges of a periodic clock: time is divided into fixed-size discrete intervals.





#### Asynchronous

Events – e.g. the loading of a register -- can happen at at arbitrary times.



#### Globally Timed

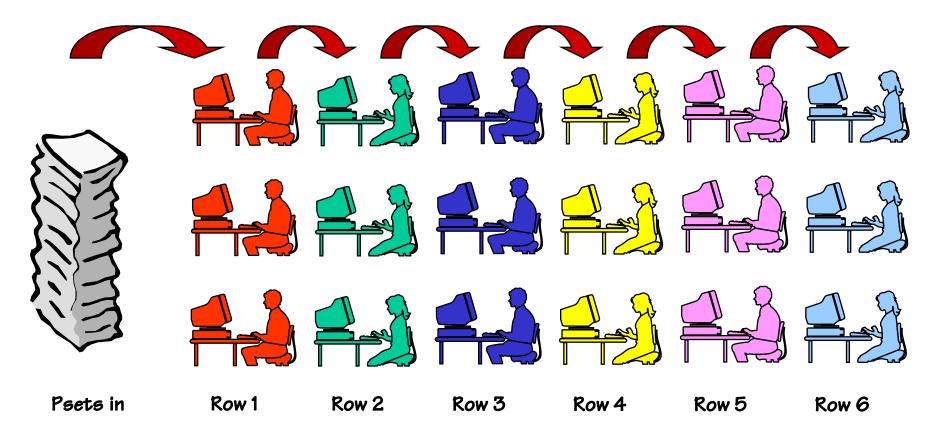
Timing dictated by centralized FSM according to a fixed schedule.



Locally Timed

# "Classroom Computer"

There are lots of problem sets to grade, each with six problems. Students in Row 1 grade Problem 1 and then hand it back to Row 2 for grading Problem 2, and so on... Assuming we want to pipeline the grading, how do we time the passing of papers between rows?



Comp 411 L16 - Pipelining 74

	Synchronous	Asynchronous
Globally Timed		
Locally Timed		

Comp 411

#### Synchronous

#### Asynchronous

Globally Timed Teacher picks time interval long enough for worst-case student to grade toughest problem. Everyone passes psets at end of interval.

Locally

#### **Synchronous**

#### Asynchronous

Globally Timed

Teacher picks time interval long enough for worst-case student to grade toughest problem. Everyone passes psets at end of interval.

Locally Timed

Students raise hands when they finish grading current problem. Teacher checks every 10 secs, when all hands are raised, everyone passes psets to the row behind. Variant: students can pass when all students in a "column" have hands raised.

#### **Synchronous**

#### Asynchronous

Globally Timed Teacher picks time interval long enough for worst-case student to grade toughest problem. Everyone passes psets at end of interval.

Teacher picks variable time interval long enough for current students to grade current set of problems. Everyone passes psets at end of interval.

Locally Timed Students raise hands when they finish grading current problem. Teacher checks every 10 secs, when all hands are raised, everyone passes psets to the row behind. Variant: students can pass when all students in a "column" have hands raised.

#### **Synchronous**

#### Asynchronous

Globally Timed

Teacher picks time interval long enough for worst-case student to grade toughest problem. Everyone passes psets at end of interval.

Teacher picks variable time interval long enough for current students to grade current set of problems. Everyone passes psets at end of interval.

Locally Timed

Students raise hands when they finish grading current problem. Teacher checks every 10 secs, when all hands are raised, everyone passes psets to the row behind. Variant: students can pass when all students in a "column" have hands raised.

Students grade current problem, wait for student in next row to be free, and then pass the pset back.

## Control Structure Taxonomy

Easy to design but fixed-sized interval can be wasteful (no data-dependencies in timing)

Synchronous

Large systems lead to very complicated timing generators... just say no!

Asynchronous

Globally Timed Centralized clocked FSM generates all control signals.

Central control unit tailors current time slice to current tasks.

Locally Timed Start and Finish signals generated by each major subsystem, synchronously with global clock.

The best way to build large systems that have independent components.

Each subsystem takes asynchronous Start, generates asynchronous Finish (perhaps using local clock).

The "next big idea" for the last several decades: a lot of design work to do in general, but extra work is worth it in special cases

## Summary

- Latency (L) = time it takes for the results from a given input to arrive at outputs
- Throughput (T) = rate at each new outputs appear
- For combinational circuits:  $L = t_{PD}$  of circuit, T = 1/L
- For K-pipelines (K > 0):
  - always have register on output(s)
  - K registers on every path from input to output
  - $T = 1/(t_{PD,REG} + t_{PD})$  of slowest pipeline stage +  $t_{SETUP}$ 
    - more throughput  $\rightarrow$  split slowest pipeline stage(s)
    - use replication/interleaving if no further splits possible
  - L=K/T
    - pipelined latency ≥ combinational latency