

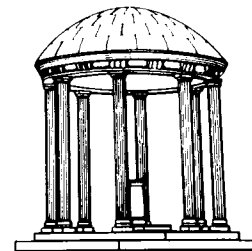
Relief Texture Mapping

TR00-009
March 3, 2000



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RELIEF TEXTURE MAPPING

by


Manuel Menezes de Oliveira Neto

A dissertation submitted to the faculty of the University of North Carolina at Chapel Hill in partial fulfillment of the requirements of the degree of Doctor of Philosophy in the Department of Computer Science.


Chapel Hill

March 2000

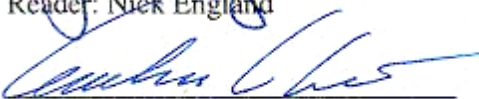
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ABSTRACT

Manuel Menezes de Oliveira Neto

Relief Texture Mapping

(Under the supervision of Professor Gary Bishop)

This dissertation presents an extension to texture mapping that supports the representation of 3-D surface details and view motion parallax. The results are correct for viewpoints that are static or moving, far away or nearby. In this approach, a *relief texture* (texture extended with an orthogonal displacement per texel) is mapped onto a polygon using a two-step process. First, it is converted into an ordinary texture using a surprisingly simple 1-D forward transform. The resulting texture is then mapped onto the polygon using standard texture mapping. The 1-D warping functions work in texture coordinates to handle the parallax and visibility changes that result from the 3-D shape of the displacement surface. The subsequent texture-mapping operation handles the transformation from texture to screen coordinates.

The pre-warping equations have a very simple 1-D structure that enables the pre-warp to be implemented using only 1-D image operations along rows and columns and requires interpolation between only two adjacent texels at a time. This allows efficient implementation in software and should allow a simple and efficient hardware implementation. The texture-mapping hardware already very common in graphics systems efficiently implements the final texture mapping stage of the warp.

I demonstrate a software implementation of the method and show that it significantly increases the expressive power of conventional texture mapping. It also dramatically reduces the polygon count required to model a scene, while preserving its realistic look. This new approach supports the representation and rendering of three-dimensional objects and immersive environments, and naturally integrates itself with popular graphics APIs. An important aspect of this research is to provide a framework for combining the photo-realistic promise of image-based modeling and rendering techniques with the advantages of polygonal rendering.

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ACKNOWLEDGEMENTS

Very special thanks go to my advisor, Gary Bishop, for his enthusiasm, friendship and for the many delightful discussions we've had. His insightful remarks and suggestions were invaluable for me during this project and are a source of inspiration that will last much longer than this dissertation.

I would also like to warmly acknowledge the other members of my dissertation committee: Professors Frederick Brooks, Nick England, Anselmo Lastra, Steve Molnar and Lars Nyland. All these gentlemen have contributed valuable suggestions to this work. Lars was especially encouraging to me during the planning stages of this thesis.

Professors Anatólio Laschuk and Elian Machado deserve warm recognition for their earlier support and inspiration.

The following people have provided me with various kinds of assistance as I worked in this project. David McAllister assisted with the production of some animations. Voicu Popescu lent me a 3-D Studio MAX[®] plug-in. Paul Rademacher provided GLUI. Cassio Ribeiro designed *Relief Town*. David, Lars, Voicu, Anselmo and Chris McCue provided the reading room data set. Jason Smith let me borrow his voice and accent for video submissions, and George Stackpole proofread this dissertation. *de Espona Infografica* created most of the 3-D models I used.

During the undertaking of this project, the Computer Science Department faculty and staff provided a wonderful working environment. Janet Jones and Mary Whitton deserve special recognition for their assistance.

Additionally, I would like to acknowledge the Brazilian Research Council (CNPq – process # 200054/95) for supporting me during my graduate studies. DARPA and NSF have also provided some funding.

Finally, I want to extend my heartfelt gratitude to my mother and to Ninha, for showing me so much love, and to my wife, Ana, for all her love, patience and support.

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To my lovely wife,
To all our children,
And to their children, too.

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Chapter 1 – INTRODUCTION

Texture mapping has long been one of the most successful techniques in high-quality image synthesis [Catmull74]. It can be used to change the appearance of surfaces in a variety of ways by mapping color, adding specular reflection (environment maps [Blinn76]), causing vector normal perturbation (bump mapping [Blinn78]) and adding surface displacements [Cook84], among others. While its meaning can be very broad, the expression *texture mapping* will be reserved to refer to its most common use, the mapping of surface color. Mapping of other attributes, such as bumps and displacements, will be referred to explicitly.

By adding 2-D details to object surfaces, conventional texture mapping can be used to correctly simulate a picture on a wall or the label on a can. The planar-projective transform of texture mapping has a very convenient inverse formulation, which allows direct computation of texture element coordinates from screen coordinates, leading to efficient implementation as well as accurate resampling. Unfortunately, texture mapping



Figure 1-1. Town rendered using conventional texture mapping. Each façade and brick wall is represented by a single texture.

is not as effective for adding 3-D details to surfaces. Its fundamental limitation, the lack of *view-motion parallax*¹, causes a moving observer to perceive the underlying surface as locally flat. Such flatness also becomes evident when the surface is observed from an oblique angle (Figure 1-1).

The most popular approaches for representing surface details are *bump mapping* [Blinn78] and *displacement mapping* [Cook84]. Bump mapping simulates the appearance of wrinkled surfaces by performing small perturbations on the direction of the surfaces' normals. It produces very realistic effects, but the technique assumes that the heights of the bumps are negligibly small when compared to the extent of the associated surface and it needs to be used in conjunction with per-pixel lighting. Since the surface itself is not modified, silhouette edges appear unchanged and self-occlusions, which would be caused by real bumps, are ignored. Surface displacements or displacement maps [Cook84] specify the amounts by which a desired surface locally deviates from a smooth surface. In this case, the geometry is actually changed and often rendered as a mesh of micro-polygons. Displacement maps can be used to create faithful representations for continuous surfaces, but the associated rendering cost has prevented them from being used in interactive applications.

This dissertation introduces an extension to texture mapping that supports the representation of three-dimensional surface detail and view-motion parallax. This new approach, called *relief texture mapping*, results from a factorization of the 3-D image warping equation of McMillan and Bishop [McMillan97] into a pre-warp followed by conventional texture mapping. The pre-warp is applied to images with per-texel² displacements, called *relief textures*, transforming them into regular images by handling only the parallax effects resulting from the direction of view and the displacement of the texture elements; the subsequent texture-mapping operation handles scaling, rotation, and the remaining perspective transformation. The sizes of the displacements can vary arbitrarily and the results produced by the technique are correct for moving or static observers standing far away or nearby the represented surfaces. Since relief textures

¹ The way the view of a scene changes as a result of viewer motion.

² Texture element

contain some geometric information about the surfaces they represent, they can be used as modeling as well as rendering primitives.

The pre-warping step is described by very simple equations and can be implemented using 1-D image operations along rows and columns, requiring interpolation between only two adjacent texels at a time. The final texture mapping stage is efficiently implemented using conventional texture-mapping hardware.

Relief texture mapping significantly increases the expressive power of conventional texture mapping and drastically reduces the polygonal count required to model a scene while preserving its realistic look. The results obtained with the use of this technique are, in most cases, virtually indistinguishable from the rendering of the more complex geometric models. Figure 1-2 shows the use of relief texture mapping for the same viewpoint used to create Figure 1-1. Although each façade and brick wall is represented with a single relief texture, the perception of three-dimensionality is remarkable. For instance, notice the bricks standing out of the wall to the right and the protruding dormers on the house to the left. In the original model of this town, each house was represented using several thousand polygons while the relief texture-mapped model of Figure 1-2 contains just seven polygons per house, four of which are used to model the walls and the back of the roof.



Figure 1-2. Same view as in Figure 1-1 rendered using relief texture mapping. Notice the bricks standing out and the protruding dormers.

1.1 Hybrid Systems

In recent years, image-based modeling and rendering (IBMR) techniques have gained considerable attention in the graphics community because of their potential to create very realistic images. One of the major benefits of image-based techniques is the ability to capture details related to the imperfections of the real world that graphics researchers still do not know how to model and render [Foley00]. By casting a subset of IBMR, more specifically, the rendering of height images, as an extension to texture mapping, this research presents an effective technique to construct hybrid systems that can offer much of the photo-realistic promise of IBMR while retaining the advantages of polygonal rendering.

1.2 Thesis Statement

I propose to generate valid views of continuous surfaces representing objects and scenes by warping images augmented with depth using a series of 1-D warps, and texture-mapping the results onto planar polygons. As in 3-D image warping [McMillan97], the correctness of the proposed solution is defined as being consistent with the projections of the corresponding static three-dimensional models represented in Euclidean geometry. Like all image-based rendering methods, the proposed approach is essentially a signal reconstruction solution. The central thesis statement of this research is presented below:

The expressive power of texture maps can be greatly enhanced if textures are augmented with height fields. Such extended texture maps can be pre-warped and then conventionally texture-mapped to produce correct perspective views of surfaces for viewpoints that are static or moving, far away or nearby. Moreover, the pre-warping step can be implemented using only 1-D image operations along rows and columns.

Four central issues must be addressed in order to explore the domain associated with the proposed technique. The first one is the factorization of the 3-D image warping equation [McMillan97] into pre-warping and texture mapping (a 2-D projective mapping)

stages. Chapter 3 presents a derivation of the so-called *pre-warping equations*. Such equations can be applied to arbitrary source and target camera configurations. The only assumption is that the target view is a perspective projection image. I consider parallel projection images to have some advantages over their perspective projection counterparts when images are used as modeling primitives. For this reason, pre-warping equations for both perspective and parallel projection images with depth are derived. Establishing the visibility of the original samples from arbitrary viewpoints is addressed with an adaptation of the occlusion-compatible order algorithm [McMillan97] for parallel projection images.

The second problem involves the reconstruction of a continuous signal from a discrete input. It also deals with filtering and sampling at the regular lattice of the output image. Given the special 1-D nature of the pre-warping equations, the resampling process can be performed in 1-D. This is the subject of Chapter 4, where some resampling alternatives are explored.

The third issue is the representation and visualization of complex shapes and objects. The ability to replace complicated shapes with just a few texture-mapped polygons is an important problem in geometric simplification. This topic is discussed in Chapter 5, where an algorithm for rendering objects is presented. The increasing interest for IBMR techniques has popularized the rendering of scenes from images acquired from real environments. Thus, another important aspect to be considered is the representation and rendering of such environments using relief texture mapping. This issue is also discussed in Chapter 5.

The fourth problem is concerned with the use of multi-scale representations of relief textures. Multi-scale representations in the form of texture pyramids can be used for anti-aliasing as well as to keep the rendering cost of relief textures proportional to their contribution to the final image. This subject, as well as shading of relief texture-mapped surfaces, is discussed in Chapter 6.

1.3 Results

This dissertation presents some original results that include:

- an extension to conventional texture mapping that supports the representation of 3-D surface detail and view-motion parallax,
- a factorization of the 3-D image warping equation into a pre-warp followed by a 2-D projective mapping,
- a derivation of a family of 1-D pre-warping equations for both parallel and perspective-projection source images with depth,
- a proof for the one-dimensionality of the pre-warping equations,
- a 1-D clipping algorithm to be used during the pre-warp,
- a family of 1-D resampling algorithms,
- an algorithm for rendering objects with complex shapes represented by sets of relief textures,
- an adaptation of the occlusion-compatible order algorithm of McMillan and Bishop to parallel projection images with depth,
- a demonstration that the occlusion-compatible order can be decomposed into two one-dimensional passes,
- multi-scale image representation for use with image-warping,
- algorithms for constructing scenes by assigning reusable relief textures to arbitrarily positioned and oriented polygons,
- verification that, after rotations have been factored out, 3-D warps reduce to a two-dimensional problem that can be implemented as a series of 1-D warps, regardless of the coordinate systems associated with the source and target image planes. Moreover, such one-dimensional warps do not suffer from the shortcomings associated with arbitrary serial warps (bottlenecks, foldovers and images twists), which are discussed in Chapter 2.

In addition to its original results, the following assertions will be demonstrated:

- complex geometric shapes can be modeled and rendered using the techniques described in this dissertation,

- the use of relief texture mapping can dramatically reduce the polygon count of a scene when compared to conventional geometric modeling techniques,
- the resampling process associated with the proposed method can be completely done in 1-D and is simpler than the corresponding operations for general 3-D image warping,
- relief texture mapping can be used in combination with conventional geometric models to produce complex scenes with visibility issues appropriately solved. The proposed approach naturally integrates itself with popular graphics APIs such as OpenGL [Woo97].

Finally, I expect to provide substantial evidence supporting the claim that the 1-D operations required for pre-warping and reconstruction lend themselves to a natural digital hardware implementation.

1.4 Overview of the Relief Texture Mapping Algorithm

Relief texture mapping implements view-motion parallax by pre-warping textures before mapping them onto polygons (Figures 1-3). Images with depth and a target view are taken as input. The pre-warping step is responsible for solving visibility issues, filling holes caused by disocclusion events³ using linear interpolation, and computing the non-

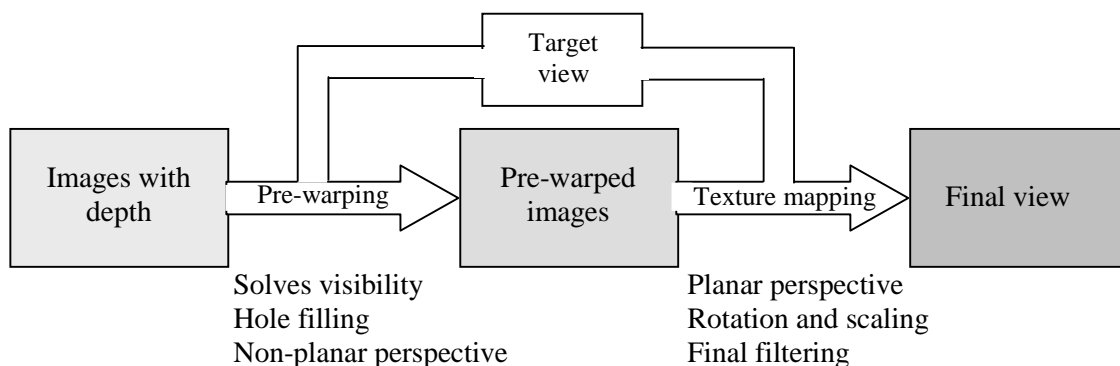


Figure 1-3. Relief-texture-mapping algorithm. Images with depth and the desired viewpoint are given as input to the pre-warping phase responsible for solving visibility and hole filling. The resulting image is used as input for a conventional texture-mapping operation that will produce the final image.

³ Exposures of surfaces not represented in the source images.

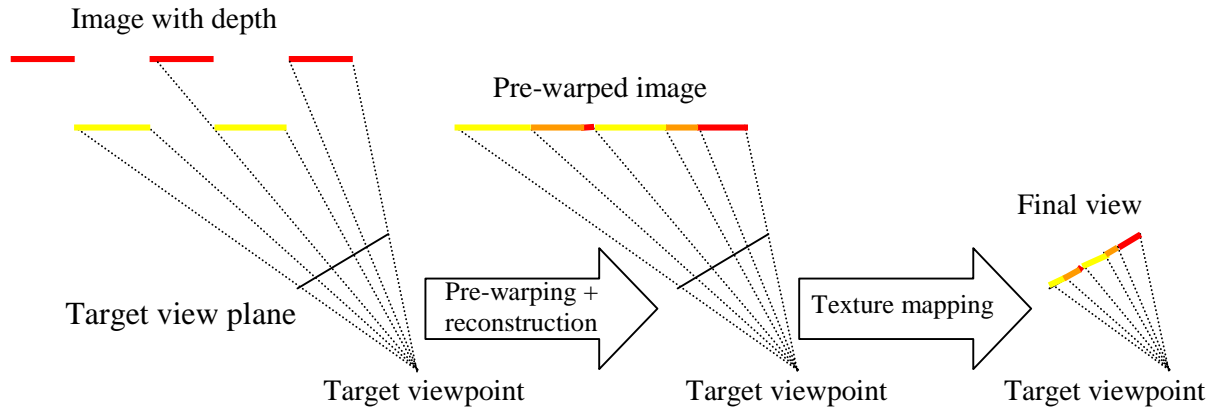


Figure 1-4. 2-D illustration of the steps of the relief texture-mapping algorithm. The pre-warping solves visibility issues and fills holes. The resulting image is used as input for the texture mapping operation that produces the final image.

planar component of the perspective distortion. The pre-warp is performed in the coordinate system of the source images and visibility issues are solved with respect to the target viewpoint. The final view is obtained by conventionally texture mapping the resulting pre-warped images onto polygons that match the dimensions, position and orientation of the image planes associated with the input images. The texture mapping stage of the algorithm implements rotation, scaling, the planar component of the perspective distortion and the final filtering. During the pre-warp, depth values associated with source texels can be interpolated and used to achieve correct occlusions in scenes containing multiple, and possibly interpenetrating, objects. Figure 1-4 provides a simple 2-D example illustrating the steps of the algorithm, which the reader should be able to relate to the stages of the pipeline shown in Figure 1-3.

1.5 Related Work

The technique described in this dissertation is derived from the 3-D image warping method presented in Leonard McMillan’s doctoral dissertation [McMillan97]. Due to its central role in this research, 3-D image warping will receive a detailed discussion in Chapter 3. Other related approaches have been classified according to their major features and are presented next.

1.5.1 Image warping methods

Sprites with depth [Shade98] enhance the descriptive power of traditional sprites with out-of-plane displacements per pixel. Such a technique is based on a factorization usually referred to in the computer vision literature as *plane-plus-parallax* [Sawhney94]. Given its importance and its close links with the 3-D image warping equation, such a factorization will also be considered in detail in Chapter 3. The idea behind sprites with depth is to use a two-step-rendering algorithm to compute the color of pixels in the target image from pixels in a source image. In the first step, the displacement map associated with the source image is forward mapped using a 2-D transformation to compute an intermediate displacement map. In the second pass, each pixel of the desired image is transformed by a homography (planar perspective projection) and the resulting coordinates are used to index the displacement map computed in the first pass. The retrieved displacement values are then multiplied by the epipole⁴ of the target image and added to the result of the homography. Such coordinates are used to index the color of the destination pixels.

Although such an approach may, at first, seem similar to the mine, given that both methods use images extended with orthogonal⁵ displacements, it differs in some fundamental aspects. Sprites with Depth are an approximation to the 3-D image warping process. They do not take advantage of available hardware and its 2-D image reconstruction is more involved and prone to artifacts than the ones presented here. Relief texture mapping, on the other hand, is based on an exact factorization of the 3-D image warping equation [McMillan97], takes advantage of texture mapping hardware, uses an efficient image reconstruction strategy and naturally integrates with popular graphics APIs. In Figure 1-2, relief textures were pre-warped in software and texture-mapped using OpenGL [Woo97].

⁴ The projection of one camera's center of projection onto the image plane of another camera.

⁵ The techniques described in this dissertation can also be used with source perspective projection images with depth. This issue will be discussed in Chapter 3.

1.5.2 View-dependent texture mapping

View-dependent texture mapping consists of compositing multiple textures based on the observer's viewpoint, and mapping them onto polygonal models. In [Debevec96], a model-based stereo algorithm is used to compute depth maps from pairs of images. Once a depth map associated with a particular image has been computed, new views can be re-rendered using several image-based rendering techniques. Debevec, Yu, and Borshukov [Debevec98] use visibility preprocessing, polygon-view maps, and projective texture mapping to produce a three-step, view-dependent texture mapping algorithm that reduces the computational cost and produces smoother blending when compared to the work described in [Debevec96].

1.5.3 One-dimensional Perspective Projection

Robertson [Robertson87] showed how hidden-point removal and perspective projection of height images can be performed on rows and columns. His approach explores the separability of perspective projection into orthogonal components. First, the image is rotated to align its lower edge with the lower edge of the viewing window. Then, a horizontal compression is applied to each scanline so that all points that may potentially occlude each other fall along the same column. 1-D vertical perspective projection is applied to the columns of the intermediate image in back-to-front order, thus performing hidden-point removal. Finally, 1-D horizontal perspective projection is applied to the resulting image, incorporating compensation for the compression performed in the second step [Robertson87].

1.5.4 Extension for handling visibility

A *nailboard* [Schaufler97] is a texture-mapped polygon augmented with a small depth value per texel specifying how much the surface of an object deviates from the polygon for a specific view. The idea behind nailboards is to take advantage of frame-to-frame coherence in smooth sequences. Thus, instead of rendering all frames from scratch, more complex objects are rendered to separate buffers. The contents of such buffers are used to create partially transparent textured polygons with per-texel deviations from the

objects' surfaces. These sprites are re-used as long as the geometric and photometric errors remain below a certain threshold [Schaufler97]. An error metric is therefore required. When a nailboard is rendered, the depth values associated with each texel are added to the depth of the textured polygon in order to solve visibility among other nailboards and conventional polygons.

1.6 Discussion

Impostors have been used to improve a system's frame rate by reducing the amount of geometry that needs to be rendered. Some of the most common examples of impostors include the use of texture-mapped polygons [Maciel95] and levels of detail [Heckbert97]. While the use of conventionally texture-mapped polygons is very effective in reducing a scene's polygonal count, they have limited application due the lack of view-motion parallax. This research presents a new class of dynamic texture-mapped impostors that are virtually indistinguishable from the geometric models they represent, even when the viewer is very close.

An interesting property of relief textures is the ability to adjust their rendering cost to match their contribution to the final image. Thus, for instance, a surface that is far away from the viewer can be rendered as a conventional texture or using a low-resolution level of its associated relief texture pyramid. On the other hand, as the viewer approaches the represented surface (*e.g.*, when the viewer crosses the plane containing the texture-mapped polygon), the relief texture can be rendered as a mesh of micro-polygons. The user application can select the most appropriate rendering strategy for each situation.

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Chapter 2 – Separable Transformations

The notion of image warping is at the center of this dissertation and of several other image-based rendering approaches. This chapter provides formal definitions for important related concepts, such as *images* and *warp maps*, which serve as foundations for this and subsequent chapters. Its main purpose is to discuss the decomposition of two-dimensional warps into a series of one-dimensional operations and the advantages of using such 1-D transformations over their 2-D counterparts. The difficulties associated with the use of 1-D warps (need for an inverse solution, *bottlenecks*, *image twists*, and *foldovers*) and their main causes are examined. Examples illustrating the robustness of the relief texture-mapping algorithm with respect to these problems are provided. This chapter also introduces the concept of an *ideal separable transform*, *i.e.*, a 1-D warp in which source image coordinates can be transformed independently from each other. The chapter ends with a discussion of one-dimensional intensity resampling, its advantages, limitations, and applicability to 3-D image warping.

2.1 Images and Warp Maps

A *continuous image* is a map $i:U \rightarrow C$, where $U \subset \mathfrak{R}^2$ is called the support of the image, and C is a vector space, usually referred to as its color space [Gomes97]. Besides color, the elements of C may carry information about other image attributes such as transparency, depth, etc. A digital image $i_d:U' \rightarrow C'$ is a sampled version of a continuous image, where $U' = \{(x_i, y_j) \in U : x_i = i \cdot \Delta x, y_j = j \cdot \Delta y; i, j \in Z\}$ is an orthogonal uniform lattice, Δx and Δy are positive real numbers, and $C' \subset C$ is a quantization of C [Gomes97].

A *2-D warp map* $w:U \rightarrow W \subset \mathfrak{R}^2$ is a geometric transformation that deforms the support of an image, thus producing a new one. Usually, its input is referred to as *source*

image while its output is called *destination* or *target image*. When w causes no superposition of points in the target image, the map is called *injective* and an inverse transformation $w^{-1}:W \rightarrow U$ exists. Although some simple warping filters implement invertible transformations, such a property does not hold in general. In the digital case, the warp map $w_d:U' \rightarrow W \subset \mathfrak{R}^2$ distorts the input lattice, usually causing its orthogonality and uniformity to be lost. An inverse warp is defined as $w_d^{-1}:W' \rightarrow U \subset \mathfrak{R}^2$, where W' is an orthogonal uniform lattice associated with the target image. It is important to note that the technique usually referred to as 3-D image warping [McMillan97] is, in fact, a 2-D warp map. Although from a formal standpoint such a name may be misleading, it will be used here due to its widespread acceptance.

Given a warp map w and real numbers $\lambda > 1$ and $0 < \mu < 1$, w is called an *expansion* or *expanding transformation* if $|w(X) - w(Y)| \geq \lambda|X - Y|$ for all $X, Y \in U$. Likewise, w is called a *contraction* or *contracting transformation* if $|w(X) - w(Y)| \leq \mu|X - Y|$ for all $X, Y \in U$. A transformation that preserves distances, *i.e.*, $|w(X) - w(Y)| = |X - Y|$ for all $X, Y \in U$, is called an *isometry* [Gomes97]. A special kind of isometry is the *identity* transformation $w(X) = X$ for all $X \in U$. In general, warping transformations are not pure expansions, contractions or isometries. Very often, some regions are locally expanded, whereas others are locally contracted or remain unchanged. These special geometric transformations have particular importance in image warping. Expanded or magnified regions require some kind of interpolation and are less susceptible to aliasing artifacts. On the other hand, contracted or minified areas are more prone to aliasing, requiring appropriated filtering. Regions where distances are preserved usually require both reconstruction and filtering to account for a possible grid misalignment between the source and target digital images. When the isometry is the identity function, the transformation is unnecessary. This last observation can be explored to achieve considerable speed up in certain cases and will be discussed in Chapter 3.

2.2 Parallel and Serial Warps

Catmull and Smith [Catmull80] showed how affine and perspective transformations onto planar, bilinear and biquadratic patches could be decomposed into two 1-D transformations (shears) over rows and columns. Later, Smith [Smith87] showed that texture mapping onto planar quadric and superquadric surfaces, and planar bicubic and biquadratic image warps are also two-pass transformable. He coined the expressions *parallel warp* and *serial warp* which refer to the original 2-D map and to the composition of 1-D transforms that accomplishes a similar result, respectively.

Assuming the row pass takes place first, a two-pass serial warp⁶ is accomplished

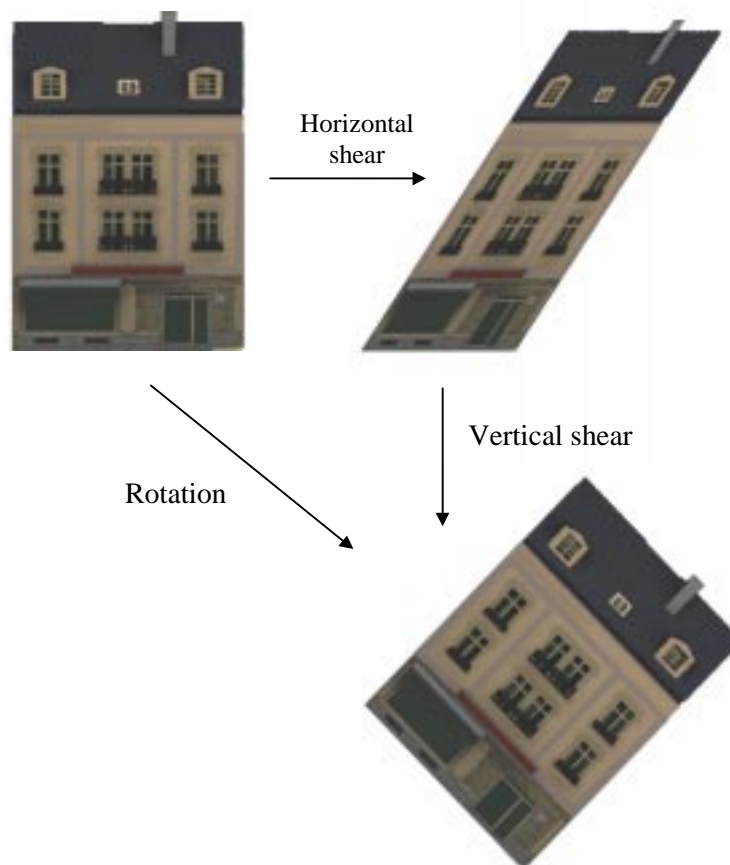


Figure 2-1. Two-pass affine transformation: 45 degrees rotation by applying two shear operations along the rows and columns of the image.

⁶ Some approaches use more than two passes [Paeth90].

by a *horizontal shear* followed by a *vertical shear* operation applied to the image. The horizontal pass shifts the elements of the rows by variable amounts. Likewise, the vertical pass moves the elements along the columns of the resulting image. Figure 2-1 illustrates the technique for the case of an affine transformation. This discussion will focus on digital images.

In the case of a parallel warp, coordinates (u_s, v_s) in the source image are mapped to coordinates (u_t, v_t) in the target image: $(u_t, v_t) = w(u_s, v_s) = (H(u_s, v_s), V(u_s, v_s))$, where $H, V : U' \rightarrow \mathfrak{R}$. In general, input samples are mapped to arbitrary locations in the output image (Figure 2-2). The equivalent serial transformation can be obtained by defining a composite mapping $v \circ h$, where h preserves the v_s coordinates of its input pixels: $h(u_s, v_s) = (h_1(u_s, v_s), v_s) = (u_t, v_s)$, and v preserves the u_s coordinates of its transformed points: $v(u_s, v_s) = (u_s, v_2(u_s, v_s)) = (u_s, v_t)$. The composition $v(h(u_s, v_s))$ must produce the desired result. However, since v_2 is defined with respect to the coordinate system of the source image and the original value of u_s is no longer available after the first pass, one needs to compute h_1^{-1} and obtain the target coordinates as $v(h(u_s, v_s)) = (u_t, v_2(h_1^{-1}(u_t, v_s), v_s)) = (u_t, v_2(u_s, v_s)) = (u_t, v_t)$.

Usually, finding a closed-form solution for h_1^{-1} is not easy and, sometimes, it does not exist at all [Catmull80]. In such a case, and also when there are multiple such solutions, numerical techniques are preferred [Smith87]. The computation of h_1^{-1} can be

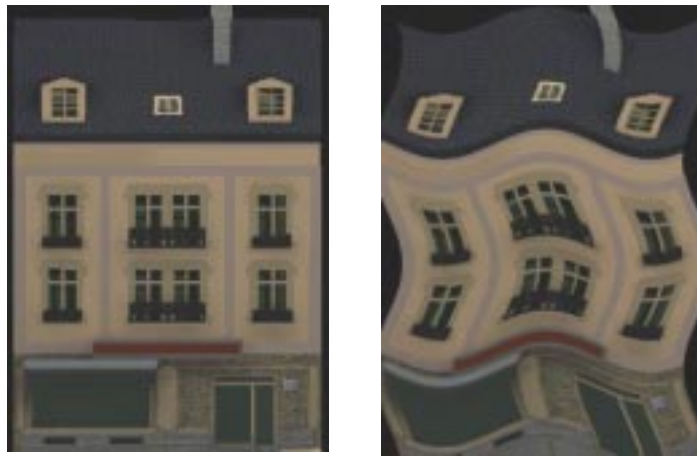


Figure 2-2. 2-D image warping.

avoided if the original coordinates of pixels in the source image are stored in a lookup table. This idea was originally suggested by Catmull and Smith [Catmull80] and later explored by Wolberg and Boult [Wolberg89].

The decomposition of a mapping into a series of independent 1-D operations presents several advantages over the original 2-D transform [Fant86] [Wolberg90]. First, the resampling problem becomes simpler. Reconstruction, area sampling and filtering can be efficiently done using only two pixels at a time. Secondly, it lends itself naturally to digital hardware implementation. Thirdly, the mapping is done in scanline order both in scanning the input and output images. This leads to efficient data access and considerable savings in I/O time. Also, “the approach is amenable to stream-processing techniques such as pipelining and facilitates the design of hardware that works at real-time video rates” [Wolberg90].

2.3 Difficulties Associated with Serial Warps

The basic idea behind serial warps is to move pixels to their final positions in the target image by shifting them along rows and then along columns (or vice versa), so that, at each pass, one of the coordinates assumes its final value. Thus, besides the difficulty of finding closed-form solutions for h_1^{-1} , serial warps suffer from two major problems, namely *bottlenecks* and *foldovers* [Catmull80]. A bottleneck is characterized by the collapse of the intermediate image into an area much smaller than the final one. This can happen if the first pass is not a one-to-one mapping (*i.e.*, not injective). When it occurs, the final image is usually disrupted. Even if the second pass restores its final shape, some color information has already been lost. A foldover, on the other hand, is characterized by self-occlusions of non-planar surfaces.

2.3.1 Bottlenecks

The major sources of bottlenecks are *image rotations* and *perspective distortions* [Wolberg90]. For instance, consider rotating an image by 90 degrees using a serial warp and assume the horizontal pass takes place first. In this case, all pixels along each row get mapped to the same column, causing the intermediate image to collapse into a diagonal

line (Figure 2.3). Contractive perspective distortions during the first pass may also lead to bottlenecks if the second pass is expansive [Wolberg90]. In combination with rotations, perspective distortions can cause the intermediate image to twist, leading to loss of information and introducing severe artifacts in the target image.

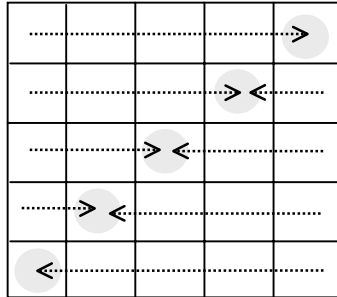


Figure 2.3. Example of serial warp bottleneck: the image collapses to a diagonal line after the first pass of a 90 degrees clockwise rotation.

Switching the orders between the horizontal and vertical passes, or transposing the image before applying a complementary transformation can be used to minimize the effects of bottlenecks or, in some cases, eliminate them [Catmull80]. These simple solutions seem to work for mappings involving planar surfaces, but no formal proof of its effectiveness has ever been presented. Since the occurrence of bottlenecks is associated with the compression of the intermediate image, its area is usually used as a measure of “bottleneckness”. For instance, Catmull and Smith [Catmull80] compute the area of images produced by the first pass using four different strategies:

- a) transform rows first;
- b) transform columns first;
- c) rotate the image by 90 degrees and then transform rows first;
- d) rotate the image by 90 degrees and then transform columns first.

Warping the first and last pixels of each row (column) and adding the lengths of the resulting spans approximates the area of the intermediate image. The approach producing the biggest value is selected for the final warping.

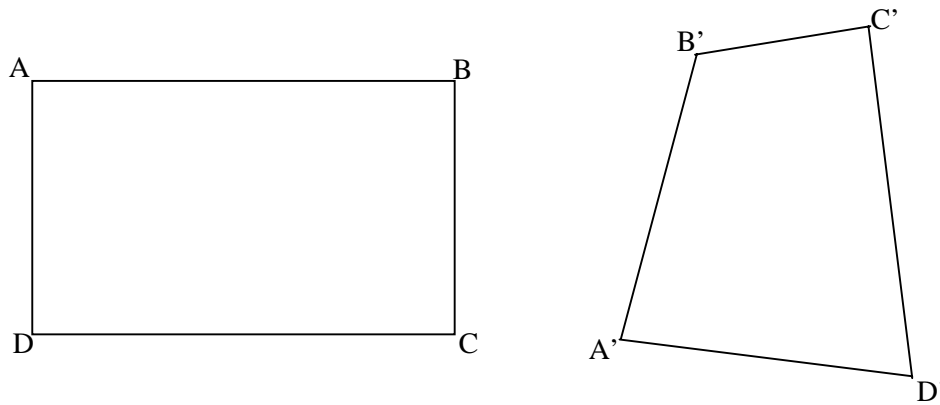


Figure 2-4. A source image, indicated by a rectangle (left). The same source image shown in perspective (right).

Another solution, proposed by Wolberg and Boult [Wolberg89], consists of performing a full horizontal-first warp of both the source image and its transpose, and composing the target image with the best pixels from both results. Image transposition is performed by a 90 degree clockwise rotation in order to avoid the need to reorder pixels left to right [Wolberg89]. In this approach, the warp is defined by a set of three spatial lookup tables (for X, Y and Z) provided by the user. The authors acknowledge the fact that the occurrence of bottlenecks is intimately related to the order in which the 1-D warps are performed. They claim, however, that the need for a vertical-first strategy can be avoided by upsampling the lookup tables before performing the vertical pass. In order to guide the image composition step, the four corners of each transformed pixel are used to compute a bottleneck measure. The goal is to minimize the angles between the edges of the transformed quadrilateral and the rectilinear grid of the output image. The function $b = \cos \theta \cos \phi$, computed on a per pixel basis for both warped images, is used for this purpose, where θ expresses the horizontal deviation from the output grid and ϕ , the

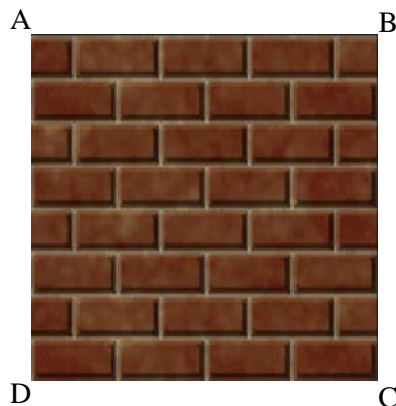


Figure 2-5. Source range image.

vertical deviation. Pixels presenting the least deviation are used to composite the final image.

In order to make the implications of bottlenecks more concrete, a simple example is presented next. Figure 2-4 (right) is a perspective view of a rectangle (left). The vertices of both quadrilaterals are identified by capital letters, evidencing the existence of a counter-clockwise rotation. Notice in Figure 2-4 (right) that vertex A' is to the left of B' (similar to the relationship between A and B), but D' is to the right of C' (as opposed to their counterparts D and C). Likewise, vertex A' is above D', but B' is below C'. Figure 2-5 shows a texture⁷ to be warped to the polygon shown in Figure 2-4 (right).

Consider performing the perspective mapping from the rectangle onto the quadrilateral shown in Figure 2-4 using serial warps. If the horizontal pass is applied first, the intermediate image gets twisted as a result of the change of relative order between the columns of C' and D' (Figure 2-6). Figure 2-7 shows the twisted output produced by the

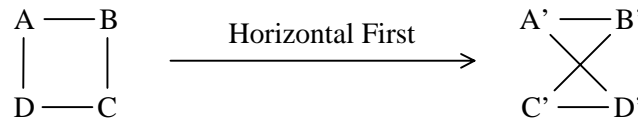


Figure 2-6. The result of the horizontal-first pass gets twisted.

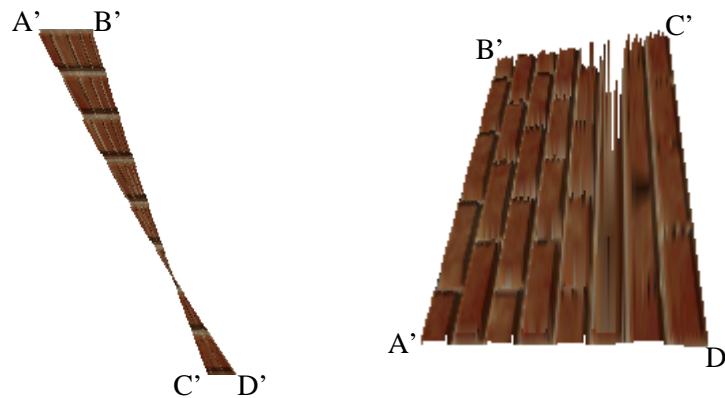


Figure 2-7. The result of the horizontal pass gets twisted as a result of the change in relative order between the columns of C' and D' (left). Final image produced by a serial warp (horizontal first) (right).

⁷ Actually, a parallel projection image with depth that will be warped to fit the desired view. A detailed discussion about these kinds of images and how they are warped is provided in Chapter 3.

horizontal warp (left) and the resulting target image (right).

If, however, the vertical pass is executed first, a different twist will introduce artifacts in the final image (Figure 2-8), brought about this time by the change in the relative order between the rows with B' and C'. Figure 2-9 shows the output produced by the vertical warp (left) as well as the resulting target image (right). Such results should be compared to the output of a parallel warp (mesh-based reconstruction) for the same view (Figure 2-10).

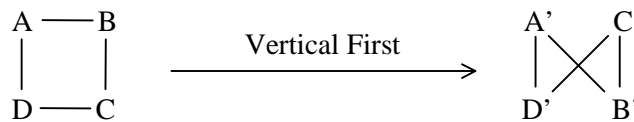


Figure 2-8. The result of the vertical-first pass also gets twisted.

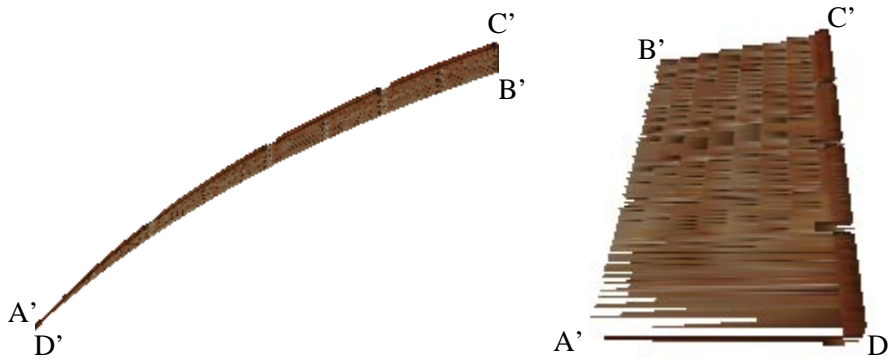


Figure 2-9. The result of the vertical pass gets twisted as a result of the change in relative order between the rows with B' and C' (left). Final image produced by a serial warp (vertical first) (right).

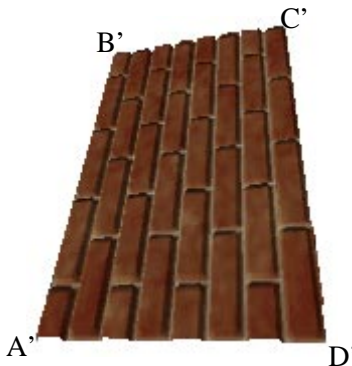


Figure 2-10. The result of parallel warp (mesh-based reconstruction) to the target image plane is free from the artifacts introduced by serial warps.

When image transposition is performed before the serial warp, such a rotation needs to be subtracted from the actual transformation. Also, rotations around the X and Y axes need to be switched. For the example shown in Figure 2-4, if the rotation happens to be in the same direction of the actual transformation (*i.e.*, counter-clockwise), bottlenecks are avoided in both serial orders (Figure 2-11). If, on the other hand, the rotation is performed in the opposite direction of the actual transformation (*i.e.*, clockwise), both serial orders cause the image to be mirrored with respect to its center, possibly introducing artifacts (Figure 2-12). Thus, for the case depicted in Figure 2-4, Wolberg and Boulton's algorithm [Wolberg89] would generate its final image by composing samples produced by the schemes shown in Figures 2-6 and 2-12.

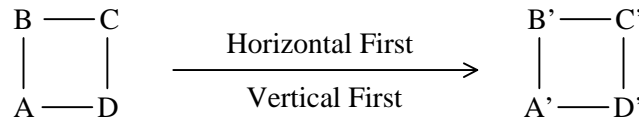


Figure 2-11. Source image is rotated by 90 degrees in the same direction of the actual transformation before applying the serial warps. Both serial orders work. Compensation for such an initial rotation is required.

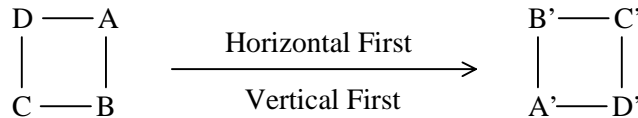


Figure 2-12. Source image is rotated by 90 degrees in the opposite direction of the actual transformation before applying the serial warps. Both serial orders will cause the image to be mirrored with respect to its center.

Whenever perspective is involved, it is difficult to tell from the transformation matrix when bottlenecks will occur [Catmull80]. Even in the presence of relatively simple viewing and modeling transformations, it seems to be equally difficult to predict whether the most appropriate rotation should be either clockwise or counter-clockwise. The quality of the results produced by the traditional approaches, however, depends on the selection of the most suitable rotation for each viewing configuration.

Figure 2-13 (right) shows the result produced by the relief texture-mapping algorithm for the example of Figure 2-4. The details of the algorithm are presented in the next chapters. Briefly, it consists of factoring the parallel warp into a serial warp followed

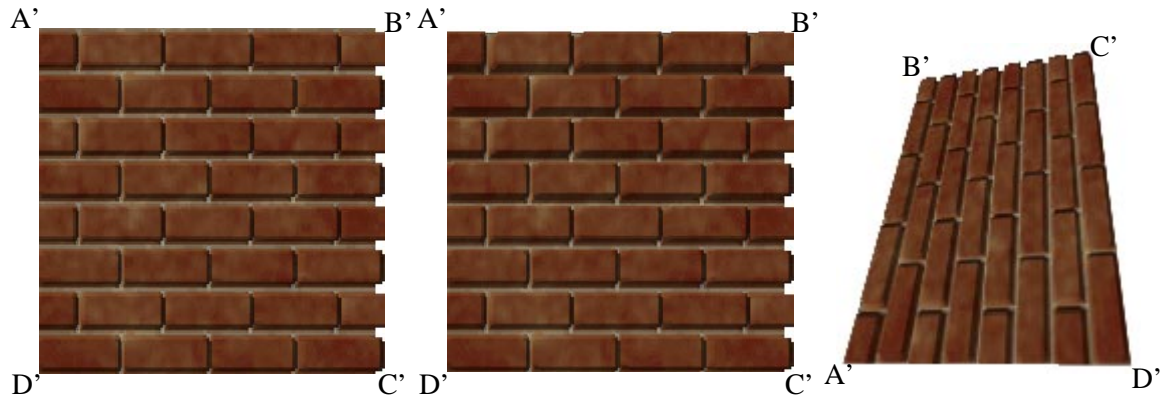


Figure 2-13. Perspective view of a brick wall (right) rendered with the relief texture-mapping algorithm. Results of the pre-warp: first (horizontal) pass (left); second pass (center).

by conventional texture mapping. Rotations are implemented using the texture mapping operation and, as a result, the serial warp is free from bottlenecks and image twists.

2.3.2 Foldovers

Foldovers [Catmull80], or *self-occlusions*, are caused by non-injective 2-D mappings. Perspective projections of non-planar patches can cause multiple samples to map to the same pixel on the screen, depending on the viewpoint. If appropriate care is not taken, samples may be overwritten during the first pass, not being available for the second one. For example, consider mapping the texture shown on Figure 2-14 (left) onto the surface described by the gray scale image to its right, where white represents height. Figure 2-15 shows a perspective view of the resulting texture-mapped surface, which partially occludes itself. If a vertical-first serial warp is used, some pixels in the upper central portion of the intermediate image may be overwritten (Figure 2-16 (left)).

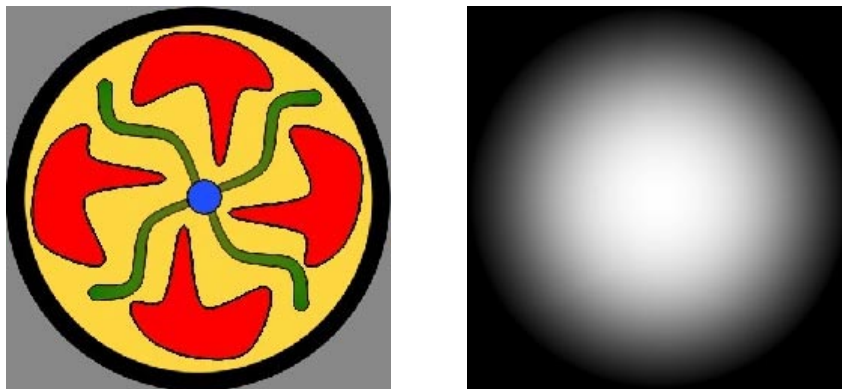


Figure 2-14. Texture (left) and a surface described by a gray scale image, where white means height (right).



Figure 2-15. Perspective view of the texture-mapped surface obtained after applying the texture shown in Figure 2-14 (left) to the surface shown to its right.

Although they should become visible after the second pass, their information has been lost and the final image presents some artifacts, as shown by the arrow in Figure 2-16 (right).

The traditional approach for handling self-occlusions in the context of serial warps is to store both color and depth information in multiple layers. During the second pass, the depth values are used to perform 1-D warps in back to front order. Catmull and Smith [Catmull80] suggest the use of multiple frame buffers, one for each fold of the surface. Such a solution may prove to be too expensive for arbitrary surfaces, which can potentially present a large number of folds. Wolberg and Boult [Wolberg89] use a more economic scheme, in which extra layers are allocated on a per column basis. In both

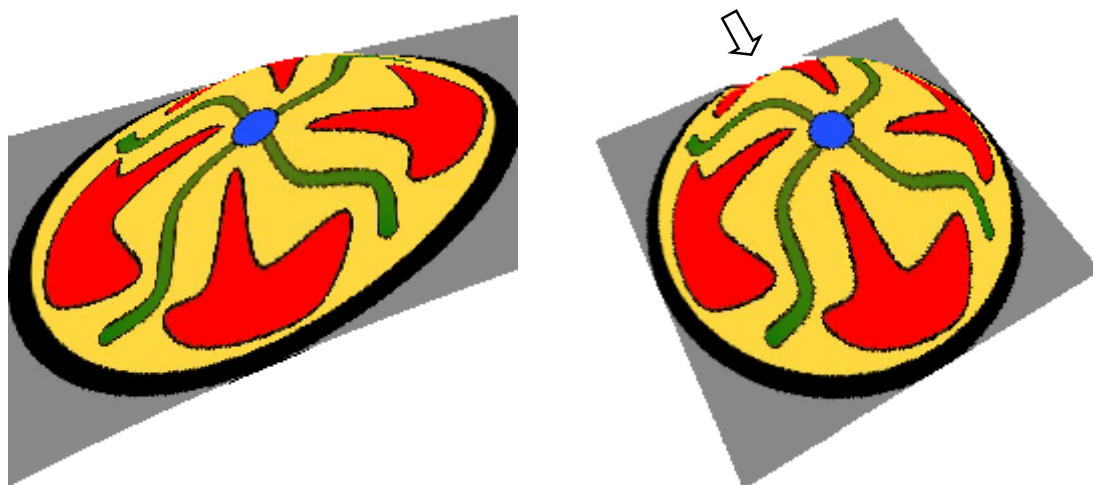


Figure 2-16. Foldover artifact (arrow) caused by a 2-pass warp. Result of the vertical pass cause pixels to be overwritten (left). Resulting image (right).

cases, appropriate filtering may require accessing multiple layers in order to produce antialiased output pixels.

Because the warping step of the relief texture-mapping algorithm only deals with some amount of perspective distortion, it is much less prone to self-occlusions. Figure 2-17 illustrates the intermediate and final results produced by the algorithm for the example of Figure 2-15. Nevertheless, a general solution for handling foldovers is presented in Chapter 4. It consists of interspersing the horizontal and vertical passes and can handle an arbitrary number of folds without requiring extra storage or depth comparisons.

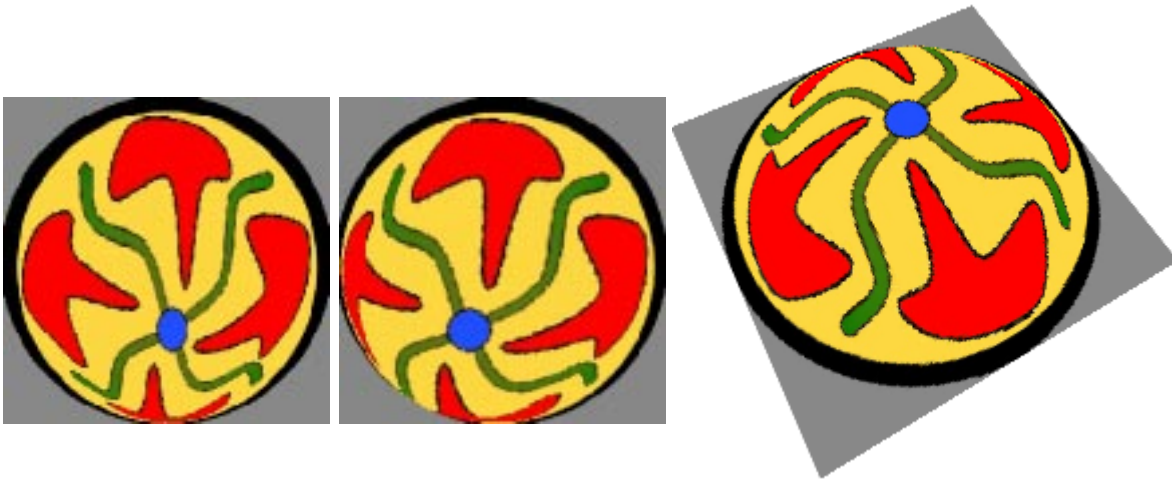


Figure 2-17. Perspective view of the texture-mapped surface rendered using relief texture mapping. Results of the vertical (left) and the horizontal pass (center). The final image is shown to the right.

2.4 The Ideal Separable Transform

The ideal separable transform can be factored as two independent functions of one variable $H, V : S' \rightarrow \mathfrak{R}$, where $S' = \{x_i \in \mathfrak{R} : x_i = i \cdot \Delta x; i \in Z, \Delta x \in \mathfrak{R}^+\}$. Thus, $(u_i, v_i) = w(u_s, v_s) = (H(u_s), V(v_s))$, not requiring the computation of h_1^{-1} or the use of lookup tables. Notice that such a transformation is expected to be faster and scale better than a regular serial warp. A family of pre-warping equations for relief texture-mapping satisfying these requirements will be presented in Chapter 3.

2.5 Intensity Resampling

The process of creating new images from discrete ones by means of spatial transformations is called *image resampling*. For the case of warped images, it consists of the following ideal steps [Heckbert89]:

- reconstruct a continuous signal from the input samples
- warp the reconstructed signal
- filter the warped signal to eliminate high frequencies that cannot be captured by the sampling rate implied by the output grid
- sample the filtered signal to produce the output image.

In practice, a continuous input signal is never actually reconstructed. In fact, only the significant sample points are evaluated by interpolating the input data. The filtering stage before the final sampling, however, is still required and is usually referred to as *antialiasing*. Nonlinear mappings, such as the ones involving perspective projection, require the use of *adaptive* or *space variant* filters, meaning that the shape and dimensions of the filter kernel change across the image.

2.5.1 One-dimensional intensity resampling

One-dimensional intensity resampling is considerably simpler than its 2-D counterpart. 1-D reconstruction reduces to linear interpolation, and antialiasing can be performed with very little extra computation, by accumulating all contributions to the currently resampled pixel [Fant86]. Figure 2-18 shows the mapping of an input row onto an output row. The black segment has slope bigger than one and corresponds to a local expansion. Dark gray segments have slopes smaller than one and are associated with local contractions. The light gray segment is a local isometry. A one-dimensional intensity resampling algorithm can be summarized as follows: if an output pixel falls completely inside an expanded region, its color is defined by appropriately sampling the interpolated color at the center of the output pixel. Otherwise, its color is obtained by weighting the contributions of the various spans covering the pixel. Thus, for instance,

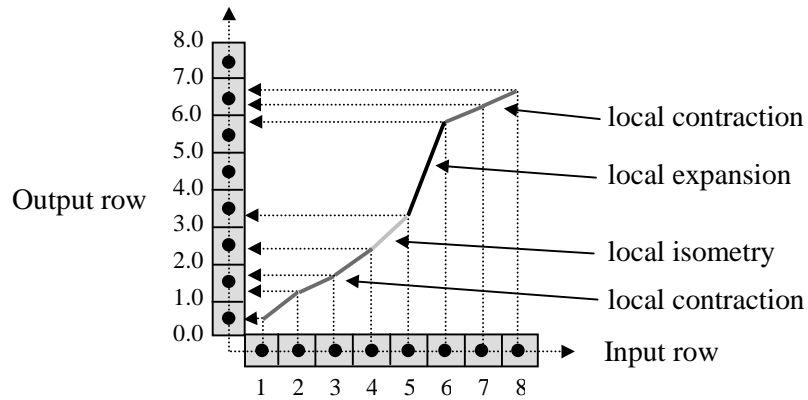


Figure 2-18 One-dimensional forward warping and intensity resampling of digital images. Adapted from [Gomes97].

the color associated with the fifth pixel in the output row is given by $color_at(4.5)$, where $color_at$ is a function that returns a linearly interpolated output color for a given parameter value. Now, let the u_i coordinates produced by function H for input pixels 1 to 4 be 0.5, 1.3, 1.7 and 2.4, respectively (Figure 2-18). Since only half of the first pixel in the output row is actually covered, its color is computed as $0.5 * color_at(0.75)$. Likewise, the color associated with the second pixel is given by $0.3 * color_at(1.15) + 0.4 * color_at(1.5) + 0.3 * color_at(1.85)$. In this case, the multiplicative factors represents pixel coverage by the various spans and the color function is evaluated at the centers of the span segments covering the pixel.

Suppose that a 2-D antialiasing process integrates all pixels in a neighborhood $N(p)$ of a source pixel p to obtain a target pixel p' . Then, in order for a serial map to perform the same filtering, all these pixels should be used in the computation of p' . But this implies that all pixels in $N(p)$ should map into the same column (row) during the first pass, so that the second pass can map all their images into p' [Smith87]. Given that this condition is seldom satisfied, “it is surprising how often the use of two serial 1-D filtering steps actually works” [Smith87].

2.5.2 Limitations of one-dimensional serial resampling

Despite the many advantages over parallel warps, the intensity resampling produced by two-pass methods is not always equivalent to a two-dimensional intensity

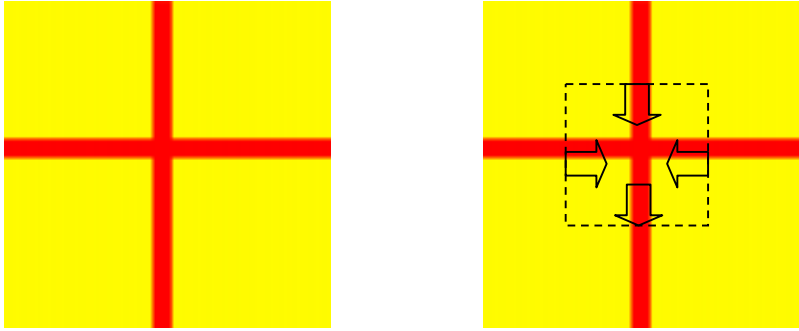


Figure 2-19. Texture presenting sharp intensity discontinuities in both horizontal and vertical directions (left). A warp defined inside the square region consisting of horizontal compression and vertical shift (right).

resampling. Although similar for affine transforms, results may vary significantly for arbitrary warps.

Serial intensity resampling performs poorly when the image resulting from the first pass presents color discontinuities in the direction of the second pass. Figure 2-19 (left) shows a texture presenting sharp intensity discontinuities in both horizontal and vertical directions. A simple 2-D warp is defined so that the interior of a squared region is compressed horizontally and shifted down (Figure 2-19 (right)). Figures 2-20 (a) to (d) illustrate the outputs of serial resamplings. Figures 2-20 (a) and (b) show the intermediate and final steps produced by a vertical-first strategy, respectively. Notice that the horizontal line gets disconnected after the first pass and cannot be recovered by the second one. The corresponding results produced by a horizontal-first approach are shown in Figures 2-20 (c) and (d). As the square is horizontally compressed into a rectangle, the horizontal line maintains its continuity. Notice that although some color discontinuities were introduced by compressing the vertical line in the central region of the texture, the resulting artifacts are much less objectionable than the ones in Figure 2-20 (b).

2.6 Discussion

3-D image-warping methods usually map relatively large variations in depth values into expanded regions, possibly introducing intensity discontinuities not present in the original texture, such as the one shown in Figure 2-20(a). The magnitude of the expansions depends on the viewing configuration. The artifacts shown in Figure 2-20 (b)

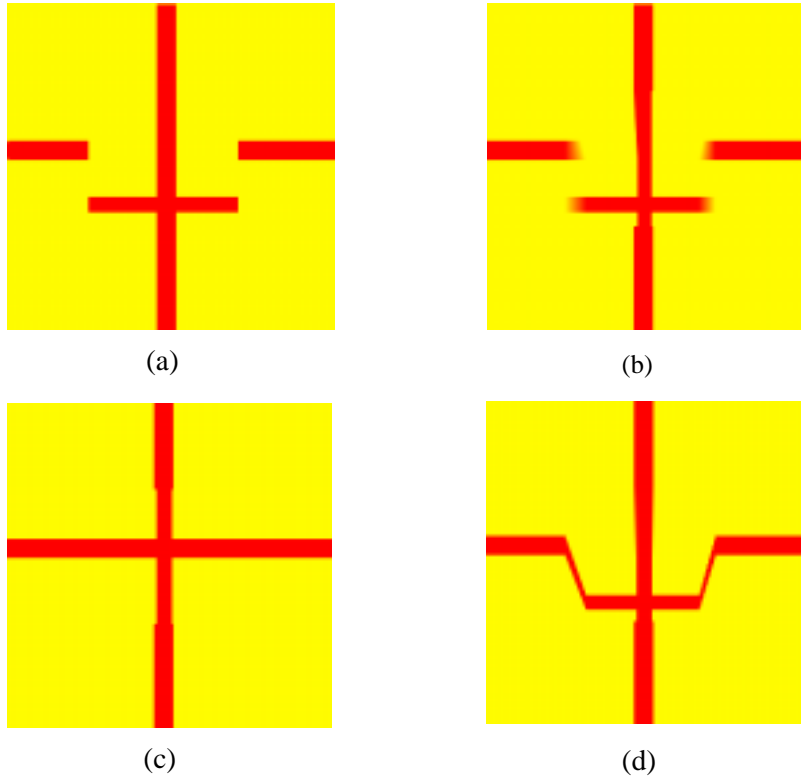


Figure 2-20. Results of the passes of a vertical-first strategy: (a) and (b). Results of the passes of a horizontal-first strategy: (c) and (d).

expose a fundamental limitation of serial resampling methods, and may mistakenly suggest that the technique is inappropriate for 3-D image-warping applications. In practice, however, sharp depth discontinuities seem to be associated with smooth color transitions or, more often, matched by abrupt color variations. None of these cases are challenges to 1-D reconstruction methods and, for them, the order in which the 1-D passes are applied (either horizontal or vertical first) seems not to be relevant. This issue will be discussed in more detail in Chapter 4.

The relief texture-mapping algorithm has several desirable features. First, the mapping from world to camera coordinate system, perspective transformation, clipping, hidden surface removal and image resampling are all performed in 1-D. Together, these operations amount to a significant portion of the whole graphics pipeline. Secondly, as a scanline algorithm, the approach is suitable for parallel implementation. In each pass, all rows (columns) can be warped independently from each other. Thus, significant levels of parallelization can be achieved.

Inverse mapping methods are usually preferred for high-quality image synthesis applications because filtering naturally fits the construction of the output images. For 1-D transformations, filtering is extremely simplified and forward mapping produces results as good as its inverse counterpart. This is extremely important in the context of 3-D image warping where forward transformations are considerably faster than inverse ones.

Chapter 3 – Relief Texture Mapping and the Pre-Warping Equations

This chapter provides a derivation for a family of ideal separable transforms called the *pre-warping equations* and explains their one-dimensional nature. Combined with texture mapping, such equations can be used to produce correct reprojections of depth images associated with arbitrary source and target camera configurations. Particular emphasis is given to the case involving *relief textures*, which are parallel projection images with depth, used as modeling and rendering primitives.

The idea behind the *relief texture-mapping* algorithm is conceptually quite simple: the 3-D image warp equation [McMillan97] is factored into a serial warp and a conventional texture mapping operation. Besides performing planar projective transformations, conventional texture mapping also handles rotations and scaling. Since the primary sources of difficulties associated with serial warps are image rotations and perspective distortions, relief texture-mapping benefits from the efficiency of such warps while avoiding their shortcomings.

3-D image warping [McMillan97] produces correct reprojections of depth images on arbitrary image planes. Its equation can be factored into a planar perspective transformation between the source and target image planes, and a per-pixel shift by the residual planar parallax. If the planar perspective transformation is applied first, the remaining per-pixel shift needs to handle changes in visibility. If, on the other hand, the residual shift is applied first, the remaining transformation is not a planar projective mapping and, therefore, cannot be implemented as a texture mapping operation. However, the observation that texture mapping is a special case of 3-D image warping [McMillan97] greatly simplifies the derivation of simple pre-warping equations, since the problem can be cast as a system involving equations that share many coefficients.

This chapter presents a proof for the one-dimensional nature of the pre-warping equations based on simple notions of Euclidean geometry. It also shows that, after rotations have been factored out, 3-D warps can be reduced to a two-dimensional problem regardless of the coordinate systems associated with the source and target image planes. Moreover, it shows that the pre-warping equations can be completely derived in 2-D using only similar triangles. A geometric interpretation for the coefficients of the pre-warping equations for relief textures is presented.

3-D image warping can take advantage of list priority rendering. An adaptation of such an algorithm for parallel projection images with depth is presented. The chapter ends with a discussion of the rendering of inside-looking-out views of environments modeled with perspective projection source images.

3.1 Images with Depth and Relief Textures

An *image with depth* is a pair $\{i_d, K\}$, where $i_d : U' \rightarrow C'$ is a digital image and K is a camera model associated with i_d . Each element of the color space C' of i_d is augmented to include a scalar value representing the distance, in Euclidean space, between the sampled point and a reference entity. If K is a perspective-projection camera model, the image is called a *perspective projection image with depth* and the reference entity is K 's center of projection. If, however, K is a parallel-projection camera model, the image is called a *parallel-projection image with depth* and the reference entity is K 's image plane.

Figure 3-1 shows a model of a perspective projection camera. Vectors \vec{a} and \vec{b} form a basis for the image plane. The lengths of these vectors are, respectively, the

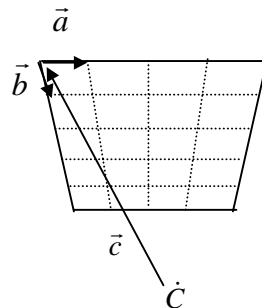


Figure 3-1. Perspective pinhole camera model.

horizontal and vertical sample spacing in Euclidean space. \dot{C} is the center of projection (COP) of the camera, and \vec{c} is a vector from the COP to the origin of the image plane [McMillan97].

Figure 3-2 shows the model for a parallel-projection camera. Vectors \vec{a} and \vec{b} have the same definitions as in the perspective case of Figure 3-1. Vector \vec{f} is a unit vector orthogonal to the plane spanned by \vec{a} and \vec{b} . The tails of all these vectors are at \dot{C} , the origin of the image plane.

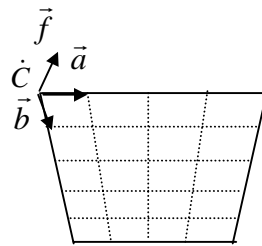


Figure 3-2. Parallel projection camera representation.

Relief textures are parallel projection images with depth, and are used as *modeling* and *rendering* primitives. Figures 3-3 illustrates the color image and depth map associated with the relief texture shown in Figure 3-4.



Figure 3-3. Color image (left) and depth map (right) associated with a relief texture. Darker regions in the depth map indicate more distant surfaces.

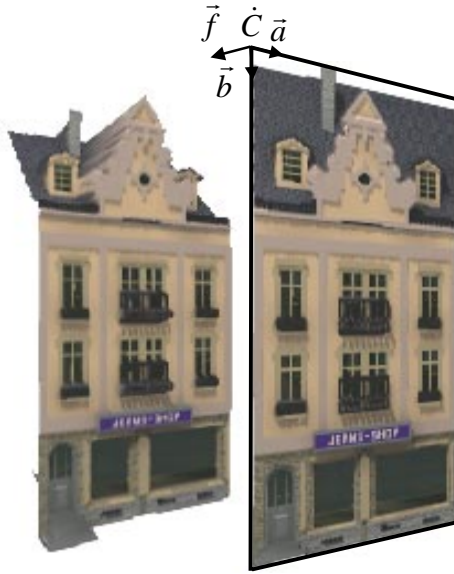


Figure 3-4. A relief texture and its reprojection viewed from an oblique angle.

3.2 3-D image Warping

Three-dimensional image warping [McMillan97] is a geometric transformation $w: U' \rightarrow W \subset \mathbb{R}^2$ that maps a source image with depth i_s onto a target image i_t . The geometric content of the scene is represented implicitly by combining depth information with the camera model associated with the source image. Thus, let \dot{x} be a point in Euclidean space whose projection into the image plane of i_s has coordinates (u_s, v_s) . Using the perspective projection camera model of Figure 3-1, the coordinates of \dot{x} can be expressed as $\dot{x} = \dot{C}_s + (\bar{c} + u_s \bar{a} + v_s \bar{b}) t_s(u_s, v_s)$ (Figure 3-5), where the scalar $t_s(u_s, v_s) = \frac{|\dot{x} - \dot{C}_s|}{|\bar{c} + u_s \bar{a} + v_s \bar{b}|}$ is the ratio between the distances from \dot{C}_s to \dot{x} and from \dot{C}_s to

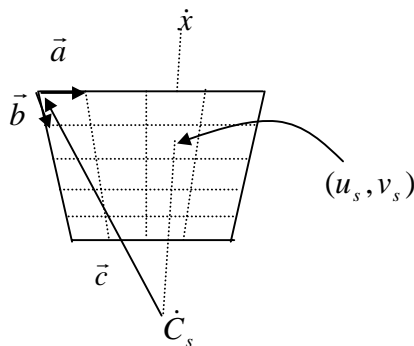


Figure 3-5. Recovering the coordinates of a point in Euclidean space from a perspective image with depth: $\dot{x} = \dot{C}_s + (\bar{c} + u_s \bar{a} + v_s \bar{b}) t_s(u_s, v_s)$.

pixel (u_s, v_s) , respectively. In matrix notation:

$$\dot{x} = \dot{C}_s + \begin{bmatrix} a_i & b_i & c_i \\ a_j & b_j & c_j \\ a_k & b_k & c_k \end{bmatrix} \begin{bmatrix} u_s \\ v_s \\ 1 \end{bmatrix} t_s(u_s, v_s) = \dot{C}_s + P \bar{x} t_s(u_s, v_s)$$

The projection of \dot{x} on an arbitrary target image can be obtained by expressing the coordinates of \dot{x} in both camera systems [McMillan97] (Figure 3-6):

$$\begin{aligned} \dot{C}_t + P_t \bar{x}_t t_t(u_t, v_t) &= \dot{x} = \dot{C}_s + P_s \bar{x}_s t_s(u_s, v_s) \\ \bar{x}_t t_t(u_t, v_t) &= P_t^{-1} [P_s \bar{x}_s t_s(u_s, v_s) + (\dot{C}_s - \dot{C}_t)] \\ \bar{x}_t &\doteq P_t^{-1} [P_s \bar{x}_s t_s(u_s, v_s) + (\dot{C}_s - \dot{C}_t)] \end{aligned} \quad (3-1)$$

where \doteq is projective equivalence, *i.e.*, the same except for a scalar multiple. Notice from Equation (3-1) that arbitrary target views can be obtained from a single source image without knowledge about the depth associated with pixels from the desired views. Dividing Equation (3-1) by $t_s(u_s, v_s)$ and distributing P_t^{-1} gives:

$$\bar{x}_t \doteq P_t^{-1} P_s \bar{x}_s + P_t^{-1} (\dot{C}_s - \dot{C}_t) \delta_s(u_s, v_s) \quad (3-2)$$

where $\delta_s(u_s, v_s) = 1/t_s(u_s, v_s)$ is called the *generalized disparity* of source pixel (u_s, v_s) [McMillan97]. Equation (3-2), called the *3-D image-warping equation* [McMillan97], provides a more intuitive interpretation of the underlying process: the target image is obtained by applying a planar perspective transformation (homography⁸)

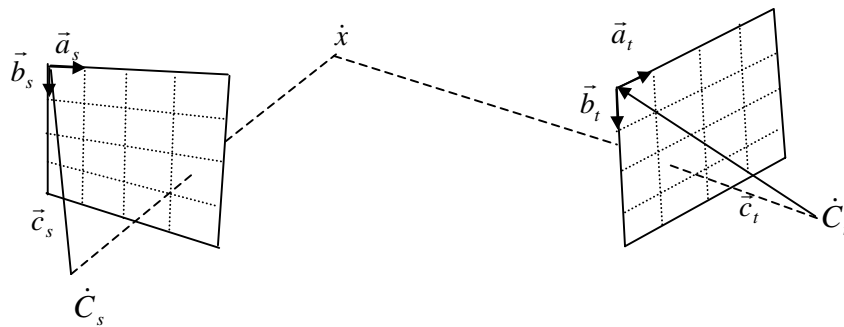


Figure 3-6. Point \dot{x} in Euclidean space projected onto both source and target image planes

⁸ Essentially, a texture mapping operation.

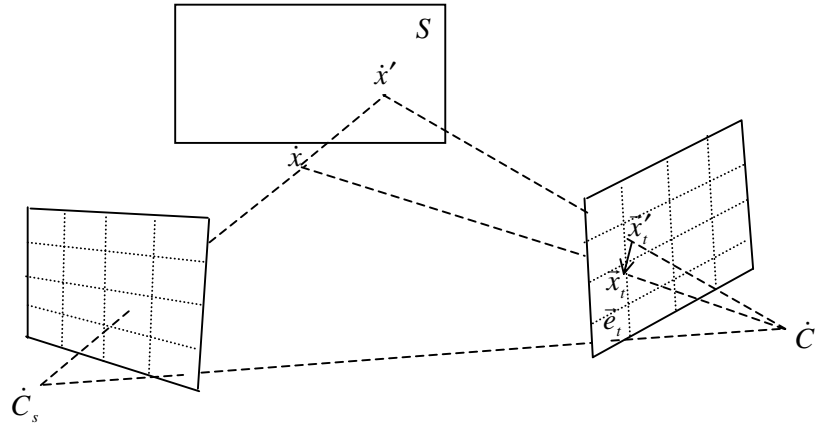


Figure 3-7. Two-view planar parallax [Sawhney94].

to the source image, and a per-pixel shift proportional to $\delta_s(u_s, v_s)$ in the direction of the *epipole* of the target view. Such a factorization, often referred to as *plane-plus-parallax* in the computer vision literature, avoids many of the inherent ambiguities and instabilities associated with the decomposition of image motion into its rotational and translational components [Irani97]. It has been used to produce improved solutions for a variety of problems in computer vision [Sawhney94] [Kumar94] [Irani96] [Irani97] and constitutes the basis for a related work on Sprites with Depth [Shade98].

The intuition behind this factorization is explained in Figure 3-7. Let x' be the intersection of the source ray, passing through the sampled point x in Euclidean space, with a reference plane S , and let \bar{x}_t and \bar{x}'_t be the projections of x and x' , respectively, into the target image plane. If $x = x'$ for all sampled points, the target image can be obtained simply by a planar perspective projection of S onto the target image plane. In general, however, not all samples are coplanar and the differences between the coordinates of \bar{x}_t and \bar{x}'_t for all such samples define an epipolar field centered at \bar{e}_t , the epipole of the target image. This property can be easily verified by noticing that x , x' , \bar{x}_t , \bar{x}'_t and \bar{e}_t are on the same epipolar plane⁹ (Figure 3-7). The values associated with the epipolar field, also referred to as *residual planar parallax* and represented in Equation (3.2) by the term $P_t^{-1}(\dot{C}_s - \dot{C}_t)\delta_s(u_s, v_s)$, can be expressed as $((-HT_z)/(Zd_t))(\bar{x}'_t - \bar{e}_t)$ [Irani96] (Figure 3-8). Here, H is the distance from x to S , Z and T_z are, respectively, the

⁹ A plane passing through the centers of projection of two cameras. The intersections of such planes with each image plane define the so-called *epipolar lines*.

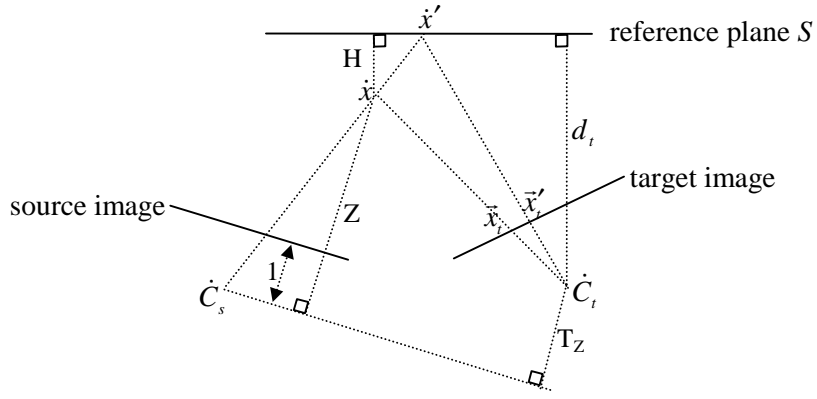


Figure 3-8. 2-D schematics for the plane-plus-parallax decomposition [Irani96].

depths of \dot{x} and the target COP with respect to the source camera, and d_t is the distance from S to the target COP (Figure 3-8). The negative sign to the left of H in the epipolar field expression indicates that \dot{x} is in front of S . The expression changes sign as \dot{x} crosses S .

3.3 Factoring the 3-D Image-Warping Equation

Both the homography and the residual planar parallax terms involve two-dimensional transformations. They are coupled in Equation (3-2) and cannot be independently applied as is. This can be checked after rewriting Equation (3-2) as

$$\begin{bmatrix} u'_t \\ v'_t \\ w'_t \end{bmatrix} = \begin{bmatrix} u_1 \\ v_1 \\ w_1 \end{bmatrix} + \begin{bmatrix} u_2 \\ v_2 \\ w_2 \end{bmatrix} \quad (3-3)$$

with

$$u_t = \frac{u_1 + u_2}{w_1 + w_2}$$

$$v_t = \frac{v_1 + v_2}{w_1 + w_2}$$

where [McMillan97]

$$u_1 = Au_s + Bv_s + C = \vec{a}_s \cdot (\vec{b}_t \times \vec{c}_t)u_s + \vec{b}_s \cdot (\vec{b}_t \times \vec{c}_t)v_s + \vec{c}_s \cdot (\vec{b}_t \times \vec{c}_t) \quad (3-4a)$$

$$v_1 = Eu_s + Fv_s + G = \vec{a}_s \cdot (\vec{c}_t \times \vec{a}_t)u_s + \vec{b}_s \cdot (\vec{c}_t \times \vec{a}_t)v_s + \vec{c}_s \cdot (\vec{c}_t \times \vec{a}_t) \quad (3-4b)$$

$$w_1 = Iu_s + Jv_s + K = \vec{a}_s \cdot (\vec{a}_t \times \vec{b}_t)u_s + \vec{b}_s \cdot (\vec{a}_t \times \vec{b}_t)v_s + \vec{c}_s \cdot (\vec{a}_t \times \vec{b}_t) \quad (3-4c)$$

$$u_2 = D\delta(u_s, v_s) = (\dot{C}_s - \dot{C}_t) \cdot (\vec{b}_t \times \vec{c}_t)\delta(u_s, v_s) \quad (3-4d)$$

$$v_2 = H\delta(u_s, v_s) = (\dot{C}_s - \dot{C}_t) \cdot (\vec{c}_t \times \vec{a}_t)\delta(u_s, v_s) \quad (3-4e)$$

$$w_2 = L\delta(u_s, v_s) = (\dot{C}_s - \dot{C}_t) \cdot (\vec{a}_t \times \vec{b}_t)\delta(u_s, v_s) \quad (3-4f)$$

If the homography is followed by a shift in the direction of the epipole, this needs to handle changes in visibility. The residual planar parallax is given by Equations (3-5a) and (3-5b) and depends on both terms in Equation (3-3).

$$u_{SHIFT} = \frac{u_1 + u_2}{w_1 + w_2} - \frac{u_1}{w_1} = \frac{u_2 w_1 - u_1 w_2}{w_1(w_1 + w_2)} \quad (3-5a)$$

$$v_{SHIFT} = \frac{v_1 + v_2}{w_1 + w_2} - \frac{v_1}{w_1} = \frac{v_2 w_1 - v_1 w_2}{w_1(w_1 + w_2)} \quad (3-5b)$$

If the shift towards the epipole represented by the last term in Equation (3-3) is executed first, the expressions for the remaining transformation are obtained from Equations (3-5a) and (3-5b) simply by switching the subscripts 1 and 2. In this case, however, the resulting mapping is not a planar projective transformation and cannot be implemented as a texture-map operation.

It is interesting to note from Equations (3-4a) through (3-4f) that, if $\delta_s(u_s, v_s)$ has a constant value for all pixels of the source image, the 3-D image warping reduces to a texture mapping operation. In particular, if $\delta_s(u_s, v_s) = 0$ the coordinates of the target pixels are given by equations (3-6a) and (3-6b) [McMillan97].

$$u_t = \frac{u_1}{w_1} = \frac{Au_s + Bv_s + C}{Iu_s + Jv_s + K} \quad (3-6a)$$

$$v_t = \frac{v_1}{w_1} = \frac{Eu_s + Fv_s + G}{Iu_s + Jv_s + K} \quad (3-6b)$$

3.4 The Ideal Factorization

Equation (3-2) concisely describes the warping process. From a conventional rendering point-of-view, the mapping expressed by Equation (3-2) should ideally be factored so as to allow conventional texture mapping to be applied after the shift in the direction of the epipole. Such an approach is the opposite of the conventional plane-plus-parallax decomposition [Sawhney94], in the sense that shifts take place prior to the homography. It offers some advantages. First, it can benefit from texture mapping hardware to perform the final transformation and filtering. Secondly, the resulting equations have a very simple 1-D structure that enables the pre-warp to be implemented using only 1-D image operations along rows and columns, as will be discussed later in Chapter 4. Since no rotations are involved, the resulting serial warps do not suffer from the shortcomings associated with arbitrary serial warps and discussed in Section 2.3.

In order to obtain an ideal factorization, one needs to find a warp $p:U' \rightarrow Q \subset \mathfrak{R}^2$ so that the composition $m \circ p:U' \rightarrow W \subset \mathfrak{R}^2$, where $m:Q \rightarrow W \subset \mathfrak{R}^2$ is a standard texture-mapping transformation, is equivalent to the 3-D image warp $w:U' \rightarrow W \subset \mathfrak{R}^2$. w maps samples from a source image with depth onto the image plane of a target camera. m should map texels from the source image plane onto pixels in the target image plane as well. Thus, the fundamental issues are that, during the warp, visibility and the induced distortion be computed with respect to the target center of projection.

Thus, let $(u_i, v_i) = (u_s + \Delta u, v_s + \Delta v)$ be the intermediate coordinates obtained after shifting the coordinates of the source pixel (u_s, v_s) by $(\Delta u, \Delta v)$. The equivalence between the composed mapping $m \circ p$ and w is modeled by Equations (3-7a) and (3-7b). The correct interpretation of these equations is critical for the understanding of one of the central ideas of this dissertation. Simply put, Equations (3-7a) and (3-7b) ask the following question: *What coordinates (u_i, v_i) should the source pixels (u_s, v_s) have so that a view of such a flat distorted image on the source image plane from the target COP would be identical to a 3-D image warp of the source image onto the target image plane?* The process is illustrated in Figures 3-9 and 3-10. Since the warped image is ready to be texture-mapped onto the source image plane, it can be mapped onto a polygon that

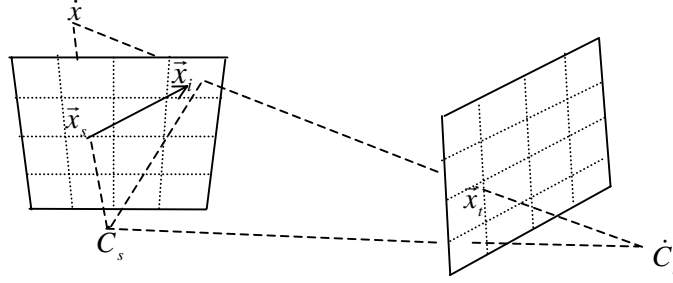


Figure 3-9. Sample \bar{x}_s is shifted to \bar{x}_i in order to match the view of \dot{x} from \dot{C}_i .

matches the dimensions, position and orientation of such a plane. This notion allows relief textures to be used as modeling primitives and will be discussed in detail in Chapter 5.

$$\frac{Au_i + Bv_i + C}{Iu_i + Jv_i + K} = \frac{Au_s + Bv_s + C + D\delta(u_s, v_s)}{Iu_s + Jv_s + K + L\delta(u_s, v_s)} \quad (3-7a)$$

$$\frac{Eu_i + Fv_i + G}{Iu_i + Jv_i + K} = \frac{Eu_s + Fv_s + G + H\delta(u_s, v_s)}{Iu_s + Jv_s + K + L\delta(u_s, v_s)} \quad (3-7b)$$

The desired warp p is obtained by solving Equations (3-7a) and (3-7b) for u_i and

v_i :

$$u_i = \frac{(A(FK - GJ) + B(IG - EK) + C(EJ - IF))u_s + (B(GL - HK) + C(HJ - FL) + D(FK - GJ))\delta(u_s, v_s)}{(A(FK - GJ) + B(IG - EK) + C(EJ - IF)) + (A(FL - HJ) + B(HI - EL) + D(EJ - IF))\delta(u_s, v_s)}$$

$$v_i = \frac{(A(FK - GJ) + B(IG - EK) + C(EJ - IF))v_s + (A(HK - GL) + C(EL - IH) + D(GI - EK))\delta(u_s, v_s)}{(A(FK - GJ) + B(IG - EK) + C(EJ - IF)) + (A(FL - HJ) + B(HI - EL) + D(EJ - IF))\delta(u_s, v_s)}$$

Dividing both numerators and denominators by $(A(FK - GJ) + B(IG - EK) + C(EJ - IF))$, produces the simple one-dimensional equations:

$$u_i = \frac{u_s + q_1\delta(u_s, v_s)}{1 + q_3\delta(u_s, v_s)} \quad (3-8a)$$

$$v_i = \frac{v_s + q_2\delta(u_s, v_s)}{1 + q_3\delta(u_s, v_s)} \quad (3-8b)$$

or

$$\begin{bmatrix} u'_i \\ v'_i \\ w'_i \end{bmatrix} = \begin{bmatrix} u_s \\ v_s \\ 1 \end{bmatrix} + \delta(u_s, v_s) \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

where q_1 , q_2 and q_3 are constants for a given configuration of source and target cameras and, together with $\delta(u_s, v_s)$, determine the amount of change $(\Delta u, \Delta v)$ in the coordinates of the source pixels along epipolar lines.

The *pre-warping equations* (3-8a) and (3-8b) solve visibility and perform some of the perspective transformation. The texture-mapping step is responsible for the remaining perspective distortion, as well as for some scaling and rotation (Figure 3-10).

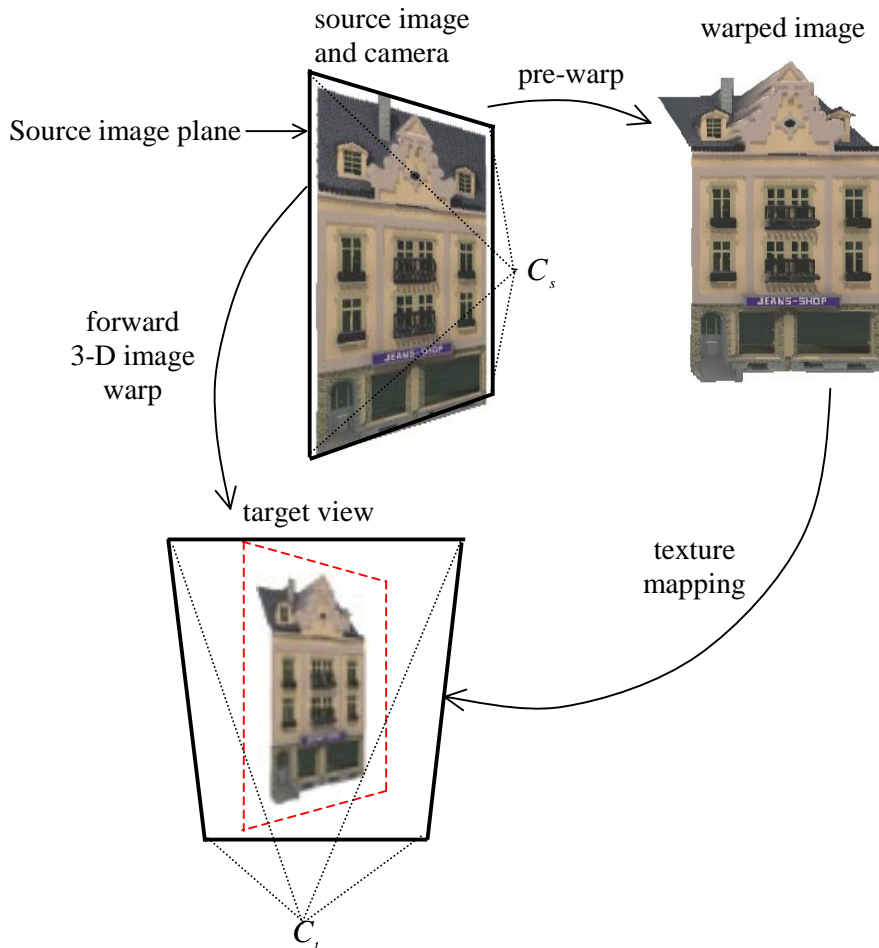


Figure 3-10. 3-D image warping is equivalent to a pre-warp of the source image followed by conventional texture mapping. The borders of the polygon that matches the source image plane are shown in dashed lines.

3.5 Simpler Coefficients

The amount of shift $(\Delta u, \Delta v)$ to be applied to a source texel only depends on the position of the target COP (and not on any other target camera parameters), on the position and orientation of the source image plane, and on the distance from the associated sample to the source image plane (Figure 3-11). Therefore, one can freely specify the parameters \vec{a}_t , \vec{b}_t and \vec{c}_t , which define a temporary target camera used only for the purpose of the pre-warp and which usually differs from the virtual camera used for the visualization of the final scene. By appropriately choosing such parameters, it is possible to eliminate several of the coefficients in Equations (3-7a) and (3-7b) by forcing the corresponding scalar triple products shown in Equations (3-4a) to (3-4f) to have the form $\vec{v} \cdot (\vec{v} \times \vec{w})$ or $\vec{w} \cdot (\vec{v} \times \vec{w})$. Such a procedure leads to a drastic simplification of the expressions used to compute coefficients q_1 , q_2 and q_3 . For instance, the canceling process can be made very effective by letting $\vec{a}_t = \alpha \vec{a}_s$, $\vec{b}_t = \beta \vec{b}_s$ and $\vec{c}_t = \gamma (\dot{C}_s - \dot{C}_t)$, for non-zero $\alpha, \beta, \gamma \in \Re$. This is equivalent to making source and target image planes parallel to each other, with no relative rotation between them, and to putting the source center of projection at the origin of the target image plane (Figure 3-12 (left)). In such a case, coefficients B, D, E, H, I and J become zero and Equations (3-8a) and (3-8b) become

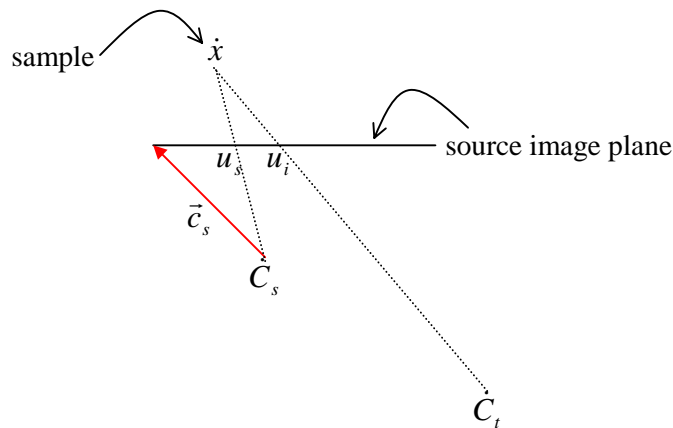


Figure 3-11. The amount of shift to be applied by the pre-warp to the coordinates of a source pixel does not depend on the target image plane (not shown).

$$u_i = \frac{u_s - \frac{CL}{AK} \delta(u_s, v_s)}{1 + \frac{L}{K} \delta(u_s, v_s)} \quad (3-9a)$$

$$v_i = \frac{v_s - \frac{GL}{FK} \delta(u_s, v_s)}{1 + \frac{L}{K} \delta(u_s, v_s)} \quad (3-9b)$$

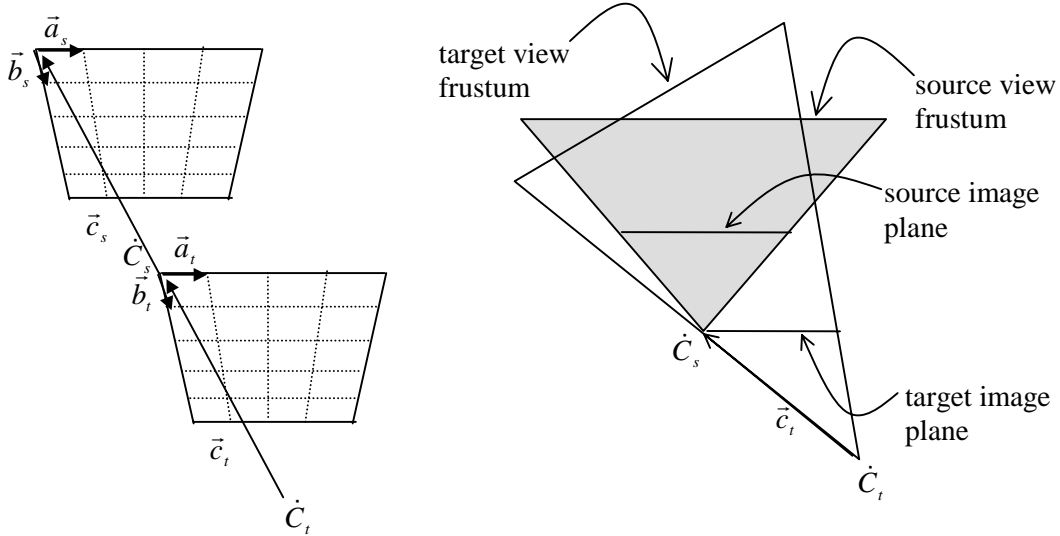


Figure 3-12. Left: configuration involving two perspective cameras leading to simplified pre-warping equations. Right: 2-D representation of the configuration on the left.

3.6 Pre-warping Equations for Relief Textures

The use of parallel projection images as modeling primitives presents some advantages. For instance, the sampling density is constant across the entire image. Also, one can take advantage of the perpendicular relationship between the sampling rays and the image plane of a parallel projection image to produce a simple algorithm to render these image-based representations. The details of such an algorithm are presented in Chapter 5.

Given a parallel projection source camera, the coordinates of a point \dot{x} in Euclidean space are given by:

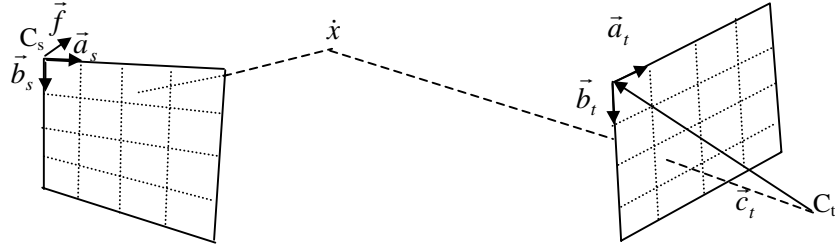


Figure 3-13. Computing the projection of point \hat{x} into a perspective target camera from its coordinates in a parallel projection source camera.

$$\hat{x} = \dot{C}_s + \begin{bmatrix} a_{si} & b_{si} & f_i \\ a_{sj} & b_{sj} & f_j \\ a_{sk} & b_{sk} & f_k \end{bmatrix} \begin{bmatrix} u_s \\ v_s \\ displ(u_s, v_s) \end{bmatrix} = \dot{C}_s + P'_s \vec{x}'_s$$

where $displ(u_s, v_s)$ is the orthogonal displacement, or height, associated with source pixel (u_s, v_s) . Expressing the coordinates of \hat{x} using both a perspective target camera and a parallel projection source camera (Figure 3-13), the 3-D warping equation becomes:

$$\begin{aligned} \dot{C}_t + P_t \vec{x}_t t_t(u_t, v_t) &= \dot{C}_s + P'_s \vec{x}'_s \\ t_t(u_t, v_t) P_t \vec{x}_t &= P'_s \vec{x}'_s + (\dot{C}_s - \dot{C}_t) \\ \vec{x}_t &\doteq P_t^{-1} (P'_s \vec{x}'_s + (\dot{C}_s - \dot{C}_t)) \end{aligned} \quad (3-10)$$

Again, \doteq is projective equivalence. Thus, the 3-D warping equations involving a parallel projection source image and a perspective projection target image are given by:

$$u_t = \frac{Au_s + Bv_s + D + C' displ(u_s, v_s)}{Iu_s + Jv_s + L + K' displ(u_s, v_s)} \quad (3-11a)$$

$$v_t = \frac{Eu_s + Fv_s + H + G' displ(u_s, v_s)}{Iu_s + Jv_s + L + K' displ(u_s, v_s)} \quad (3-11b)$$

where coefficients A, B, D, E, F, H, I, J and L have the same definition as in Equations (3-4a) to (3-4f) and

$$C' = \vec{f} \cdot (\vec{b}_t \times \vec{c}_t)$$

$$G' = \vec{f} \cdot (\vec{c}_t \times \vec{a}_t)$$

$$K' = \vec{f} \cdot (\vec{a}_t \times \vec{b}_t)$$

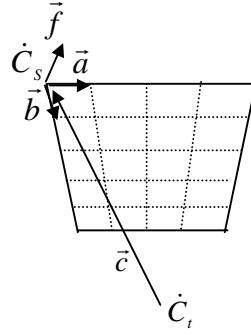


Figure 3-14. Parallel and perspective projection cameras sharing the same image plane (origin, \vec{a} and \vec{b} vectors).

Notice that due to the change in the order of the coefficients D , H and L in Equations (3-11a) and (3-11b), the system originally defined by Equations (3-7a) and (3-7b) should be rewritten as:

$$\frac{Au_i + Bv_i + D}{Iu_i + Jv_i + L} = \frac{Au_s + Bv_s + D + C' \text{displ}(u_s, v_s)}{Iu_s + Jv_s + L + K' \text{displ}(u_s, v_s)} \quad (3-12a)$$

$$\frac{Eu_i + Fv_i + H}{Iu_i + Jv_i + L} = \frac{Eu_s + Fv_s + H + G' \text{displ}(u_s, v_s)}{Iu_s + Jv_s + L + K' \text{displ}(u_s, v_s)} \quad (3-12b)$$

The condition that eliminates coefficients B , D , E , H , I and J , i.e., $\vec{a}_t = \alpha \vec{a}_s$, $\vec{b}_t = \beta \vec{b}_s$ and $\vec{c}_t = \gamma(\vec{C}_s - \vec{C}_t)$, is trivially satisfied by letting source and temporary target image planes coincide, including their origins and basis vectors. In this case, the subscripts of all vectors can be dropped without risk of confusion (Figure 3-14). After removing the zero coefficients and solving for u_i and v_i :

$$u_i = \frac{u_s + k_1 \text{displ}(u_s, v_s)}{1 + k_3 \text{displ}(u_s, v_s)} \quad (3-13a)$$

$$v_i = \frac{v_s + k_2 \text{displ}(u_s, v_s)}{1 + k_3 \text{displ}(u_s, v_s)} \quad (3-13b)$$

where $k_1 = \frac{C'}{A} = \frac{\vec{f} \cdot (\vec{b} \times \vec{c})}{\vec{a} \cdot (\vec{b} \times \vec{c})}$, $k_2 = \frac{G'}{F} = \frac{\vec{f} \cdot (\vec{c} \times \vec{a})}{\vec{b} \cdot (\vec{c} \times \vec{a})}$ and $k_3 = \frac{K'}{L} = \frac{\vec{f} \cdot (\vec{a} \times \vec{b})}{(\vec{C}_s - \vec{C}_t) \cdot (\vec{a} \times \vec{b})} = \frac{\vec{f} \cdot (\vec{a} \times \vec{b})}{\vec{c} \cdot (\vec{a} \times \vec{b})}$. In matrix notation

$$\begin{bmatrix} u'_i \\ v'_i \\ z'_i \\ w'_i \end{bmatrix} = \begin{bmatrix} 1 & 0 & k_1 & 0 \\ 0 & 1 & k_2 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & k_3 & 1 \end{bmatrix} \begin{bmatrix} u_s \\ v_s \\ displ(u_s, v_s) \\ 1 \end{bmatrix}$$

Equations (3-13a) and (3-13b) are called the *pre-warping equations for relief textures*. The per-textel divisions can be avoided by quantizing the displacement values (in a pre-processing step) and storing the reciprocal of the denominator of Equation (3-13a) in a lookup table. This issue will be discussed in Section 4.4.

3.6.1 The one-dimensional nature of the pre-warping equations

Although, at first glance, it may seem surprising that the pre-warping equations are one-dimensional, the geometric intuition behind this fact is actually very simple and leads to the computation of even simpler coefficients. It follows directly from the separability of the perspective projection into orthogonal components. Recall that the pre-warp simply reprojects a range image with respect to a different center of projection. This notion of separability into orthogonal components will be explained in the context of relief textures for the horizontal shift. The same reasoning applies to the vertical case, as well as to perspective projection images.

Proposition 3-1. Given three planes Π_1 , Π_2 and Π_3 , so that no two of them are parallel to each other, if the intersection between Π_1 and Π_2 is parallel to Π_3 , then the lines resulting from the intersections between Π_1 and Π_3 , and between Π_2 and Π_3 are parallel.

Proof. Let AB be parallel to plane Π_3 , but not in Π_3 , and let Π_1 be any plane through AB intersecting Π_3 in CD (Figure 3-15). AB and CD cannot meet; otherwise, AB would meet Π_3 . Since AB and CD are in Π_1 , AB and CD are parallel [Euclid]. Now, consider Π_2 , a plane through AB intersecting Π_3 in EF. Using the same argument, one concludes that AB and EF are parallel. Since both CD and EF are in Π_3 , by transitivity, CD and EF are parallel.

If AB is in Π_3 , then AB = CD = EF and therefore, CD is parallel to EF.

(q.e.d)

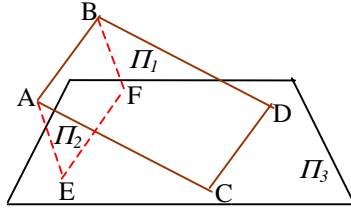


Figure 3-15. AB is parallel to Π_3 . The intersection of Π_3 with plane Π_1 passing through AB is parallel to AB [Euclid]. CD and EF are parallel.

Notice that the parallelism between CD and EF is independent of any coordinate system associated with Π_3 . In other words, the basis vectors used to span Π_3 are not required to be orthogonal. Thus, let Π_3 be an image plane and let AB in Π_1 be parallel to Π_3 (i.e., all points in AB are equidistant to Π_3), such that the intersection CD between Π_1 and Π_3 has constant “column” coordinate in the coordinate system associated with Π_3 (Figure 3-16). According to Proposition 3-1, each non-empty intersections between Π_3 and any plane containing AB is parallel to CD and, therefore, also has constant “column” coordinate in the coordinate system of Π_3 . Thus, the intersection between Π_3 and the plane defined by AB and a center of projection \dot{C} has constant “column” coordinate in such a coordinate system. In the context of the pre-warping, such an observation leads to the following conclusion: *for a given camera configuration and displacement/disparity value, the amount of horizontal (vertical) shift to be applied to a texture element is independent of its row (column) coordinate.* More specifically, the amount of shift only depends on the position of the target COP, on the position and orientation of the source

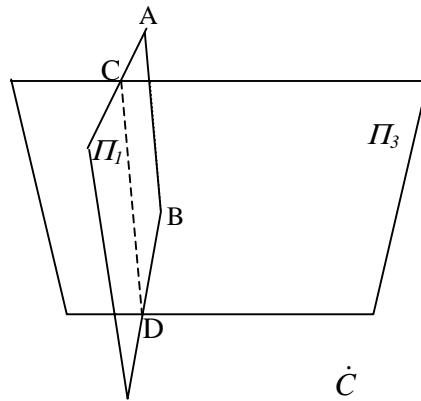


Figure 3-16. Intersection CD between Π_3 and Π_1 has constant “column” coordinate in the coordinate system of Π_3 . Π_1 contains AB, which is parallel to Π_3 . The intersection between Π_3 and the plane defined by AB and \dot{C} is parallel to CD.

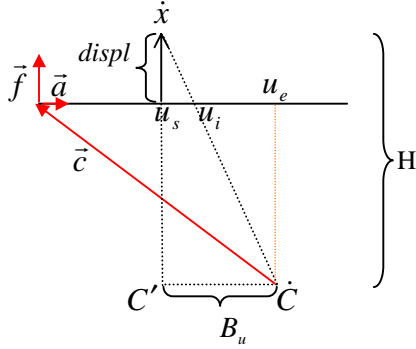


Figure 3-17. Top view of a relief texture with point \hat{x} projecting at column u_i as observed from \hat{C} . Triangles $C' \hat{C} \hat{x}$ and $u_s u_i \hat{x}$ are similar. u_e is the column of the epipole.

image plane, and on the distance from the sample to the source image plane. This allows the computation of the amount of shift to be carried out in two dimensions only and independently of the coordinate system associated with the image plane.

Figure 3-17 shows a 2-D view of a relief texture observed from \hat{C} . The pre-warp maps point \hat{x} , originally at column u_s , to column u_i . Thus, the amount of horizontal shift applied to the corresponding texel is $\Delta u = u_i - u_s$. By similar triangles (Figure 3-17):

$$\frac{\Delta u}{B_u} = \frac{displ}{H}$$

$$\Delta u = \frac{B_u}{H} displ \quad (3-14)$$

where B_u is the distance from u_e (the horizontal coordinate of the epipole¹⁰) to u_s :

$$B_u = u_e - u_s \quad (3-15)$$

Expressing u_e in source pixel coordinates

$$u_e = \frac{-\vec{c} \cdot \vec{a}}{\|\vec{a}\|^2} = \frac{-\vec{c} \cdot \vec{a}}{\vec{a} \cdot \vec{a}}$$

H , on the other hand, is given by

$$H = \vec{c} \cdot \vec{f} + displ \quad (3-16)$$

¹⁰ In the case of a parallel projection image, the epipole is obtained by orthographically projecting the COP of the other camera into the parallel projection image plane.

The absolute value of Δu is directly proportional to B_u , the horizontal distance from the epipole to the source texel, and inversely proportional to H , the depth of \dot{x} with respect to \dot{C} . The sign of B_u , in Equation (3-15), guarantees that the shift is always towards the epipole. For a given viewing configuration, the amount of horizontal shift applied to a source texel only depends on its original column and displacement values.

Substituting Equations (3-15) and (3-16) into Equation (3-14) gives

$$\Delta u = \frac{(u_e - u_s)displ}{\vec{c} \cdot \vec{f} + displ} \quad (3-17)$$

Computing Δu directly from the pre-warping equation (3-13a)

$$\begin{aligned} \Delta u &= \frac{u_s + k_1 displ}{1 + k_3 displ} - u_s \\ \Delta u &= \frac{(k_1 - u_s k_3) displ}{1 + k_3 displ} \end{aligned} \quad (3-18)$$

Notice that Equations (3-17) and (3-18) express the same quantity. The former is obtained after dividing both numerator and denominator of Equation (3-18) by k_3 . A direct inspection of these two expressions reveals that

$$k_3 = \frac{1}{\vec{c} \cdot \vec{f}} \quad (3-19)$$

and

$$k_1 = u_e k_3 \quad (3-20)$$

It is easy to verify that $\frac{1}{\vec{c} \cdot \vec{f}} = \frac{\vec{f} \cdot (\vec{a} \times \vec{b})}{\vec{c} \cdot (\vec{a} \times \vec{b})}$ (see Equation 3-13). Since \vec{f} is a unit

vector perpendicular to the plane spanned by vectors \vec{a} and \vec{b} ,

$$\vec{f} = \alpha(\vec{a} \times \vec{b}) \quad (3-21)$$

where

$$|\alpha| = \frac{1}{\|\vec{a} \times \vec{b}\|}$$

Therefore,

$$\frac{\vec{f} \cdot (\vec{a} \times \vec{b})}{\vec{c} \cdot (\vec{a} \times \vec{b})} = \frac{(1/\alpha)\vec{f} \cdot \vec{f}}{(1/\alpha)\vec{c} \cdot \vec{f}} = \frac{1}{\vec{c} \cdot \vec{f}}$$

Using similar derivations, the amount of vertical shift can be shown to be

$$\Delta v = \frac{B_v}{H} displ$$

where

$$B_v = v_e - v_s$$

and

$$v_e = \frac{-\vec{c} \cdot \vec{b}}{\vec{b} \cdot \vec{b}}$$

Thus,

$$\Delta v = \frac{(v_e - u_s) displ}{\vec{c} \cdot \vec{f} + displ} \quad (3-22)$$

Computing Δv directly from the pre-warping equation (3-13b)

$$\Delta v = \frac{(k_2 - v_s k_3) displ}{1 + k_3 displ} \quad (3-23)$$

From Equations (3-22) and (3-23), one obtains

$$k_2 = v_e k_3 \quad (3-24)$$

where v_e is the row coordinate of the epipole.

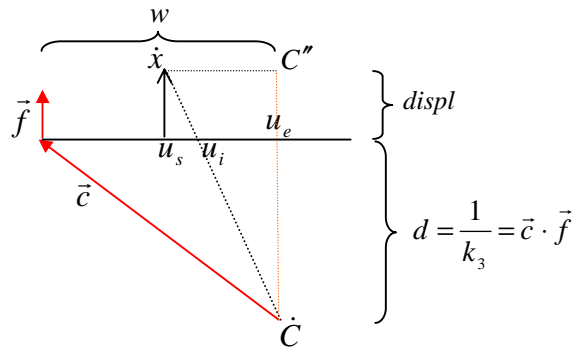


Figure 3-18. Another geometric interpretation for the amount of shift applied to point \hat{x} as observed from \dot{C} . Triangles $\hat{x} C'' \dot{C}$ and $u_s u_i \hat{x}$ are similar. u_e is the column of the epipole.

3.6.2 Geometric interpretation of the coefficients of the pre-warping equations for relief textures

Equations (3-19), (3-20) and (3-24) provide a clear geometric interpretation for the coefficients of the pre-warping equations for relief textures. Figure 3-18 shows the same situation depicted in Figure 3-17 in terms of the new coefficients. According to Equations (3-19) and (3-20)

$$k_3 = \frac{1}{d} \quad (3-25)$$

and

$$k_1 = u_e k_3 = \frac{w}{d} \quad (3-26)$$

Thus, k_3 is the reciprocal of the distance from the target center of projection (\dot{C}) to the shared image plane. k_1 is the ratio between the horizontal coordinate of the epipole and the distance from \dot{C} to the shared image plane. Likewise, k_2 is the ratio between the vertical coordinate of the epipole and the distance from \dot{C} to the shared image plane.

3.6.3 A Useful identity

Proposition 3-2. Given three arbitrary vectors \vec{a} , \vec{b} and \vec{c} in three-space, such that \vec{a} is not a null vector, the following relationship holds:

$$(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c}) = -\|\vec{a} \times \vec{b}\|^2 \frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{a}}$$

Proof: From the expressions for k_1 , the following equality holds

$$\frac{\vec{f} \cdot (\vec{b} \times \vec{c})}{\vec{a} \cdot (\vec{b} \times \vec{c})} = u_e k_3 = \frac{-\vec{c} \cdot \vec{a}}{(\vec{a} \cdot \vec{a})(\vec{c} \cdot \vec{f})} \quad (3-27)$$

Substituting Equation (3-21) into the numerator of the left-hand side of Equation (3-27) and rearranging the terms of the corresponding denominator

$$\frac{\alpha(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c})}{(\vec{a} \times \vec{b}) \cdot \vec{c}} = \frac{-\vec{c} \cdot \vec{a}}{(\vec{a} \cdot \vec{a})(\vec{c} \cdot \vec{f})}$$

Rewriting $(\vec{a} \times \vec{b})$ from the denominator as $(1/\alpha)\vec{f}$ (using Equation (3-21)) produces

$$\frac{\alpha(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c})}{(1/\alpha)\vec{f} \cdot \vec{c}} = \frac{-\vec{c} \cdot \vec{a}}{(\vec{a} \cdot \vec{a})(\vec{c} \cdot \vec{f})}$$

Using the commutative property of the dot product

$$\frac{\alpha^2(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c})}{\vec{c} \cdot \vec{f}} = \frac{-\vec{a} \cdot \vec{c}}{(\vec{a} \cdot \vec{a})(\vec{c} \cdot \vec{f})}$$

Finally, multiplying both sides by $\frac{\vec{c} \cdot \vec{f}}{\alpha^2}$ and realizing that $|\alpha| = \frac{1}{\|\vec{a} \times \vec{b}\|}$ (Equation (3-21)), produces the desired result

$$(\vec{a} \times \vec{b}) \cdot (\vec{b} \times \vec{c}) = -\|\vec{a} \times \vec{b}\|^2 \frac{\vec{a} \cdot \vec{c}}{\vec{a} \cdot \vec{a}} \quad (\text{q.e.d.})$$

3.6.4 Pre-warping equations for perspective projection source images: a geometric derivation

A geometric derivation of pre-warping equations for configurations involving two perspective projection cameras is presented next. Although the form of the resulting equations is slightly different from that of Equations (3-9a) and (3-9b), the value of such a derivation resides in providing some geometric intuition about the pre-warping process for perspective projection source images. Figure 3-19 shows a top view of a scene consisting of a point \dot{x} observed from both source and target COPs. The corresponding

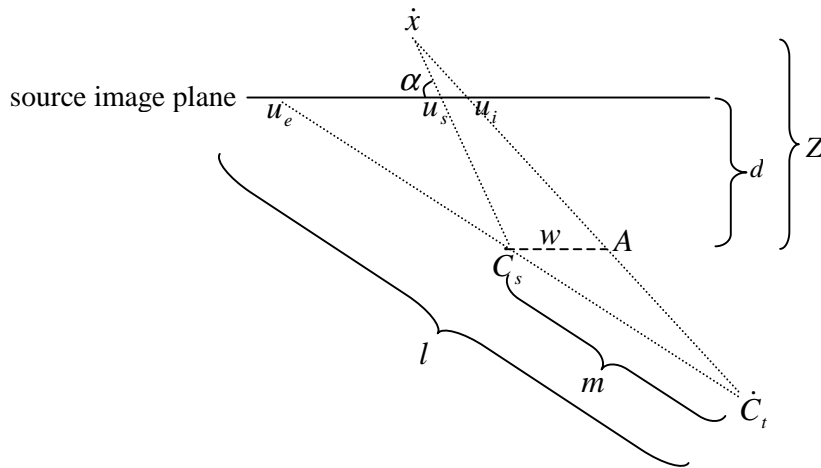


Figure 3-19. 2-D view of a scene showing a source camera and target COP. Point \dot{x} projecting at columns u_s and u_i as observed from \dot{C}_s and \dot{C}_t , respectively.

projections onto the source image plane are u_s and u_i , respectively. u_e is the column coordinate of the epipole, d is the distance from the source COP to its image plane, and Z is the depth of \dot{x} with respect to \dot{C}_s . The distances from \dot{C}_i to \dot{C}_s and from \dot{C}_i to u_e are m and l , respectively. According to the definition of generalized disparity [McMillan97], $\delta(u_s, v_s) = d / Z$.

Triangles $u_s \dot{x} u_i$ and $\dot{C}_s \dot{x} A$ are similar and so are triangles $\dot{C}_s A \dot{C}_i$ and $u_e u_i \dot{C}_i$. From the first pair of similar triangles, one has

$$\frac{(u_i - u_s)}{w} = \frac{(Z - d) / \cos \alpha}{Z / \cos \alpha}$$

$$(u_i - u_s) = w(1 - \delta(u_s, v_s)) \quad (3-28)$$

Equation (3-28) expresses the amount of horizontal shift applied to source column u_s . $\delta(u_s, v_s) = 1$ when \dot{x} is on the image plane, in which case its projection does not move. As \dot{x} switches from one side of the source image plane to another, the sense of the shift is reversed.

From triangles $\dot{C}_s A \dot{C}_i$ and $u_e u_i \dot{C}_i$:

$$\frac{w}{(u_i - u_e)} = \frac{m}{l} = c.$$

Thus,

$$w = (u_i - u_e)c. \quad (3-29)$$

Substituting Equation (3-29) into Equation (3-28) and solving for u_i

$$u_i = \frac{u_s + cu_e(\delta(u_s, v_s) - 1)}{1 + c(\delta(u_s, v_s) - 1)}. \quad (3-30)$$

Using a similar derivation, one obtains

$$v_i = \frac{v_s + cv_e(\delta(u_s, v_s) - 1)}{1 + c(\delta(u_s, v_s) - 1)}. \quad (3-31)$$

3.7 Occlusion-Compatible Order for Parallel Projection Images with Depth

The occlusion-compatible order of McMillan and Bishop [McMillan97], essentially a painter's style algorithm, specifies possible orders in which the pixels of a perspective projection image with depth can be warped, so that correct visibility is achieved for arbitrary viewpoints without explicit depth comparison. Algorithm 3-1 summarizes the procedure.

find the projection of the target COP into the source image plane (the epipole);
divide the source image into at most four sheets based on the coordinates of the epipole;
if the target COP is behind the source COP then
 for each resulting sheet
 warp its samples from the epipole towards the borders of the sheet;
else
 for each resulting sheet
 warp its samples from the borders of the sheet towards the epipole;

Algorithm 3-1. Occlusion-compatible order for perspective projection source images.

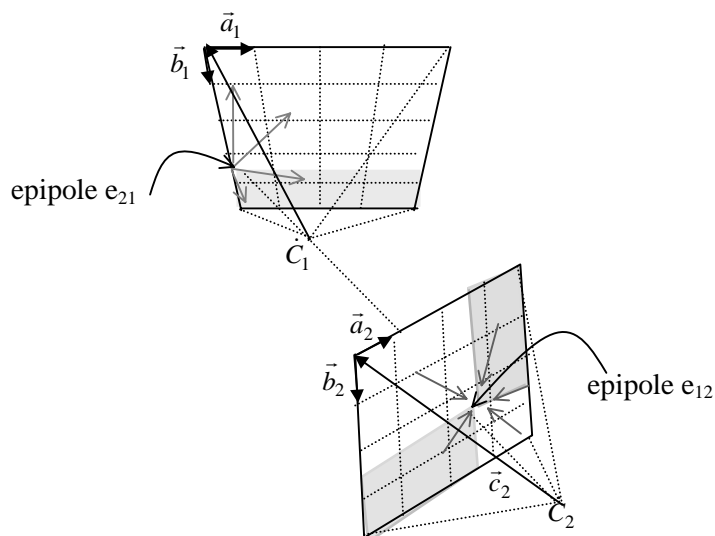


Figure 3-20. Occlusion-compatible order. The epipole divides the source image in at most four sheets. The arrows indicate the order in which pixels should be warped.

Figure 3-20 shows two images playing both source and target image roles. The white and gray regions represent the sheets when the image acts as source; the arrows define the warping order. The intuition behind the algorithm is illustrated in Figure 3-21 for the case in which the target center of projection is behind the source COP: whenever multiple samples fall along a target ray, the one whose corresponding pixel is furthest from the epipole is the closest to the desired center of projection and, therefore, may safely overwrite previously warped samples. Thus, source pixels should be warped from the epipole towards the borders of the image.

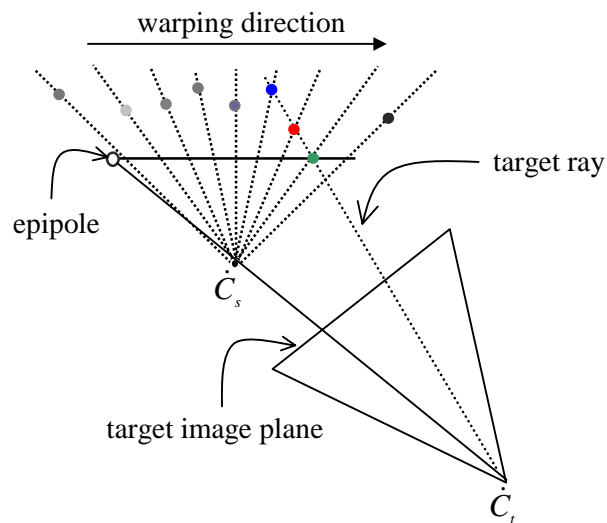


Figure 3-21. Occlusion-compatible order: geometric intuition. The target center of projection is behind the source COP. Whenever multiple samples fall along the same target ray, the one whose projection into the source image plane is furthest from the epipole is the closest to the target center of projection.

In the pinhole camera model, all rays must pass through its COP. This causes the projection of a point to reverse the side (left/right, top/bottom) of its projection on the image plane as the point crosses the plane (parallel to the image plane) that contains the center of projection (Figure 3-22). Thus, although the spatial relationship among the samples in the source image remains the same, the reference point for the algorithm (the epipole) may reverse sides as the desired viewpoint moves (Figure 3-22). In such a case, the order in which the samples are warped needs to be reversed accordingly. This explains the two possible enumerations of the occlusion-compatible order algorithm. The COP of a parallel projection image, on the other hand, is at infinity. Thus, the epipole

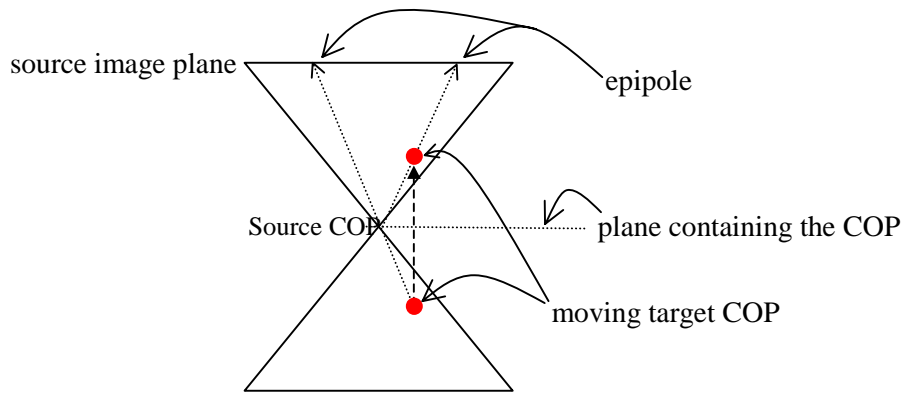


Figure 3-22. Pinhole camera model: the projection of a point changes sides (left/right, top/bottom) in the image plane as the point crosses the plane parallel to the source image plane that contains the COP.

does not change as the target COP is moved perpendicularly with respect to the source image plane (Figure 3-23). Therefore, a single order exists for warping the source pixels.

When parallel projection images with depth are used as source images, samples falling along a target ray define similar triangles (Figure 3-24). The sides of such triangles are, respectively, the depth associated with the sample, the distance between the corresponding pixel and the intersection of the target ray with the source image plane, and the distance between such an intersection and the sample itself. Due to triangle similarity, the sample whose corresponding pixel is closest to the epipole is also closest

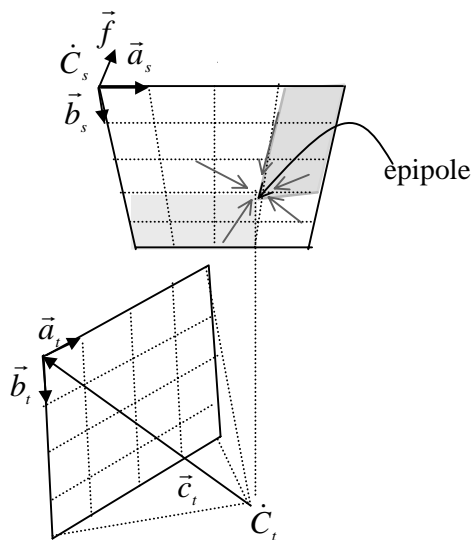


Figure 3-23. Occlusion-compatible order for parallel projection images. Samples are always warped from the borders towards the epipole.

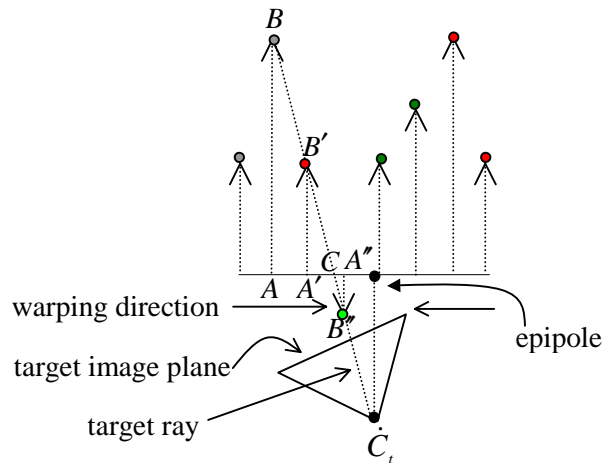


Figure 3-24. Occlusion-compatible order for parallel projection images: geometric intuition. Triangles ABC , $A'B'C$ and $A''B''C$ are similar. Similarity of triangles guarantees that an occlusion compatible order is obtained by always warping samples from the borders towards the epipole.

to the desired viewpoint (Figure 3-24). Thus, an occlusion-compatible order for parallel projection images with depth is obtained by always warping pixels from the borders towards the epipole and is summarized in Algorithm 3-2.

find the projection of the target COP into the source image plane (the epipole);
divide the source image into at most four sheets based on the coordinates of the epipole;
for each resulting sheet
 warp its samples from the borders of the sheet towards the epipole;

Algorithm 3-2. Occlusion-compatible order for parallel projection source images.

3.7 Pre-Warping Equations for Inside-Looking-Out Cells

Often, texture-mapped impostors are used to replace distant portions of a scene [Maciel95]. Another commonly used technique to speedup the rendering of large geometric databases, and particularly suited for indoor environments, consists of partitioning the space into cells and rendering exclusively the cell containing the current viewpoint. In this case, only objects that fall inside the current cell need to be rendered as polygons, while the outside environment is rendered as texture maps. Image warping

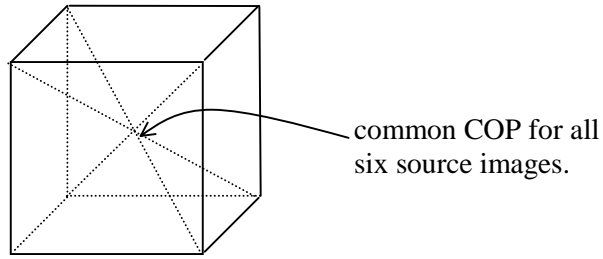


Figure 3-25. Cell with six perspective projection images acquired from its center and covering the whole field of view.

techniques can be used to account for parallax effects of objects outside the cell [Rafferty98]. Alternatively, the faces of the cell can be replaced by images acquired from its center [Aliaga99] (Figure 3-25). As the viewpoint changes, such images are warped and texture mapped onto the faces of a box. Notice that such a situation is exactly the one modeled by the system of Equations (3-7a) and (3-7b) and, therefore, the warping can be performed using the much simpler Equations (3-9a) and (3-9b).

Figure 3-25 shows a cell with six inside looking outside perspective projection images covering the whole field of view and acquired from the center of the cube. From any target viewpoint inside the cube, correct views of the external environment can be obtained by warping the subset of images associated with its visible faces (Figure 3-26). Equations (3-9a) and (3-9b) can be used to pre-warp such images that are then texture-mapped onto the corresponding faces of the cube to produce correct views of the scene. The complete procedure is detailed in Algorithm 3-3.

for each visible face f_i do

pre-warp f_i to its own image plane using Equations (3-29a) and (3-29b)

texture map the resulting image onto the corresponding face of the box.

Algorithm 3-3. Inside looking outside views of a box cell.

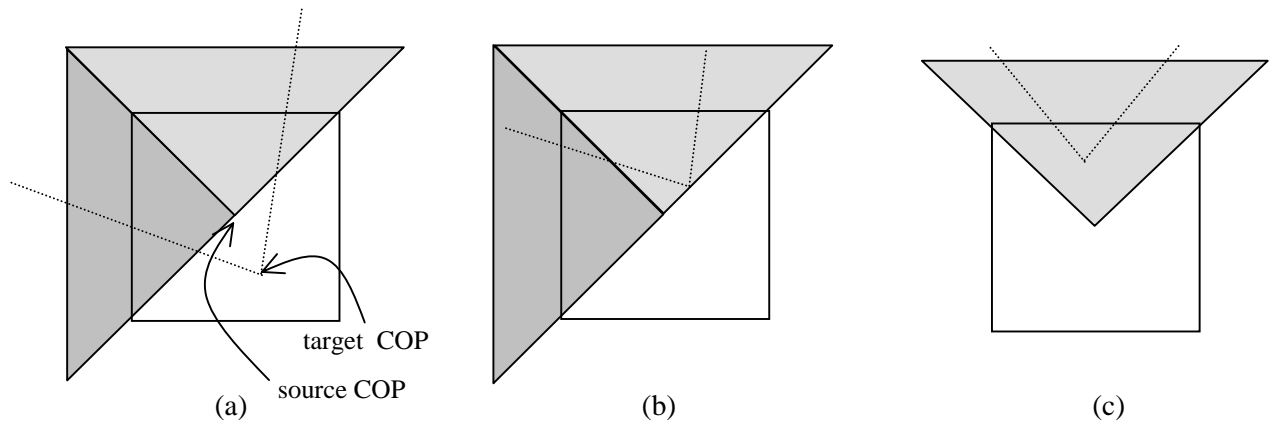


Figure 3-26. 2-D representation of a target view inside a cell. The dotted lines represent the desired field of view. Shaded regions represent view frusta associated with visible faces.

3.8 Discussion

The pre-warp is done in texture space along rows and columns of the source and intermediate images. The information about the cameras' position and orientation, essential for the transformation, are added to the process via coefficients k_i and q_i .

For easy address calculation, mip-map pyramids constrain texture dimensions to powers of two. Simply storing both source and intermediate textures observing such dimensions allows one to take advantage of trilinear filtering capability available in current graphics accelerators. Whereas the color and depth maps shown in Figure 3-3 were resized to look more natural, the actual square maps used to produce some of the



Figure 3-27. Actual 256 by 256-texel color image and depth map used to create illustrations for this chapter.

images in this chapter are shown in Figure 3-27. Vectors \vec{a} and \vec{b} associated with the relief texture are automatically rescaled to compensate for the distortion. In this example, the rescaling led to higher sampling density along the horizontal dimension (see Figure 3-4). The resulting pre-warped textures are also square and some scaling is required to produce correct-looking pictures. This is accomplished automatically by the texture mapping operation.

Whenever $displ(u_s, v_s) = 0$ (or $\delta(u_s, v_s) = 0$) then no transformation is necessary. Figure 3-28 shows multiple views of a relief texture-mapped brick wall. Texels representing bricks have zero displacement, whereas the mortar is at some depth. The images on the left correspond to three views of the wall; the ones on the right are the

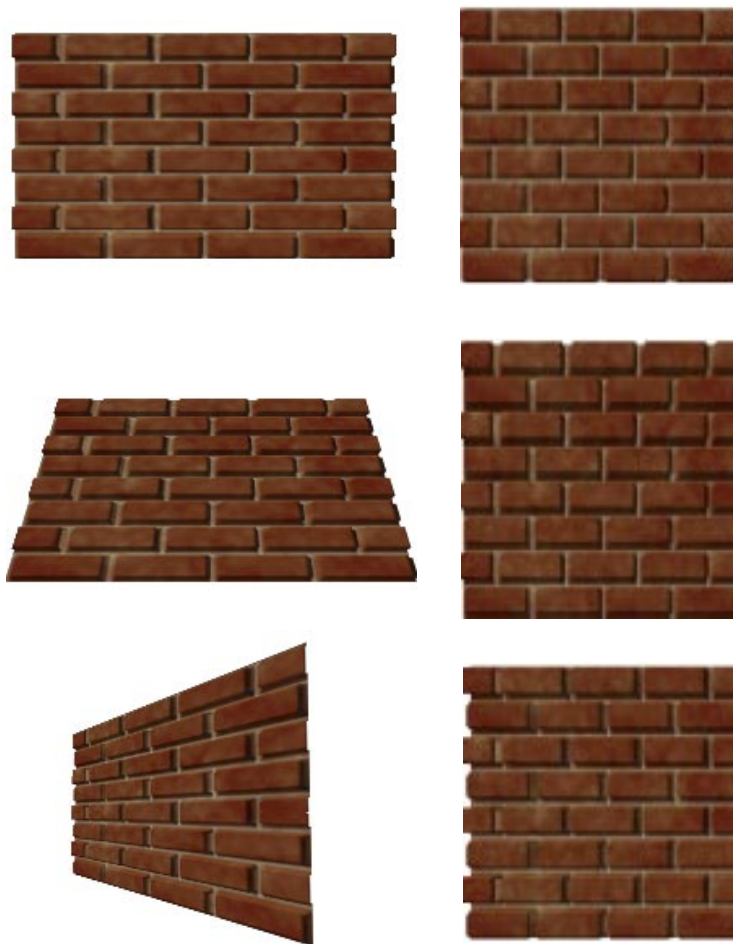


Figure 3-28. Three views of a relief texture-mapped brick wall. The images on the left show polygons texture-mapped with the corresponding pre-warped images shown to the right. Brick texels have zero displacement and, therefore, do not move.

associated pre-warped textures. Notice that although the images on the left present significant visual differences, pixels representing bricks did not move at all in the pre-warped textures on the right. When a large number of pixels have zero displacement, such as in the example of Figure 3-28, speed-ups of over 100% were verified in the current software prototype by just avoiding unnecessary transformations.

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Chapter 4 – Image Resampling from Relief Textures

The process of spatially transforming discrete images, usually referred to as image resampling, is composed of two stages: image reconstruction and a later sampling at the pixel grid of the output image (section 2.4). Chapter 3 described the warping map, *i.e.*, how to determine the coordinates of infinitesimal points in the destination image from points in the source image. Here, we discuss issues regarding reconstruction, antialiasing and the final sampling.

The simplest and most common approaches to reconstruction are splatting [Westover90] and meshing. Splatting requires spreading each input pixel over several output pixels to assure full coverage and proper interpolation. This usually involves splat-shape calculation and blending of overlapping samples. Meshing, on the other hand, requires rasterizing a quadrilateral for each pixel in the $N \times N$ input image. The multiple writes (to possibly incoherent memory positions) of splatting and the setup overhead of rasterizing tiny quadrilaterals make both approaches very expensive.

The one-dimensional structure of the pre-warping equations allows resampling to be implemented as a two-pass process using 1-D operations along rows and columns.

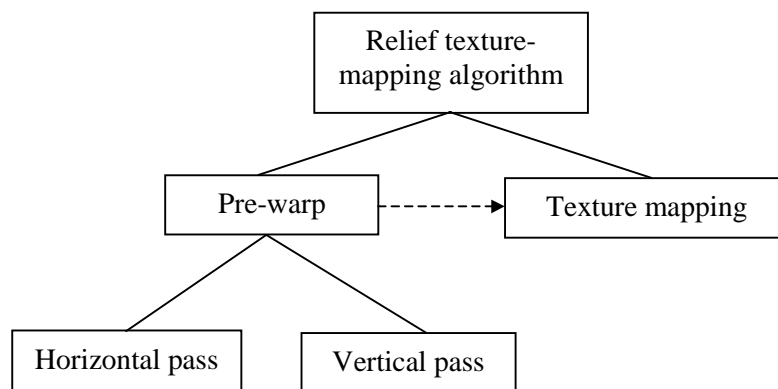


Figure 4-1. Structure of the two-pass 1-D relief texture-mapping algorithm.

Figure 4-1 shows the structure of the two-pass 1-D relief texture-mapping algorithm. The reader should make a clear distinction between the two steps of the algorithm: pre-warping followed by texture mapping, and the two phases used to implement the pre-warp step itself. Such phases consist of a horizontal pass and a vertical pass (Figure 4-1).

The notion of serial resampling is introduced using a simple two-pass strategy. This is followed by a detailed discussion of the limitations of such an approach and ways to overcome them. Some filtering issues and the effects of using quantized displacement values on the quality of the reconstructed images are discussed at the end of the chapter.

4.1 Two-Pass 1-D Resampling

Assume the horizontal pass is completed before beginning the vertical one; either order is acceptable and either may be advantageous, as discussed in section 2.3. Equation (3-13a) shows that the pre-warped coordinate u_i depends only on its input counterpart u_s and on the input texel displacement. Therefore, each row of the input texture can be processed independently, with all its texels going to a corresponding row in the intermediate texture. The elements of each row are processed in occlusion-compatible order by starting with the element furthest from the epipole and working towards the epipole. For simplicity's sake, consider the case when the epipole is to the right and the warp is proceeding left to right. Figure 4-2 shows a pseudocode for warping one texel without antialiasing. Antialiasing is implemented using the method described in section 2.5.1. The *get* and *put* operations in the pseudocode are *reads* and *writes* of the texel at the indicated index (column, in this case) position. The output element index is computed based on the input element index and on the displacement of the texel, according to

```

get  $I_{in}$ ,  $C_{in}$ ,  $D_{in}$ 
 $I_{next} = \text{Equation\_3-13a}(I_{in}, D_{in})$ 
for ( $I_{out} = \text{integer}(I_{prev}+1)$ ;  $I_{out} \leq I_{next}$ ;  $I_{out}++$ )
    linearly interpolate  $C_{out}$  between  $C_{prev}$  and  $C_{in}$ 
    linearly interpolate  $D_{out}$  between  $D_{prev}$  and  $D_{in}$ 
    put  $C_{out}$ ,  $D_{out}$  at  $I_{out}$ ,
 $I_{prev}=I_{next}$ ;  $C_{prev}=C_{in}$ ;  $D_{prev}=D_{in}$ 

```

Figure 4-2. Pseudocode for left-to-right warp and resampling of one texel with coordinate u (or v), index I , color C and displacement D . No antialiasing is treatment is shown for simplicity.

Equation (3-13a). The color and displacement values between the previous and current input texels are linearly interpolated, sampled at each output texel center, and the sampled values are then stored for use by the next pass. After the horizontal warp has processed all rows, the same algorithm (replacing Equation (3-13a) with Equation (3-13b)) is applied to the columns of the resulting image, thus handling the vertical shifts of texels. Figure 4-3 illustrates the entire process for one texel (*B*). Figure 4-3(a) shows two adjacent texels *A* and *B* and their positions after the pre-warp (dashed circles). The first texel of each row is moved to its final column (Figure 4-3(b)) and, as the subsequent texels are warped, color and displacement values are interpolated during rasterization (Figure 4-3(c)). Notice that adjacent texels are usually warped to adjacent positions and the situation shown in Figure 4-3(c) is used to stress the interpolation scheme. This situation may happen, though, if the adjacent samples are at the boundary of a depth discontinuity. Let texel *C* be above to texel *B* after all rows have been warped (Figure 4-3(d)). During the second pass, the interpolated displacement values are used to compute the final row coordinates of all texels. Along columns, each texel is moved to its final row (Figure 4-3(e)) and colors are interpolated (Figure 4-3(f)).

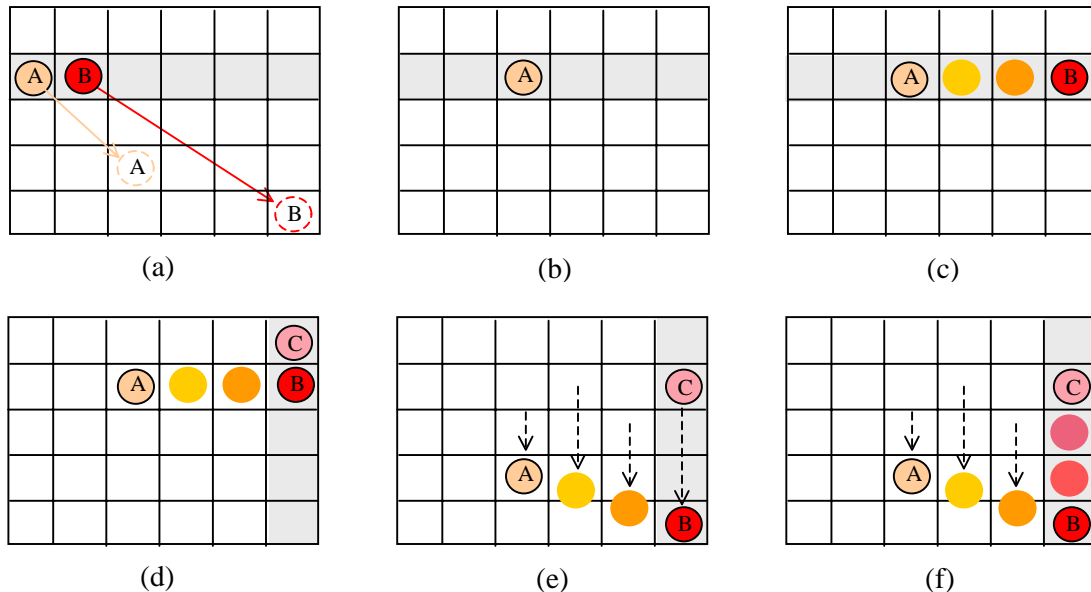


Figure 4-3. Warping of one texel. (a) Source texels *A* and *B* and their final position indicated by dashed circles. (b) The first texel of the current row is moved to its final column. (c) Next texel is moved to its final column and color and displacement values are interpolated during rasterization. (d) After all rows have been warped, texel *C* is adjacent to texel *B*. (e) Vertical pass: row coordinates are computed using the interpolated displacement values and, along each column, texels are moved to their final rows. (f) Color values are interpolated during rasterization.



Figure 4-4. (a) Color image associated with a relief texture (256x256 texels). (b) View of a building generated with the two-pass 1-D warping and resampling algorithm.

I have compared the results of this simple resampling algorithm to a pre-warp that uses hardware-based triangular mesh rasterization because the latter is the most common reconstruction method used in computer graphics. The results are almost indistinguishable in most cases. The differences are limited to interpolated regions across depth discontinuities and, therefore, areas not represented in the source images. Figure 4-4(b) shows a view of a building façade reconstructed using the algorithm. The limitations of this 1-D resampling strategy are discussed next.

4.1.1 Limitations of the straightforward two-pass 1-D warp

The two-pass algorithm presented in Figure 4-2 is prone to some occlusion and color interpolation artifacts. Moreover, straight lines may get curved in regions interpolated across depth discontinuities (Figure 4-5). Whereas the first two kinds of problems require changes in the resampling strategy, the curved distortion has a simpler solution.

4.1.1.1 Self-occlusion errors

The straightforward two-pass implementation may, theoretically, cause information to be lost in the presence of self-occlusions. For instance, consider the example shown in Figure 4-6. Most of the plane has zero displacement, causing the

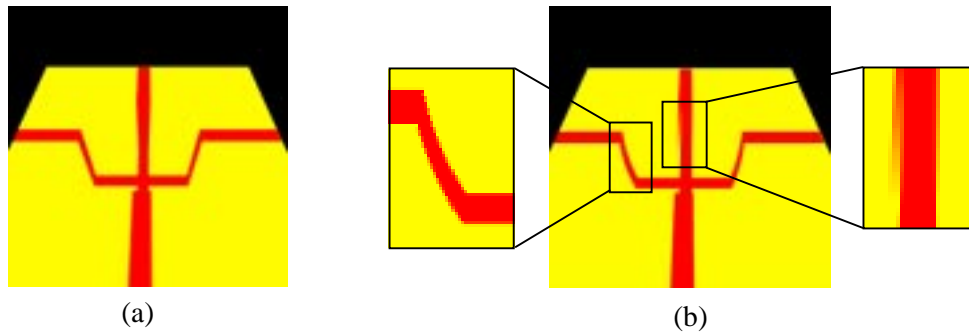


Figure 4-5. Oblique view of a surface. (a) Mesh-based versus (b) two-pass 1-D reconstruction (horizontal pass first). Reconstruction artifacts in (b): straight lines appear curved (left) and wrong color interpolation (right) may happen in interpolated regions across depth discontinuities.

corresponding samples to have the same coordinates in both the input and pre-warped textures. Texels in the diagonal slot are below the plane and should move to the left and down because the epipole is in the lower left corner. The amount of induced shift during the horizontal pass may cause some of these texels to be overwritten by others in the same row. If this happens, the vertical pass has no information about the occluded texels and thus cannot move them down to the final locations. In Figure 4-6, a pair of dotted lines represents a hypothetical path associated with a texel and illustrates the problem. In practice, however, self-occlusions seem not to introduce noticeable artifacts in the pre-warped textures. A general solution for this problem, obtained by interspersing the horizontal and vertical passes, is presented in section 4.2.

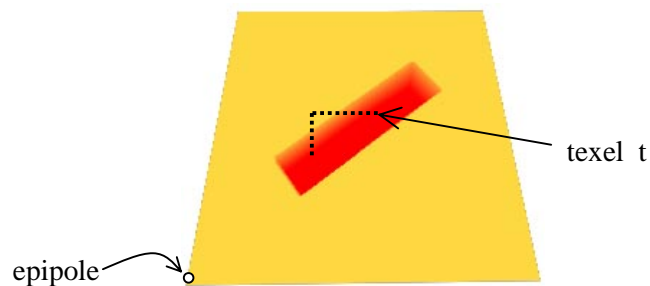


Figure 4-6. Potential self-occlusion. The diagonal area corresponds to a deep region, whereas the rest of the surface has zero displacement. Following a hypothetical path, texel t will move to the left during the horizontal pass and be overwritten by another texel with zero displacement. In this case, no information will be available for the vertical pass.

4.1.1.2 Color interpolation errors

The two-pass 1-D reconstruction scheme of Figure 4-2 can cause color interpolation errors across depth discontinuities as shown in Figure 4-5(b). Such artifacts occur because the pre-warp does not preserve image topology. Thus, consider the four neighboring pixels *A*, *B*, *C* and *D* shown in Figure 4-7. In this example, the vertical-first strategy (Figures 4-7(a), (b) and (c)) produces the same local result as the rasterization of the corresponding warped quadrilateral. The horizontal-first strategy (Figures 4-7(a), (d) and (e)), on the other hand, introduces color artifacts. The dashed lines in Figure 4-7(e) represent improper color interpolation across the boundaries of the warped quadrilateral. Although such boundaries can be preserved with the use of additional data structures, the introduction of this extra complexity would diminish some of the advantages resulting from the simplicity of 1-D warps. In practice, such extra care seems to be unnecessary most of the time. We will return to this point in the discussion section at the end of this chapter.

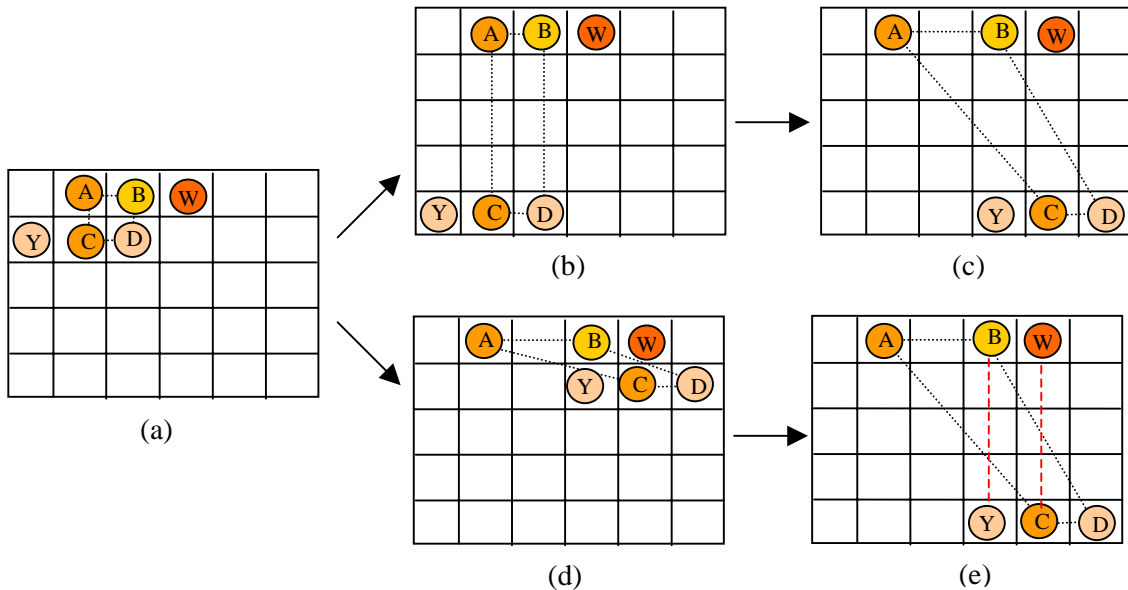


Figure 4-7. Two ways to perform a serial warp: (a) input texels; (b) and (c) results of the steps produced by a vertical-first strategy; (d) and (e) results of the steps produced by a horizontal-first strategy. The dashed lines in (e) represent improper color interpolation across the boundaries of the warped quadrilateral ABCD.

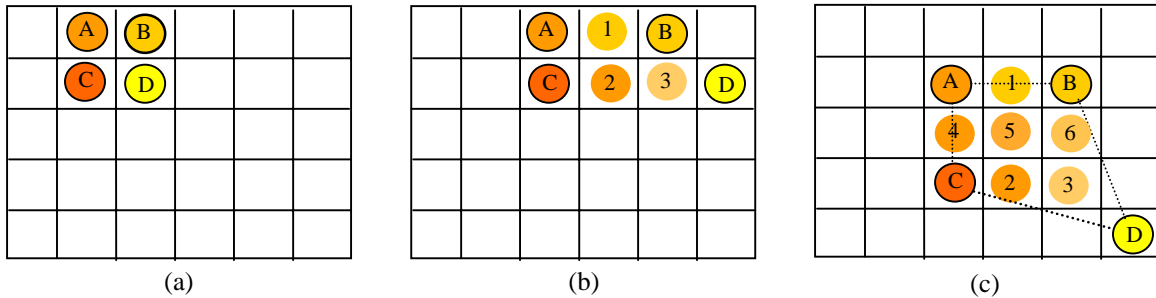


Figure 4-8. Serial warp and reconstruction: (a) source image with four texels highlighted. (b) Intermediate image obtained after the horizontal pass. Interpolated texels are labeled with numbers. (c) Pre-warped image obtained after the vertical pass.

4.1.1.3 Non-linear distortion

Straight lines may get curved in interpolated regions across depth discontinuities under the two-pass 1-D algorithm (Figure 4-5(b)). Although no texture information is in fact available for such expanded areas, the non-linear distortion introduced by the warp does not correspond to an “expected interpolation behavior”. These artifacts may be noticed if the expanded regions are relatively large and the texture contains regular patterns, such as straight lines, across the discontinuities.

The cause of the distortions is the use of a *linear fractional transformation*¹¹ for computing the remaining texel coordinate during the second pass. Thus, for instance, consider a serial warp and reconstruction of the four adjacent texels depicted in Figure 4-8. The horizontal pass takes place first and the epipole is at the lower right corner. Figure 4-8(b) shows the state immediately after the horizontal pass is concluded. According to the pseudocode in Figure 4-2, linearly-interpolated values of color and displacements are sampled and stored at the center of each texel. Although, ideally, performing a serial warp should be equivalent to performing a parallel warp of the same texels, results may differ across regions undergoing local expansion. For example, consider texels *C* and *D* in Figure 4-8. Interpolated displacement values are computed for texels labeled 2 and 3 (Figure 4-8(b)) using Equation (4-1).

$$displ(t) = (1-t)displ_C + (t)displ_D, \quad t \in [0,1] \quad (4-1)$$

¹¹ The ratio of two linear functions.

During the vertical pass, the interpolated displacement values are used to obtain the final row coordinates as

$$v_p(t) = \frac{v_C + k_2 displ(t)}{1 + k_3 displ(t)}$$

$$v_p(t) = \frac{v_C + k_2((1-t)displ_C + (t)displ_D)}{1 + k_3((1-t)displ_C + (t)displ_D)} \quad (4-2)$$

where $displ(t)$ is the interpolated displacement value computed using Equation (4-1), and $v_p(t)$ is the row coordinate of a texel in the resulting pre-warped image (Figure 4-8(c)). The non-linearity of Equation (4-2) introduces the kind of distortion exhibited in the example of Figure 4-5(b). Figure 4-9 shows a plot obtained by evaluating Equation (4-2) with some arbitrary values for v_C , k_2 , k_3 , $displ_C$ and $displ_D$. Only texels C and D have correct coordinates in the pre-warped image.

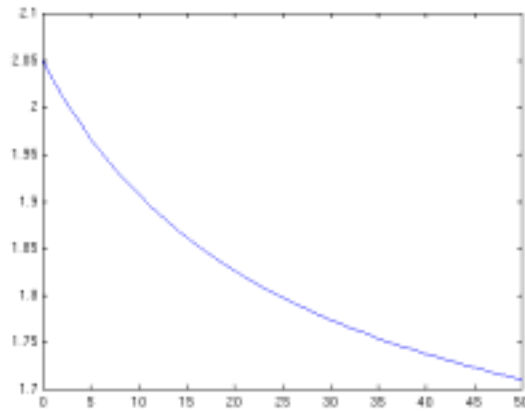


Figure 4-9. Graph of Equation (4-2) using $v_C = v_D = 4$, $k_2 = 3.57$, $k_3 = 2.33$, $displ_C = 1.62$ and $displ_D = 5.5$. $v_p(0) = 2.049$ and $v_p(1) = 1.819$

Figure 4-10 shows the texture and depth map associated with the relief texture used to produce the images in Figure 4-5. The black region in the depth map represents zero displacement, whereas the white square corresponds to unit depth. Figure 4-11 shows the intermediate and pre-warped textures associated with Figure 4-5(b). Notice that the horizontal pass produces a texture with horizontal contraction in the region corresponding to the deep box (Figure 4-11(a)). The amount of contraction can be estimated by the relative change in width of the vertical line. In such an image, all texels



Figure 4-10. Texture (a) and depth map (b) associated with a relief texture of a quadrilateral with a deep box at the center. Black represents zero displacement and white represents deep unit displacement.

are correctly mapped to their final columns. The error is introduced only during the second pass, as a consequence of the non-linear behavior of Equation (4-2). The relatively large dimensions of the expanded region (Figure 4-11(b)) allow the distortion to be noticed.

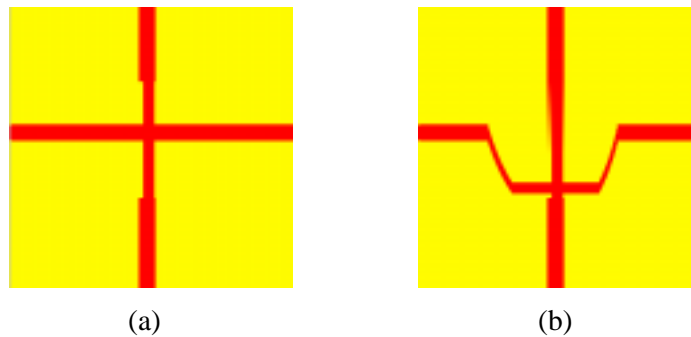


Figure 4-11. Stages of the pre-warped texture for the example shown in Figure 4-5(b). Result of the horizontal pass (a), and pre-warped image (b).

4.1.2 Correcting the non-linear effects of the interpolation

Several approaches can be used to fix the non-linear distortions introduced by the straightforward two-pass 1-D reconstruction. All solutions involve some changes in the computation performed during the first stage of the pre-warp, which will be assumed to be the horizontal pass. A vertical-first strategy is similar, only involving the renaming of a few variables.

Let C and D be the previous and current texels under the horizontal pass, respectively (Figure 4-8). Since serial warps involve three distinct image spaces, namely

source, intermediate and pre-warped image spaces, subscripts will be used to identify them. Subscripts will also be used to identify each texel. Thus, for example, coordinates (u_{sC}, v_{sC}) refer to texel C in the source image space. Likewise, (u_{iD}, v_{iD}) refer to texel D in the intermediate image space, and (u_{pC}, v_{pC}) refer to texel C in the pre-warped image space. The use of texel labels (e.g., C and D) instead of the expressions *previous* and *current* is intended to avoid ambiguity, since *previous* is a relative concept used in both horizontal and vertical passes, but referring to different entities in each case.

4.1.2.1 Asymmetric two-pass algorithm

The most straightforward solution to avoid non-linear distortions is to replace interpolation and storage of displacement values with interpolation and storage of final row coordinates during the first pass. Such row values then become immediately available for the second pass, which only interpolates color attributes. Figure 4-12 shows the pseudocode for a first-pass left-to-right horizontal asymmetric warp of one texel without antialiasing.

There are some advantages in computing both coordinates of pre-warped texels in the first step of the algorithm. Besides eliminating the distortion, the denominator of Equations (3-13a) and (3-13b) is evaluated only once per texel instead of twice. The extra cost of computing the final row coordinate during the first pass is compensated for by not having to compute it during the second pass.

$$v_p(t) = (1-t)v_{pC} + tv_{pD}, \quad t \in [0,1] \quad (4-3)$$

```

get Uin, Vin, Cin, Din
Unext = Equation_3-13a(Uin, Din)
Vnext = Equation_3-13b(Vin, Din)
for (Uout = integer(Uprev+1); Uout ≤ Unext; Uout++)
    linearly interpolate Cout between Cprev and Cin
    linearly interpolate Vout between Vprev and Vin
    put Cout, Vout at Uout
Uprev=Unext; Vprev=Vnext; Cprev=Cin

```

Figure 4-12. Pseudocode for a first-pass left-to-right horizontal asymmetric warp and resampling of one texel with coordinates (U, V) , color C and displacement D . No antialiasing.

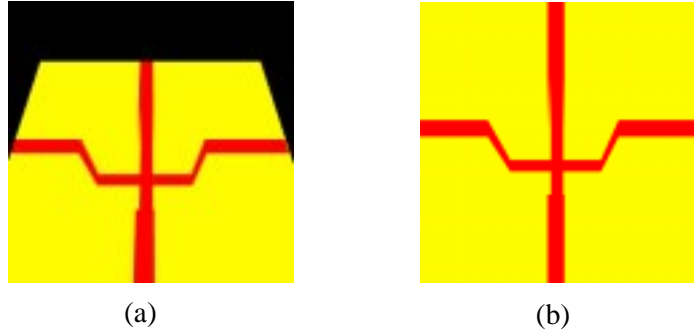


Figure 4-13. Reconstruction created with the two-pass asymmetric algorithm. (a) Same view as in Figure 4-5 with curved lines corrected. (b) Corresponding pre-warped texture.

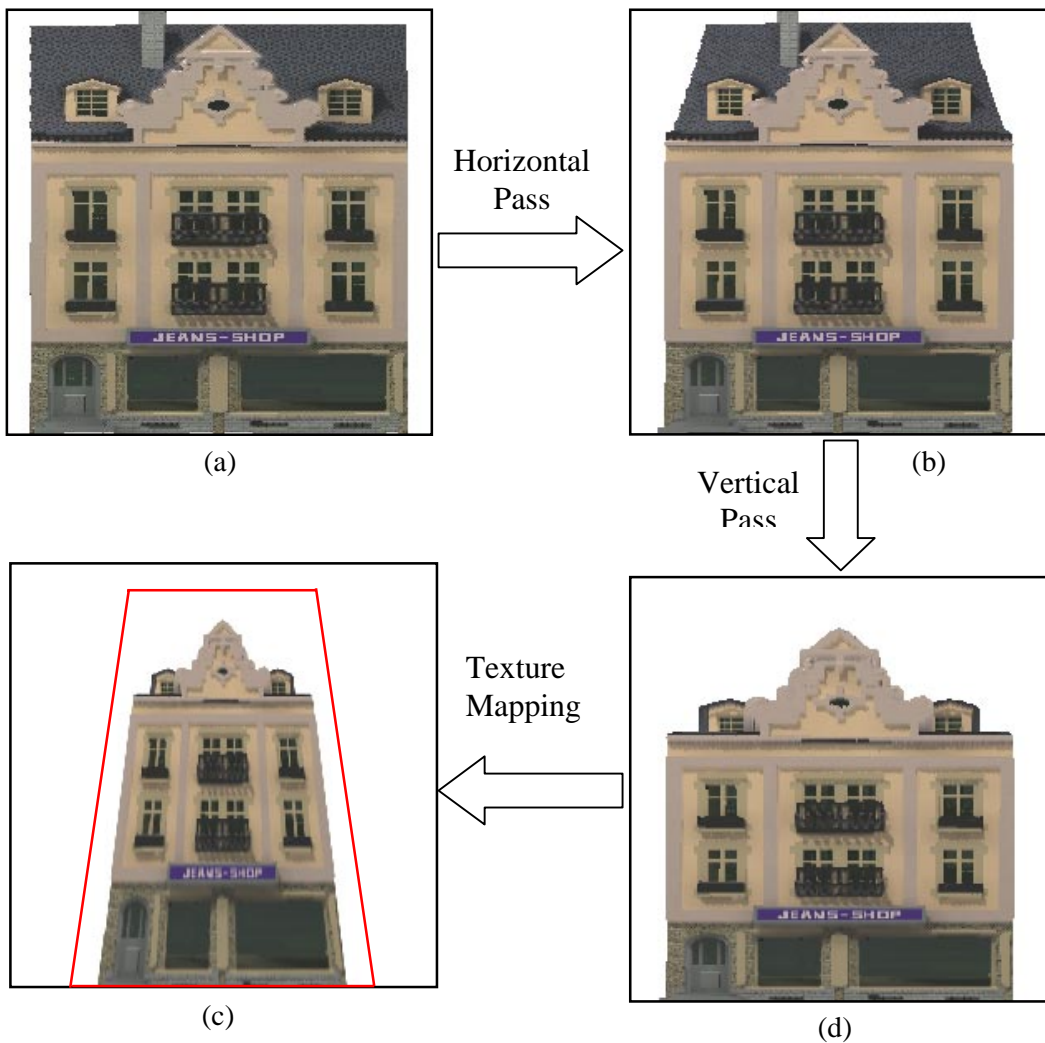


Figure 4-14. Stages of the relief texture-mapping algorithm created with the asymmetric two-pass approach. (a) Source relief texture. (b) Intermediate image produced by the horizontal pass. (c) Pre-warped texture obtained after the vertical pass. (d) Final view, showing the borders of the texture-mapped polygon.

Once the row coordinates of two

adjacent texels have been computed, they are linearly interpolated (Equation (4-3)). Figure 4-13 shows the result obtained with the asymmetric two-pass algorithm for the surface showed in Figure 4-5. The non-linear distortions have been eliminated. This asymmetric two-pass algorithm was used to create most of the illustrations shown in this dissertation and to produce the supporting animations. The use of other reconstruction strategies is acknowledged explicitly. Figure 4-14 illustrates all stages of the relief texture-mapping algorithm.

4.1.2.2 Two-pass algorithm with displacement compensation

Alternatively, several approaches can be used to compute corrected displacement values for the interpolated texels in the first pass so that the desired coordinates are obtained in the second pass. For instance, substituting the pre-warping equation

$v_{pD} = \frac{v_{sD} + k_2 displ_D}{1 + k_3 displ_D}$ into Equation (4-3) produces

$$v_p(t) = (1-t)v_{pC} + t \left(\frac{v_{sD} + k_2 displ_D}{1 + k_3 displ_D} \right) \quad (4-4)$$

Equation (4-4) is clearly linear since k_3 and $displ_D$ are both constants, and it can be rewritten as:

$$v_p(t) = \frac{v_{pC} + (v_{sD} - v_{pC})t + ((1-t)v_{pC}k_3 + tk_2)displ_D}{1 + k_3 displ_D} \quad (4-5)$$

Recall from Equation (4-2) that the interpolated row coordinate of a texel during the vertical pass is given by:

$$v_p(t) = \frac{v_{sC} + k_2 displ(t)}{1 + k_3 displ(t)} \quad (4-6)$$

The corrected displacement can then be obtained by solving Equations (4-5) and (4-6) for $displ(t)$:

$$\frac{v_{pC} + (v_{sD} - v_{pC})t + ((1-t)v_{pC}k_3 + tk_2)displ_D}{1 + k_3 displ_D} = \frac{v_{sC} + k_2 displ(t)}{1 + k_3 displ(t)}$$

$$displ(t) = \frac{v_{sC}(1 + k_3 displ_D) - v_{pC} - t(v_{sD} - v_{pC} + k_2 displ_D) - (1-t)v_{pC}k_3 displ_D}{v_{pC}k_3 - (1 + k_3 displ_D)k_2 + t(v_{sD} - v_{pC} + k_2 displ_D)k_3 + (1-t)v_{pC}k_3^2 displ_D} \quad (4-7)$$

or, more compactly

$$displ(t) = \frac{\beta_1 - t\beta_2 - (1-t)\beta_3}{\gamma_1 + t\gamma_2 + (1-t)\gamma_3} \quad (4-8)$$

where

$$\begin{aligned} \beta_1 &= v_{sC}(1 + k_3 displ_D) - v_{pC} \\ \beta_2 &= v_{sD} - v_{pC} + k_2 displ_D \\ \beta_3 &= v_{pC} k_3 displ_D \\ \gamma_1 &= v_{pC} k_3 - (1 + k_3 displ_D) k_2 \\ \gamma_2 &= v_{sD} - v_{pC} + k_2 displ_D \\ \gamma_3 &= v_{pC} k_3^2 displ_D \end{aligned}$$

Notice that most of the terms in these constants can be reused, such as $k_2 displ_D$, $k_3 displ_D$ and $(1 + k_3 displ_D)$. The corresponding equations for a vertical-first strategy are obtained directly from Equations (4-7) by replacing v with u , and k_2 with k_1 .

The corrected displacement values, $displ(t)$, are computed in the first pass using Equation (4-8) and stored at the interpolated texels. During the second pass, the final row coordinates are obtained using Equation (4-6). Figure 4-15 shows the pseudocode for a first-pass left-to-right horizontal warp with displacement compensation. Figure 4-16 shows the result of a simulation for the same parameters used in Figure 4-9 and superimposed on top of the original graph.

```

get Uin, Vin, Cin, Din
Unext = Equation_3-13a(Uin, Din)
Vnext = Equation_3-13b(Vin, Din)
for (Uout = integer(Uprev+1); Uout ≤ Unext; Uout++)
    linearly interpolate Cout between Cprev and Cin
    Dout = Equation_4-8(Vprev, Vin, Din)
    put Cout, Dout at Uout,
Uprev=Unext; Vprev=Vnext; Cprev=Cin

```

Figure 4-15. Pseudocode for a first-pass left-to-right horizontal warp with displacement compensation.

4.2 Pipelined Resampling

The overwriting of texels during the first pass may cause self-occlusions. Although bottlenecks [Catmull80] are not an issue during the pre-warp step and, in

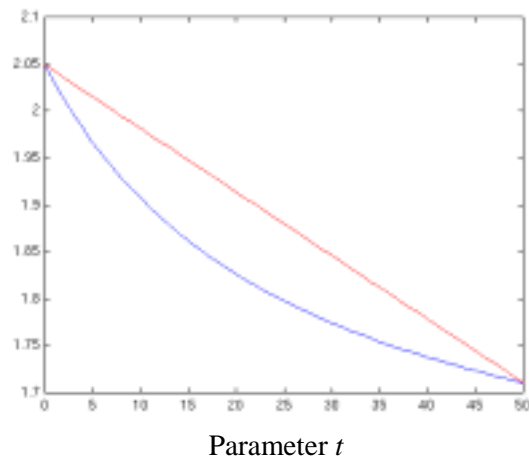


Figure 4-16. Correctly computed values superimposed on top of the graph of Figure 4-9.

practice, self-occlusions seem not to introduce noticeable artifacts in the pre-warped textures, we present a solution that is capable of handling an arbitrary number of foldovers and that does not require depth comparison. It consists of interspersing the horizontal and vertical warps. As before, assume the horizontal pass is completed first and the rows are processed in occlusion-compatible order. As the horizontal warp produces each intermediate texel, this is immediately interpolated into the appropriate column. Since each vertical warp receives and processes its texels in occlusion-compatible order, correct visibility is preserved in the output. Also, because each texel is processed immediately after its generation, no information is overwritten and self-occlusions are avoided. The steps of the algorithm are illustrated in Figure 4-17, where light gray circles represent the texels previously warped to the corresponding columns. Figure 4-18 presents a pseudocode for the pipelined warp and resampling and Figure 4-19 shows an image reconstructed with the algorithm.

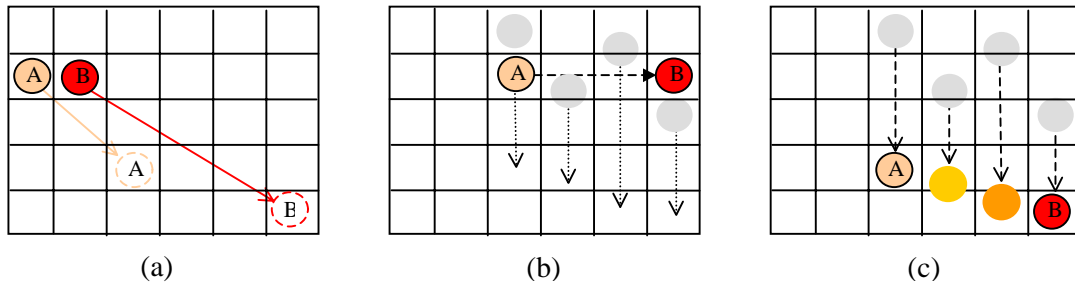


Figure 4-17. Pipelined reconstruction: (a) Two adjacent texels and their final positions after the warping (dashed circles). (b) Horizontal interpolation. Dashed lines represent linear interpolation. (c) The new values are vertically interpolated as soon as they become available.

```

get  $U_{in}, V_{in}, C_{in}, D_{in}$ 
 $U_{next} = \text{Equation\_3-13a}(U_{in}, D_{in})$ 
 $V_{next} = \text{Equation\_3-13b}(V_{in}, D_{in})$ 
for ( $U_{out} = \text{integer}(U_{prev}+1); U_{out} \leq U_{next}; U_{out}++$ )
    linearly interpolate  $C_{out}$  between  $C_{prev}$  and  $C_{in}$ 
    linearly interpolate  $V_{out}$  between  $V_{prev}$  and  $V_{in}$ 
    for ( $V_{final}=\text{integer}(V[U_{out}]+1); V_{final} \leq V_{next}; V_{final}++$ )
        linearly interpolate  $C_{final}$  between  $C[U_{out}]$  and  $C_{out}$ .
        put  $C_{final}$  at ( $U_{out}, V_{final}$ )
 $U_{prev}=U_{next}; V_{prev}=V_{next}; C_{prev}=C_{in}$ 

```

Figure 4-18. Pseudocode for left-to-right top-to-bottom pipelined warp and resampling of one texel with coordinates (U, V), color C and displacement D. $C[U_{out}]$ and $V[U_{out}]$ are the color and row coordinates of the last sample warped to column U_{out} .

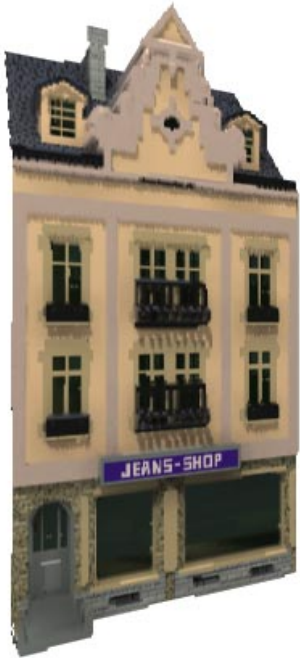


Figure 4-19. Façade of a building warped and resampled using the pipelined algorithm.

4.3 Mesh-Based Resampling

For the cases in which color interpolation errors may become a concern, the 1-D reconstruction can be replaced with a mesh-based approach. This can be done while still taking advantage of an occlusion compatible ordering. Thus, consider the situation depicted in Figure 4-20, in which the epipole splits a relief texture into four sheets. Each sheet is warped and reconstructed as a set of triangle strips as shown for the first row of

4.4 Reconstruction Using Quantized Displacement Values

As defined in Chapter 3, a relief texture is composed of color, depth and camera information. The camera data are specified by three vectors (\vec{a} , \vec{b} and \vec{f}) and a point (\dot{C}) (Figure 3-2). If the displacement associated with each texel is stored as a float, the storage requirement for the depth information matches the amount needed for the color data (32 bits per sample). In this case, transferring a relief texture from main memory to texture memory would require twice as much bandwidth and space as a regular texture. Such requirements can be reduced to essentially the same as for regular textures by storing quantized displacement values in the alpha channel of the relief texture. This strategy also helps to improve cache coherence, since the displacement and color data associated with a texel are always used together.

In a pre-processing step, the minimum and maximum displacement values associated with the relief texture are identified. A quantization step, qs , is then computed as

$$qs = \left(\frac{\text{max} - \text{min}}{254} \right)$$

and the indices for the quantized values are obtained from the actual displacements using the formula

$$qi = \text{int} \left(\frac{\text{displ} - \text{min}}{qs} \right).$$



Figure 4-22. Image associated with a relief texture of the front of a statue. The black pixels representing the background are identified by a reserved depth value and are skipped during the pre-warp.

The resulting indices range from 0 to 254, with the extra value reserved for samples that are not part of the represented object, such as in the black background region shown in Figure 4-22. Such samples are skipped during the pre-warping. The approximate displacement value for a given quantized index qi is computed as

$$displ' = min + qs * qi$$

Notice that such a strategy requires saving the values of min and qs . Since only 255 different values are used, it may be advantageous to initialize lookup tables to avoid repetitive computations as well as the per-texel divisions. Figure 4-23 presents two code fragments for initializing and accessing such tables during the pre-warp. The indices of the quantized values are used to index the tables. Notice that the use of lookup tables allows the pre-warping of one texel to be reduced to essentially two additions and two multiplications (Figure 4-3).

```

for (i=0; i<255; i++) {
    d = min + i * qs;
    r_table[i] = k1 * d;
    s_table[i] = k2 * d;
    t_table[i] = 1.0f / (1.0f + k3 * d);
}

v_t = (v_s + s_table[qi]) * t_table[qi];
u_t = (u_s + r_table[qi]) * t_table[qi];

```

Figure 4-23. Code fragments used to initialize and use lookup tables in the computation of the pre-warped coordinates. The variable qi is the index of the quantized displacement value and is stored in the alpha channel of the relief texture. Pre-warping of one texel is obtained with two additions and two multiplications.

The quantization scheme just described is not optimal. For example, if the actual displacement values are concentrated around just a few values, most indices will then be unused. In such cases, the use of a separate quantization vector could be preferred. Such a vector can be generated, for instance, with a 1-D median cut algorithm [Heckbert82]. In practice, however, the uniform quantization scheme described works very well. The discretization error introduced by depth quantization is combined with the discretization



Figure 4-24. Two views of an object rendered using quantized displacement values. Despite the geometric complexity, they are essentially indistinguishable from the renderings obtained using the actual displacements.

errors of the source and pre-warped images and undergoes the same filtering process. The results are virtually indistinguishable from the ones obtained with the use of the actual displacement values, even for complex shapes. Figure 4-24 shows two views of a statue rendered with the relief texture-mapping algorithm using quantized displacement values. Notice the details on the face and hair.

4.5 Rendering Statistics

For a typical 256x256-texel relief texture (*e.g.*, Figure 4-4(a)) mapped onto a single quadrilateral using the asymmetric two-pass approach, the current software prototype, written in C++, achieves an average frame rate of 9.42 frames per second. Such measurements were performed on a Pentium II PC running at 400MHz with an Intergraph graphics accelerator (Intense 3D RealizM II VX113A-T) with 16 MB of texture memory and 16MB of frame buffer memory. The final view of the surface was displayed on a 512x512-pixel window. The percentage of the rendering time spent with pre-warping and resampling, loading pre-warped textures into texture memory, and the actual texture mapping operation are shown in Table 4-1. Notice that since the pre-warping and resampling operations dominate the rendering time, these results are essentially independent of the dimensions of the output window.

Table 4-1: Percentage of the average rendering time associated with the steps of the relief texture-mapping algorithm (one relief texture mapped onto one quadrilateral).

Pre-warping and resampling	Loading pre-warped textures into texture memory	Actual texture mapping operation	Others
94.10%	2.65%	0.066%	3.18%

The use of quantized displacement values did not significantly reduce the rendering time in the current prototype. For relief textures of dimension 256x256 containing less than 5% background texels, the average speedup was only about 2%. Given that the pre-warping and resampling dominates the rendering time, a hardware implementation of these steps can significantly improve the performance of the entire algorithm. It can also take better advantage of displacement quantization.

4.6 Filtering Composition

A texture undergoes three resamplings during a relief texture-mapping operation: two as part of the pre-warp and a latter 2-D resampling as part of the conventional texture map stage. During the pre-warp, some regions of the image are compressed while others are expanded, requiring the 1-D filters used for reconstruction to change width along the rows and columns adaptively. Thus, these 1-D filters are adaptive asymmetric triangle¹² filters, and the whole pre-warp reconstruction consists of bilinear interpolation using an anisotropic adaptive filter. The reconstruction filter used by the relief texture-mapping algorithm is then obtained by convolving a box filter (used by texture mapping hardware) with the anisotropic filter used by the pre-warp.

It is important to make a distinction between the separability¹³ of the warping and reconstruction filters used by the pre-warp. The first, represented by the pre-warping equations (3-13a) and (3-13b), is clearly separable. The reconstruction filter, on the other

¹² A triangle filter is also known as *tent* filter, *Bartlett* filter, *Chateau* function and *roof* function.

hand, is not separable in general and causes the color interpolation artifacts discussed in section 4.1.1.2.

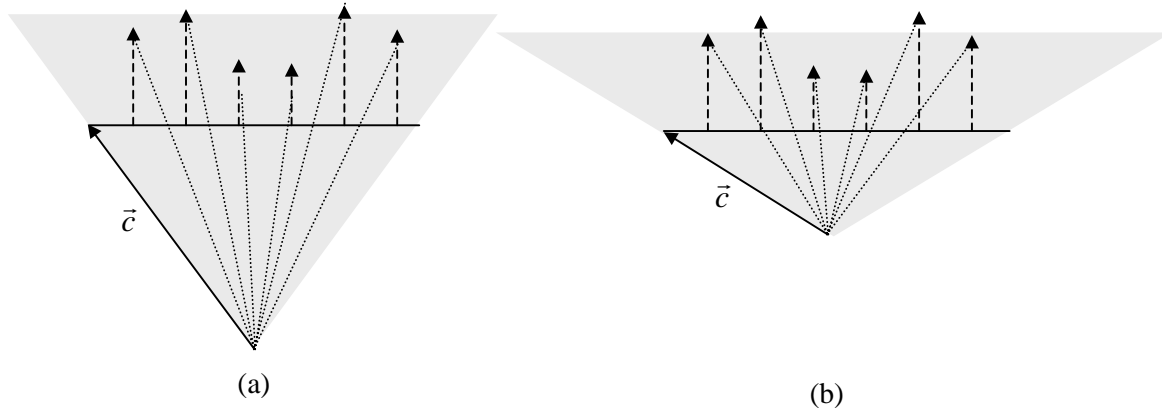


Figure 4-25. The changing field of view effect. The viewer moves with respect to a fixed source image plane causing the field of view of the temporary camera (shaded triangle) to change. The size of the projected area of the surfaces varies inversely with the changes in the field of view.

4.7 The Changing Field of View Effect

The three resampling steps mentioned in section 4.5 produce good antialiased images without introducing excessive blurring. In Section 3-6, in order to simplify the computation of coefficients k_1 , k_2 and k_3 , the image planes of both the source and the temporary¹⁴ target cameras were identified (Figure 3-14). As a result, the field of view of the temporary camera varies with the viewer's position (Figure 4-25). As the viewer moves away from the source image plane, such a field of view decreases and the projected area of the represented surfaces increases (Figure 4-25(a)). Likewise, as the viewer moves towards the source image plane, the field of view increases and the projected area of the represented surfaces decreases (Figure 4-25(b)). The relationship between the final view and the pre-warped texture is illustrated in Figure 4-26. The images shown on the left correspond to the final view of the scene while the images on the right are the output of the pre-warp process. Figures 4-26(a) and (b) correspond to the situation represented in Figure 4-25(a). The pre-warped image is minified as a consequence of the planar perspective distortion, implemented by the texture mapping

¹³ A filter is said to be *separable* if it can be decomposed into simpler functions, each of which is independently applied to distinct dimensions of the input signal.

¹⁴ Used for the purpose of the pre-warp only. See Section 3.5.

operation (Figure 4-26(a)). Figures 4-26(c) and (d) correspond to the situation depicted in Figure 4-25(b). In this case, only a small portion of the pre-warp image, visible in the final view, is magnified by the texture mapping operation (Figures 4-26(c)).

Texture minification is handled appropriately by the texture mapping operation. Unfortunately, the case in which just a small portion of a shrunk pre-warped image is magnified to produce the final view usually introduces unnecessary blur (Figure 4-26(c)).

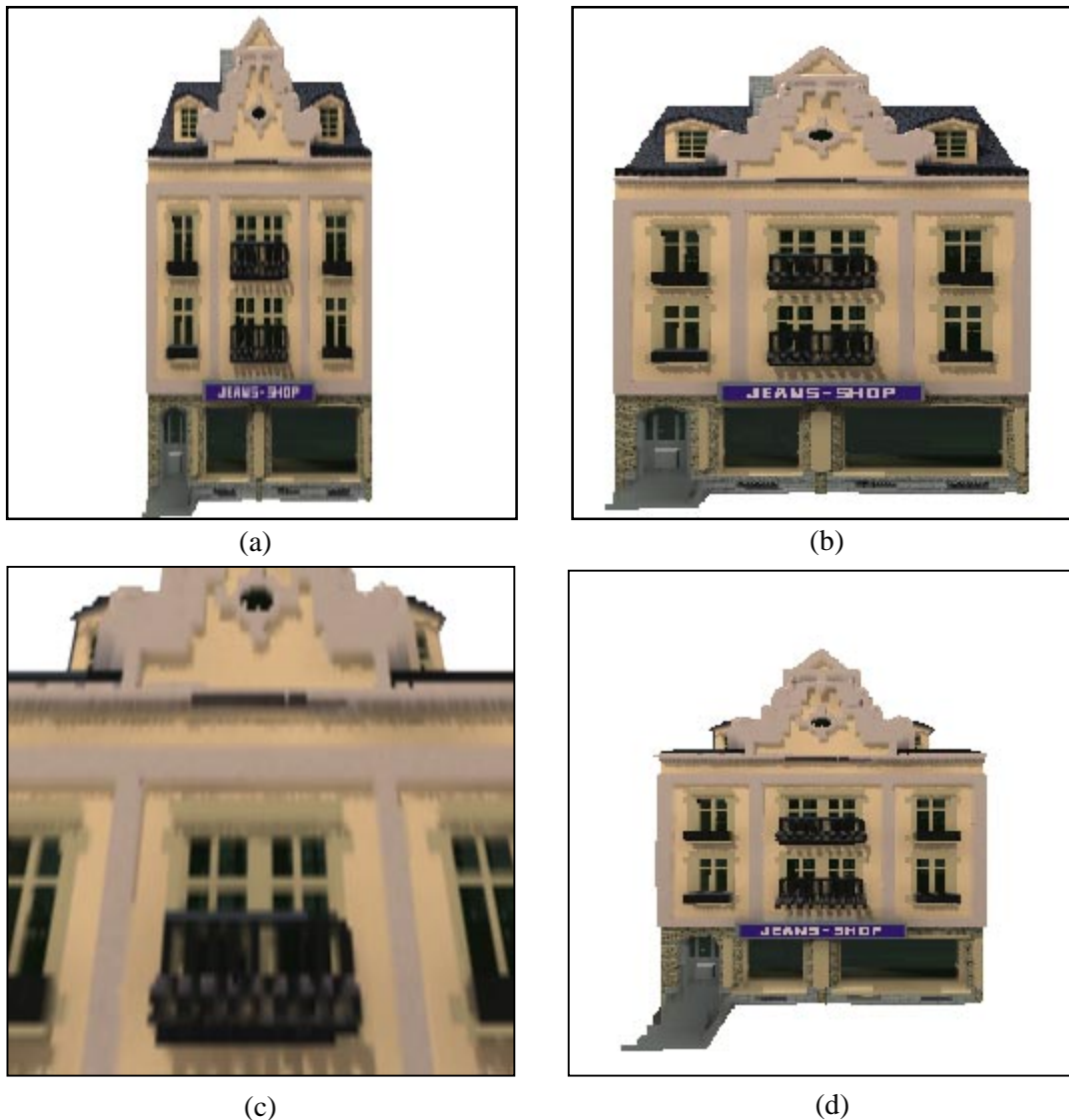


Figure 4-26. Final views (left) and associated pre-warped images (right). The dimensions of the projected image are inversely proportional to the field of view. In (c), the texture mapping operation magnified a small portion of a façade previously shrunk by the pre-warp step, thus introducing unnecessary blur.

This situation can be improved by using the full resolution of the pre-warped image to represent only its visible portion. This is equivalent to clipping the field of view of the temporary target camera against the field of view of the actual target camera and to rescaling the temporary \vec{a} and \vec{b} vectors (Figure 4-27). The implementation of these operations is explained next.

First, the normalized device coordinates (*i.e.*, the coordinates obtained after perspective projection and division by the homogeneous coordinate w) of the vertices of the source image plane are computed. By definition, the visible portion of a scene ranges from $[-1, 1]$ in normalized device coordinates in X and Y dimensions [Foley90]. Figure 4-28 illustrates the situation in 2-D for the X dimension. The new \vec{a} vector is computed as

$$\vec{a}' = \left(\frac{2}{M + m} \right) \vec{a} ,$$

where M and m are the absolute values of the maximum and minimum normalized device coordinates associated with the horizontal limits of the source image plane (Figure 4-28). The new \vec{b} vector is computed similarly using the plane's vertical limits. Notice that the origins of the source image plane and of its visible portion do not coincide in general. Also, vectors \vec{a} and \vec{b} have been rescaled. Both factors need to be compensated for during the pre-warp. This will be explained for the column coordinate. The row coordinate is similar. Thus, let r_a be the ratio between the lengths of \vec{a} and \vec{a}' , and let u_0 be the column coordinate of the origin of the visible portion of the source image plane expressed in the coordinate system of the source image itself. Recall from Equation (3-13a) that

$$u_i = \frac{u_s + k_1 \text{displ}(u_s, v_s)}{1 + k_3 \text{displ}(u_s, v_s)} .$$

Also recall that $k_1 = \frac{\vec{f} \cdot (\vec{b}_t \times \vec{c}_t)}{\vec{a}_s \cdot (\vec{b}_t \times \vec{c}_t)}$, $k_2 = \frac{\vec{f} \cdot (\vec{c}_t \times \vec{a}_t)}{\vec{b}_s \cdot (\vec{c}_t \times \vec{a}_t)}$ and $k_3 = \frac{1}{\vec{c}_t \cdot \vec{f}}$. Vectors \vec{a}_t and \vec{b}_t

have been rescaled, but since they appear in both numerator and denominator of k_2 and

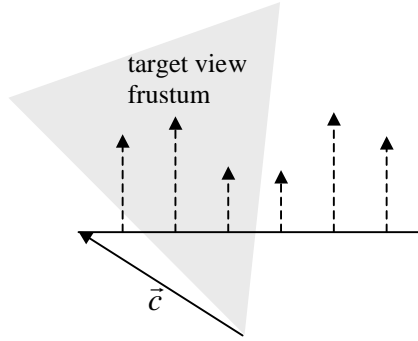


Figure 4-27. Actual target field of view (shaded). The resolution of the pre-warped texture should be used to represent only its visible portion.

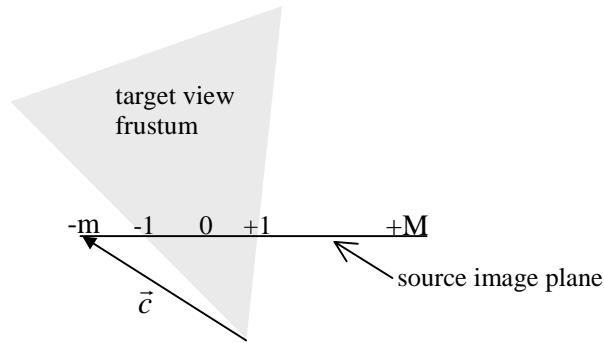


Figure 4-28. Normalized device coordinates of the vertices of the source image plane (in 2-D): $-m$ and $+M$. The visible range varies from -1 to 1 .

k_1 , respectively, the values of these coefficients remain unchanged. The pre-warping equations accounting for the change of origin and for the change in texel dimensions become

$$u'_i = \frac{(u_s - u_0)r_{\bar{a}} + k_1 \text{displ}(u_s, v_s)}{1 + k_3 \text{displ}(u_s, v_s)} \quad (4-9a)$$

$$v'_i = \frac{(v_s - v_0)r_{\bar{b}} + k_2 \text{displ}(u_s, v_s)}{1 + k_3 \text{displ}(u_s, v_s)}. \quad (4-9b)$$

Since the resulting pre-warped texture must be mapped to the visible range $[-1,1]$ (in normalized device coordinates), texture coordinates can be expressed as $t_c = 0.5 + 0.5 * NDC$, where NDC are the normalized device coordinates of the vertices of the source image plane. For the 2-D example shown in Figure 4-28, the texture coordinates associated with the left and right vertices are $t_c = 0.5 - 0.5 * m$ and $t_c = 0.5 + 0.5 * M$, respectively.

4.8 Discussion

Improper color interpolation across depth discontinuities is the major source of artifacts in images produced by two-pass 1-D reconstruction strategies. These problems are associated with high-spatial frequencies in texture patterns whose dimensions are small relatively to the expansion induced by the discontinuity. In practice, depth discontinuities are frequently associated with either smooth color changes or sharp color transitions matching the discontinuities. Both cases are satisfactorily handled by two-pass 1-D reconstruction methods. Figure 4-29(a) shows a texture whose color pattern exactly matches the depth map of Figure 4-10(b). In this case, the results produced by the 1-D approach are similar to the ones obtained with a parallel warp and resampling. Examples of the use of one-dimensional resampling involving complex shapes associated with sharp and smooth color transitions are shown in Figures 4-29 and 4-24, respectively.

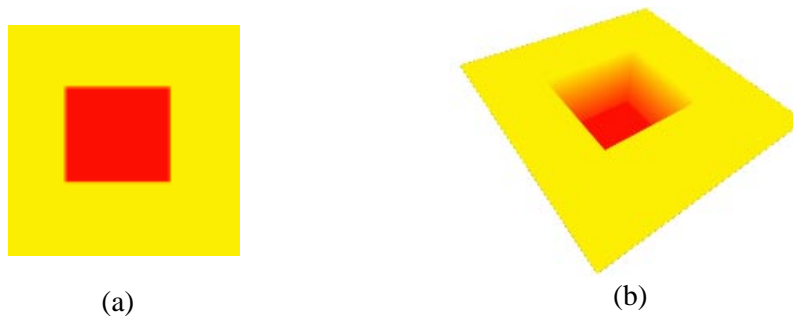


Figure 4-29. Sharp depth discontinuities matched by color change. (a) Source image used in conjunction with the depth map shown in Figure 4-10. (b) Oblique view of the resulting surface rendered with a two-pass 1-D warping and resampling algorithm. The results obtained with both orders (*i.e.*, horizontal and vertical first) are similar to a 2-D resampling.

The large number of texture lookups and the amount of computation required for texture address generation and color filtering make texture mapping the main performance bottleneck in a graphics pipeline [Hakura97]. Whereas the adoption of cache architectures for texture mapping [Hakura97] [Cox98] can significantly reduce the bandwidth requirements between texture memory and the rasterizer, it introduces the need for careful cache management. For instance, the order in which screen pixels are visited may produce arbitrary texture access patterns and, consequently, influence the cache behavior [Hakura97]. In order to reduce the number of cache misses due to texture

orientation on the screen, textures are stored in blocks or tiles [Blinn90] whose sizes should ideally correspond to one cache line. The pre-warp step does not depend on the texture orientation on the screen and can be carried out along one row (column) at a time. A cache three times¹⁵ the size of the side of the input image suffices to reduce all cache misses to cold misses during the pre-warping. A single pre-warping orientation (*e.g.*, horizontal first) can always be used for both passes if the image is appropriately transposed after the first pass. The final texture-map operation can compensate for the transposition through appropriate assignment of texture coordinates.

¹⁵ It needs to store the input row/column, the output row/column, and the interpolated row/column values used to avoid non-linear distortions.

Chapter 5 – Object and Scene Modeling and Rendering

This chapter explains the relief-texture instantiation process and describes how objects and environments can be modeled and rendered using relief texture mapping. By decoupling color and displacement data from camera information, relief textures can be arbitrarily reused. Objects are modeled using sets of six relief textures acquired from the faces of the objects' bounding boxes. The details of the rendering algorithm are presented. The chapter also discusses a way to avoid improper reconstruction of non-connected surfaces, presents a depth-buffer-modulation strategy to produce correct visibility among intersecting relief texture-mapped polygons and discusses the construction and rendering of immersive environments.

5.1 Multiple Instantiations of Relief Textures

Often, a single texture is used to add detail to multiple surfaces. For example, a brick texture might be used for all exterior walls of a building. Relief textures carry depth information on a per texel basis and have an associated parallel projection camera. While the depth data describe the deviation of the sampled points from a reference plane, the camera parameters specify the position, orientation and dimensions of the relief texture in a virtual world. More precisely, point \vec{C} determines the position; vector \vec{f} defines the orientation, and vectors \vec{a} and \vec{b} determine the dimensions (Figure 3-2). Thus, by appropriately changing the camera parameters, a relief texture can be instantiated multiple times in the same way as a conventional texture.

Let P be a polygonal representation of a scene S , and let $P_q \subseteq P$ be a set of planar quadrilaterals (quads, for short) to be texture-mapped with instances of pre-warped relief textures. The use of transparency allows the representation of non-rectangular shapes. Each quad in P_q has a corresponding relief texture R_i , whereas each texture can be

mapped onto an arbitrary number of quads simultaneously. As in conventional texture mapping, a pair (s,t) of texture coordinates is associated with each quad vertex. As the scene model is loaded, for each quad $q_i \in P_q$, camera parameters are computed based on the position, orientation and dimensions of q_i and stored with it for later use:

$$\begin{aligned} \dot{C}_i &= v_{1i}, \\ \vec{a}_i &= (v_{2i} - v_{1i}) / (rs_i * c(i)), \\ \vec{b}_i &= (v_{0i} - v_{1i}) / (rt_i * r(i)) \text{ and} \\ \vec{f}_i &= \text{normalized}(\vec{a}_i \times \vec{b}_i), \end{aligned}$$

where v_{ji} is the j^{th} vertex of q_i , $0 < rs_i \leq 1$ is the range of the texture parameter s ($rs_i = s(v_{2i}) - s(v_{1i})$), $0 < rt_i \leq 1$ is the range of the texture parameter t ($rt_i = t(v_{0i}) - t(v_{1i})$), $c(i)$ and $r(i)$ are, respectively, the number of columns and rows of the associated relief texture. rs_i and rt_i are scaling factors for the cases in which the whole parameter spaces for s and t are not used. Figure 5-1 shows a relief texture mapped onto two quads of different dimensions and orientations. The scaling factor $rs = 0.5$ produces bricks of the same size in the smaller wall, for which the values of parameter s vary from 0 to 0.5.

In general, a distinct pre-warp is required for each texture-mapped quadrilateral. At rendering time, the camera parameters associated with q_i are used in the pre-warping. This has the effect of placing the relief texture at the same position and orientation as q_i

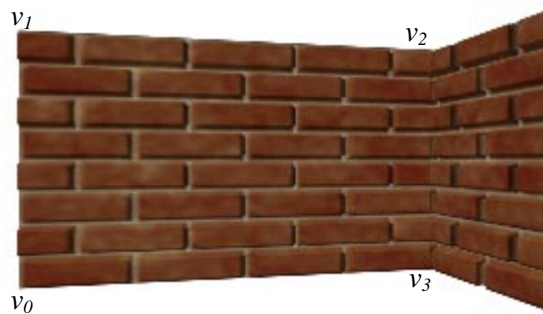


Figure 5-1. A relief texture mapped onto two polygons with different sizes and orientations. The labels indicate the vertex ordering (counter-clockwise, starting at the bottom left vertex) for the case of the larger polygon.

with respect to the desired viewpoint. The texture coordinates associated with the vertices of q_i are used to select the portion of the image to be mapped.

The current software prototype uses the OpenGL ModelView matrix [Woo97] to transform q_i 's camera parameters (\dot{C}_i , \vec{a}_i , \vec{b}_i , and \vec{f}_i) according to the current viewing configuration. The transformed values are then used to instantiate the associated relief texture. The matrix is copied and the translational component of the transformation is saved and then zeroed. The resulting rotation matrix is used to multiply the 4x4 matrix [\dot{C}_i \vec{a}_i \vec{b}_i \vec{f}_i] (with the fourth coordinate of all vectors set to 1). The translational component is then added to the transformed value of \dot{C}_i . This technique is applied to all quadrilaterals associated with relief textures. By performing the pre-warp using camera parameters computed for the quads, relief textures are not bound to any particular position or orientation and, therefore, can be arbitrarily reused.

5.2 Capturing Samples Beyond the Limits of the Source Image Plane

During the pre-warping, samples may be mapped beyond the limits of the source image plane, causing the resulting image to exhibit an incomplete view of the represented

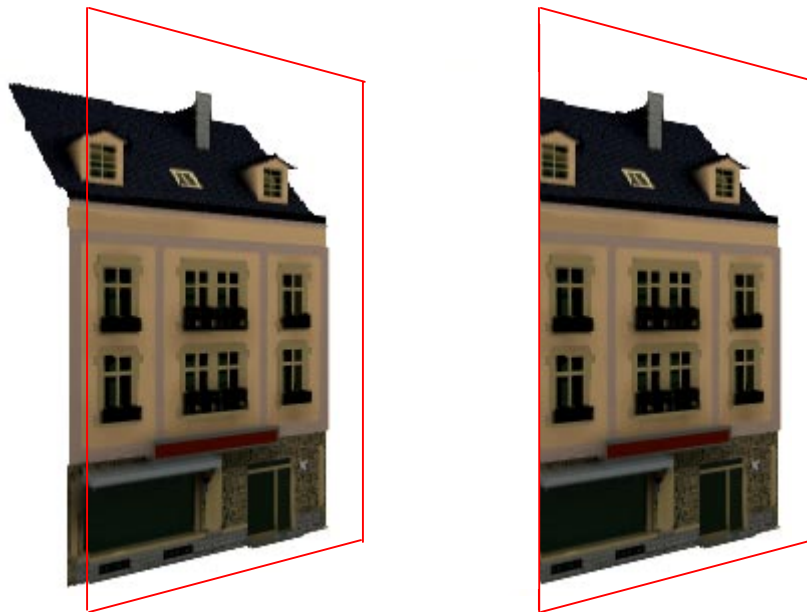


Figure 5-2. Reprojection of a relief texture representing the façade of a building. Some samples map outside of the source image plane (left). An incomplete view of the surface is obtained if outliers are ignored (right).

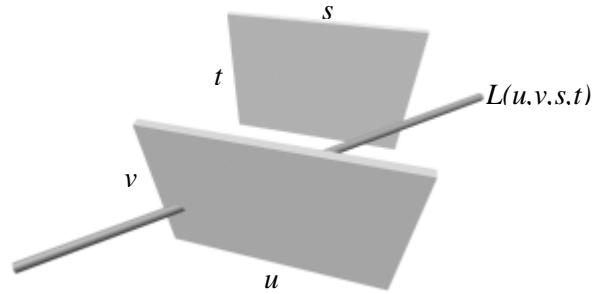


Figure 5-3. Light Field representation consisting of a single light slab. The resampling ray $L(u,v,s,t)$ intersects the s - t plane at an s coordinate whose value is outside of the interval $[0,1]$.

surface (Figure 5-2). The occurrence of such situations depends on the viewing configuration and size of the displacements. This is similar to what happens when a Light Field [Levoy96] consisting of a single light slab is used to represent a surface. In such a case, the values of the s , t , u , and v parameters may map to outside of the interval $[0, 1]$ for some resampling rays (Figure 5-3), causing the final image to miss part of the represented surface (Figure 5-4).

The problem of incomplete views can be overcome if extra polygons are texture-mapped with the outliers. This situation is illustrated in Figure 5-5. The details of the technique will be explained in the next section, in the context of the more general problem of rendering three-dimensional objects from arbitrary viewpoints.



Figure 5-4. Stanford dragon: a single slab $32 \times 32 \times 256 \times 256$ light field [LightPack]. When the values of at least one of the parameters u , v , s or t is outside of the interval $[0,1]$, the resulting image misses part of the represented surface (right).

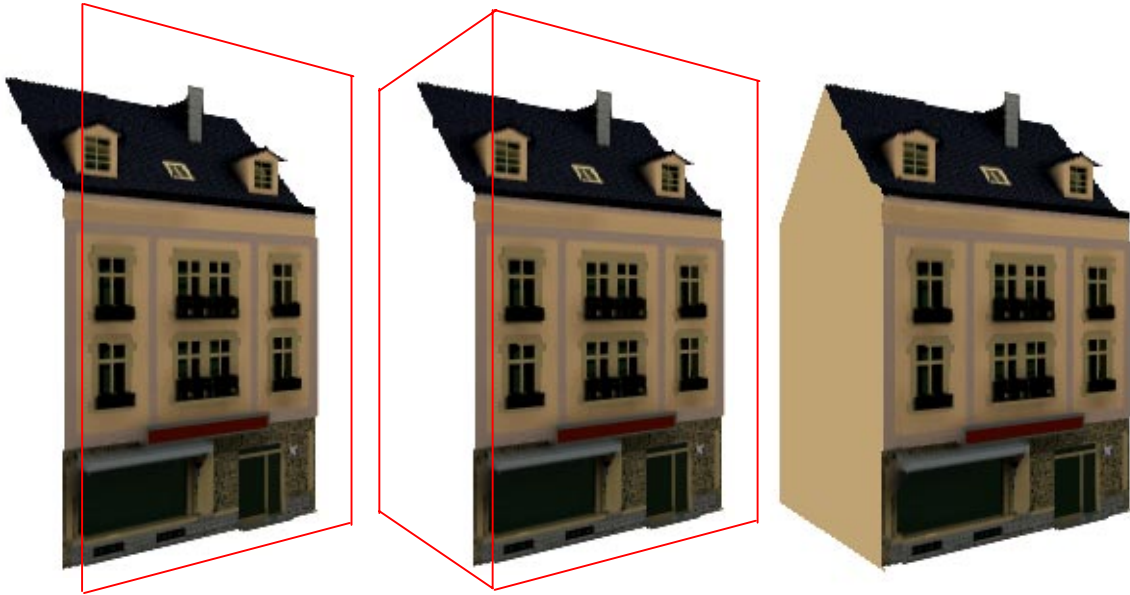


Figure 5-5. An extra quadrilateral is used to texture map outliers (center). Final view rendered with additional sidewall (right).

5.3 Object Representation

Several researchers have used image-based techniques to represent complex object shapes [Gortler96] [Levoy96] [Grossman98] [Schaufler98] [Oliveira99]. Relief textures can also be used to render complex three-dimensional shapes. Figure 5-6 shows a relief-texture representation of a statue originally modeled with 35,280 polygons. It consists of six relief textures associated with the faces of the object's bounding box. New views of an object can be obtained by pre-warping the relief textures used for its representation and mapping the resulting images onto the faces of the box. Since a single

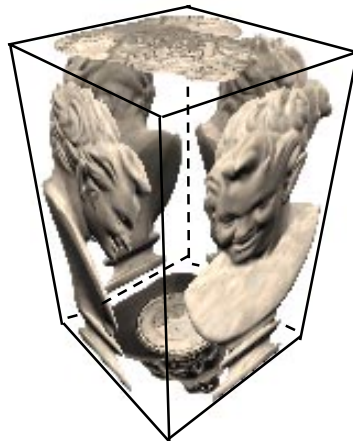


Figure 5-6. Object represented by six relief textures associated with the faces of a bounding box.

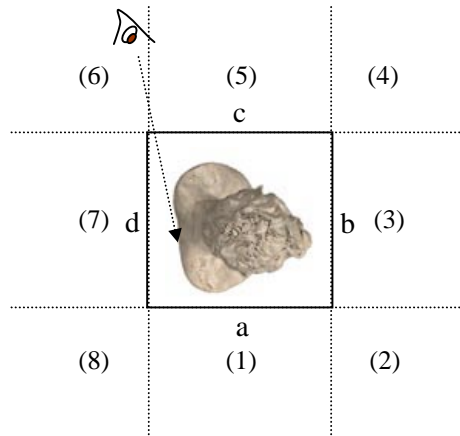


Figure 5-7. Division of the object space into numbered regions. Top view of the object shown in Figure 5-6. Samples from one face can project onto another. Letters identify the faces, and numbers identify regions used to define the faces that should be pre-warped from each region.

viewpoint is used, the images are consistent with each other. However, just warping each relief texture onto its original box face is not enough to produce the desired result. Surfaces sampled only by one face may project onto other faces, depending on the viewpoint (Figure 5-7). If such cases are not appropriately handled, holes will appear.

One solution to this problem is to fill the holes by pre-warping adjacent faces to the desired ones. The perpendicular orientation between adjacent faces allows such mappings to be performed using the same pre-warping equations (Equations (3-13a) and (3-13b)). The concept will be explained in 2-D. Its generalization to 3-D is straightforward. Figure 5-7 shows a division of the object space into numbered regions. If the viewer is in an odd region, the three closest faces are classified as *front*, *left*, and *right* with respect to the current viewpoint. Thus, for instance, if the viewer is in region (1), face *a* is *front*, face *d* is *left*, and face *b* is *right*. First, faces *left* and *right* are pre-warped to the image plane of *front*. Then *front* is pre-warped to its own image plane. If, however,

```

If viewer is in an odd region then
  pre-warp left to front's image plane;
  pre-warp right to front's image plane;
  pre-warp front to its own image plane;
else
  pre-warp left to right's image plane;
  pre-warp right to its own image plane;
  pre-warp right to left's image plane;
  pre-warp left to its own image plane;
endif

```

Figure 5-8. Pseudocode for rendering object representations.

the viewer is an even region, the two closest faces are classified as *left* and *right*. For instance, if the viewer is in region (8), face *d* is *left* and face *a* is *right*. First, *left* is pre-warped to the image plane of *right*, then *right* is pre-warped to its own image plane. Likewise, *right* is pre-warped to the image plane of *left*, and then *left* is pre-warped to its own image plane. Figure 5-8 presents the pseudocode for the algorithm. Notice that this algorithm explicitly defines a set of at most three (in the full 3-D version of the algorithm) polygons that need to be displayed.

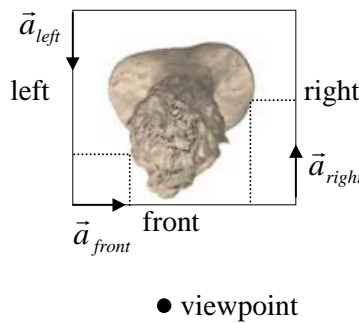


Figure 5-9. Displacement values from *left* and *right* become column values for *front*. Columns from *left* and *right* become displacement for *front*.

The perpendicular orientation between adjacent faces can be exploited to pre-warp a face to its adjacent face as if it were the adjacent face itself. When the viewer is in an odd region, the displacement values associated with *left* and *right* are converted to column indices for *front*, while their column indices can be used as displacement for *front* (Figure 5-9). Thus, *left* and *right* can be pre-warped to *front* as if they were *front*



Figure 5-10. Images associated with four of the six relief textures used to represent the statue.

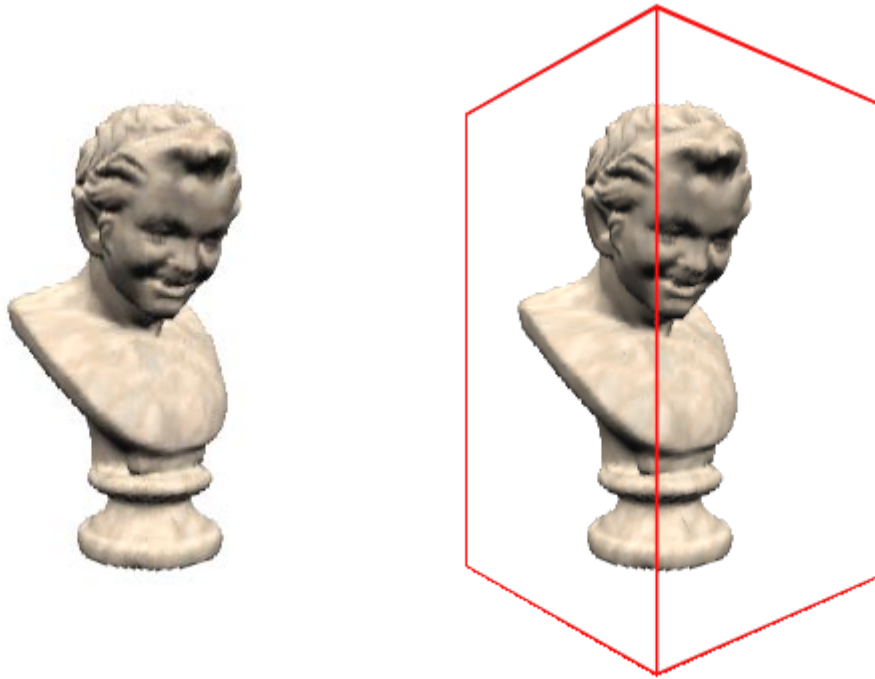


Figure 5-11. Reconstructed view of the statue obtained by texture mapping two quads (left). The lines show the boundaries of the polygons (right).

themselves. The even region is similar.

Figure 5-10 shows the images associated with four relief textures of the statue shown in Figure 5-6. Despite its complex shape, relief texture mapping produces realistic renderings of the object at interactive rates. Figure 5-11 shows the statue rendered as two texture-mapped quadrilaterals (*left* and *right*), whose boundaries are shown to the right. The corresponding pre-warped textures are shown in Figure 5-12 and illustrate the factorization of the planar perspective component from the pre-warp. Such a distortion is latter compensated for during the texture mapping stage of the algorithm. A different view of the statue is shown in Figure 5-13.

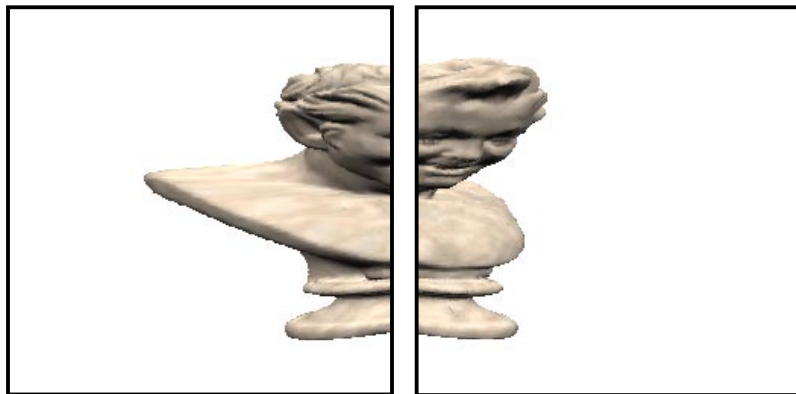


Figure 5-12. Pre-warped textures used to produce the image shown in Figure 5-11.



Figure 5-13. Another view of the statue rendered with relief textures.

5.4 Handling Surface Discontinuities

Treating each relief texture as a continuous surface may not be desirable in some situations. Improper interpolation of samples belonging to originally non-connected surfaces may lead to the occurrence of “skins”. For instance, consider the example shown in Figure 5-14. The ears and legs of the rat were stretched and improperly connected to its body. The assumption about surface continuity can be relaxed if surfaces that would otherwise be rendered as “skins” had been appropriately sampled by adjacent relief textures. In this case, interpolation between texels belonging to non-connected surfaces can be skipped during the pre-warp. A simple way to achieve this is to use depth thresholds to identify and mark such discontinuities during a pre-processing step. Figure 5-16 shows some discontinuities associated with one of the relief textures used to model the rat and responsible for the skins shown in Figure 5-14. Once discontinuities have been detected, skin-free views of the objects can be generated (Figure 5-17). In Figure 5-22, the skins between the façade and the roof of the *jeans shop* building were removed



Figure 5-14. Rendering relief textures as continuous surfaces may lead to the occurrence of “skins” (see ears and legs).

and the use of an extra conventionally texture-mapped polygon seamlessly filled the resulting hole.

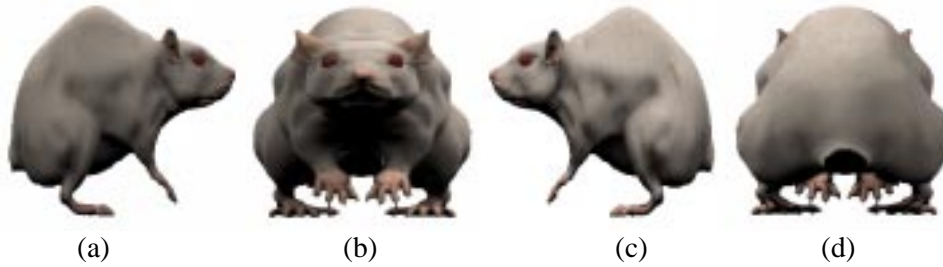


Figure 5-15. Four of the six relief textures used to model the rat of Figure 5-14. Some adjacent texels in (b) and (d) represent samples from non-connected surfaces.



Figure 5-16. Skin detection. Surface discontinuities associated with Figure 5-15(b) identified after some user-specified depth thresholds.



Figure 5-17. Same view as in Figure 5-14, rendered after surface discontinuity identification.

5.5 Correct Occlusions

The relief texture-mapping algorithm, as described so far, does not handle interpenetrating polygons appropriately. Thus, for example, consider intersecting a planar polygon with the bounding box used to represent an object. Since the intersection between two polygons defines a straight line, the resulting occlusion pattern will not

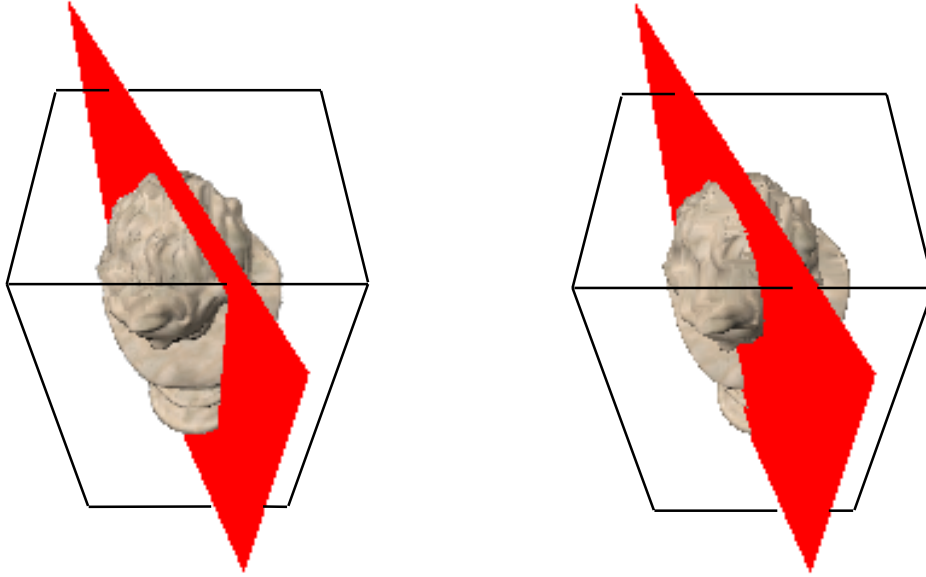


Figure 5-18. Occlusion is checked against the faces of the bounding box, introducing visibility errors characterized by straight lines in regions containing nontransparent texels (left). Correct result obtained by rendering the statue as a mesh of micro-polygons (right). Bounding box shown for comparison.

match the perceived depth of the associated relief textures and will exhibit straight lines in regions containing nontransparent texels. Figure 5-18 (left) illustrates this situation. The correct image, rendered using a mesh of micro-polygons, is shown on the right. In order to avoid this problem, corrected depth values accounting for the perceived off-the-plane displacements must be computed.

Figure 5-19 shows a 2-D view of a relief texture observed from a center of projection \dot{C} . Let \dot{x} be a point in 3-space sampled by texel t , whose coordinates in the source texture are (u_s, v_s) . The Z coordinate of \dot{x} in camera space when observed from \dot{C} is given by

$$Z_{\dot{x}} = c_1 + u_s c_2 + v_s c_3 + \text{displ}(u_s, v_s) c_4 \quad (5-1)$$

where $c_1 = \vec{c} \cdot \vec{n}$, $c_2 = \vec{a}_s \cdot \vec{n}$, $c_3 = \vec{b}_s \cdot \vec{n}$ and $c_4 = \vec{f}_s \cdot \vec{n}$ are constants for a given viewing configuration, \vec{n} is the unit vector normal to the image plane of the target camera, $\vec{c} = \dot{C}_s - \dot{C}$, and \dot{C}_s , \vec{a}_s , \vec{b}_s and \vec{f}_s are the parallel-projection camera parameters associated with the relief texture. $Z_{\dot{x}}$ can be interpolated along rows and columns in the same way as described for color in Chapter 4. Also, let (u_j, v_j) be the coordinates of texel t' obtained after pre-warping t . Notice that the perceived depth at t' is $Z_{\dot{x}}$ (Figure 5-19). Alternatively, one can compute and interpolate only the difference Δz between the

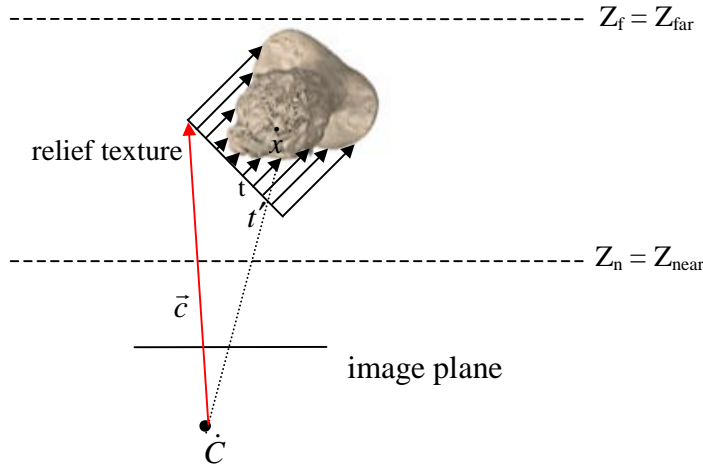


Figure 5-19. Point \hat{x} , originally associated with texel t , projects into texel t' after the pre-warping. Z_n and Z_f are the near and far clipping planes, respectively.

perceived depth and the actual depth at t' , which can be encoded using a smaller number of bits. Since t' is on the source image plane, its Z coordinate in the target camera space can be expressed as

$$Z_{t'} = c_1 + u_j c_2 + v_j c_3.$$

During the pre-warp, Δz values can be linearly interpolated along rows and columns. The interpolated values can be used to compute the amount by which the depth buffer must be changed to produce correct visibility.

The normalized Z values in device or screen coordinates for point \hat{x} is computed as [Blinn98]:

$$Z_{\hat{x} \text{ screen}} = \left(\frac{Z_f}{Z_f - Z_n} \right) \left(\frac{Z_{\hat{x}} - Z_n}{Z_{\hat{x}}} \right)$$

$$Z_{\hat{x} \text{ screen}} = k \left(\frac{Z_{\hat{x}} - Z_n}{Z_{\hat{x}}} \right), \quad (5-2)$$

where Z_n and Z_f are the near and far clipping planes, respectively, specified in the target camera's coordinate system (Figure 5-19). The amount of z-buffer correction required for texel t' is then given by

$$\Delta z_{\text{screen}} = Z_{\hat{x} \text{ screen}} - Z_{t' \text{ screen}}$$

$$\Delta z_{screen} = kZ_n \left(\frac{Z_x - Z_{t'}}{Z_x Z_{t'}} \right)$$

$$\Delta z_{screen} = kZ_n \left(\frac{\Delta z}{(Z_{t'} + \Delta z)Z_{t'}} \right). \quad (5-3)$$

Notice that all values required to compute Δz_{screen} (*i.e.*, Z_x , $Z_{t'}$, Z_n and Z_f) are available during the pre-warp.

In order to simplify depth comparisons, z-buffer hardware usually encodes the normalized Z values using integers computed as the reciprocal of Equation (5-2). Equation (5-2) maps the visible interval $[Z_n, Z_f]$ to $[0,1]$ and, therefore, bigger integer values represent depth closer to the near clipping plane, where handling visibility appropriately is critical. As a result, the fewer bits available for integer representation, the lower the z-buffer resolution, especially close to the near plane. While a 12-bit integer representation of Δz_{screen} produces results that are virtually indistinguishable from a 32-bit representation in most cases, the use of an 8-bit encoding only produces correct occlusions for a very limited distance range (Figure 5-20). This is quite unfortunate, since, otherwise, such a representation could be stored in the alpha channel of the pre-warped texture, not requiring extra bandwidth between texture memory and the rasterizer.



Figure 5-20. Relief textures rendered with z-correction using 8 bits to represent Δz_{screen} . The image to the left is virtually indistinguishable from its 32-bit counterpart. Occlusion errors start to happen as the screen-space delta values fall outside of the range appropriately covered by the available bits (right).



Figure 5-21. Relief texture rendered with z-correction using 8-bit quantized Δz values.

An efficient 8-bit encoding solution, whose results are comparable to the rendering of the actual geometric model, can still be obtained with the quantization of Δz . In this approach, one identifies the maximum and minimum Δz values after interpolation and uses the same strategy described in Chapter 4. First, a quantization step is computed as:

$$qs = \left(\frac{\max - \min}{254} \right),$$

and the indices for the quantized values are obtained from Δz using the formula

$$qi = \text{int} \left(\frac{\Delta z - \min}{qs} \right).$$

Notice that, in this case, a single index value has been reserved for transparency, thus constraining the surface to be either completely transparent or fully opaque. Extra quantization indices could be traded for extra transparency levels. The computation of the approximate delta value for a given index qi is given by

$$\Delta z' = \min + qs * qi. \quad (5-4)$$

Figure 5-21 shows the statue rendered with an interpenetrating polygon seen from different distances. In this example, Δz was interpolated and the resulting values were quantized before being used to modulate the depth buffer. The results are virtually indistinguishable from the rendering of the object as a mesh of micro-polygons.



Figure 5-22. Scene rendered using a combination of relief texture mapping and conventional techniques. The façades and the car are relief texture-mapped objects.

5.6 Scene Modeling

Relief texture mapping naturally integrates itself with polygonal rendering techniques. Figure 5-22 shows a reconstructed view of a town originally modeled using 3-D Studio MAX[®]. The façades of the buildings and some other objects such as the car present in the original model were deleted and the resulting scene was exported as a set of texture-mapped polygons. Relief texture representations of the removed objects were combined with the exported geometric model to render the scene using OpenGL (Figure 5-22).

Virtual immersive environments play an important role in graphics applications and relief texture mapping provides a natural way for modeling and rendering such environments. The ability to create relief textures from data acquired from the real world leads to the creation of realistic experiences. Thus, for instance, consider a partial model of Sitterson Hall's reading room acquired using a digital camera and a laser rangefinder [Nyland99] [McAllister99] (Figure 5-23 (left)). A relief texture representation of the reading room was obtained by reprojecting the original surfaces onto planes positioned

parallel to the walls. Such planes are indicated by dashed lines in Figure 5-23 (right). The resulting textures are shown in Figure 5-24. Black regions represent samples not available in the original data set or clipped during the construction of the relief textures.

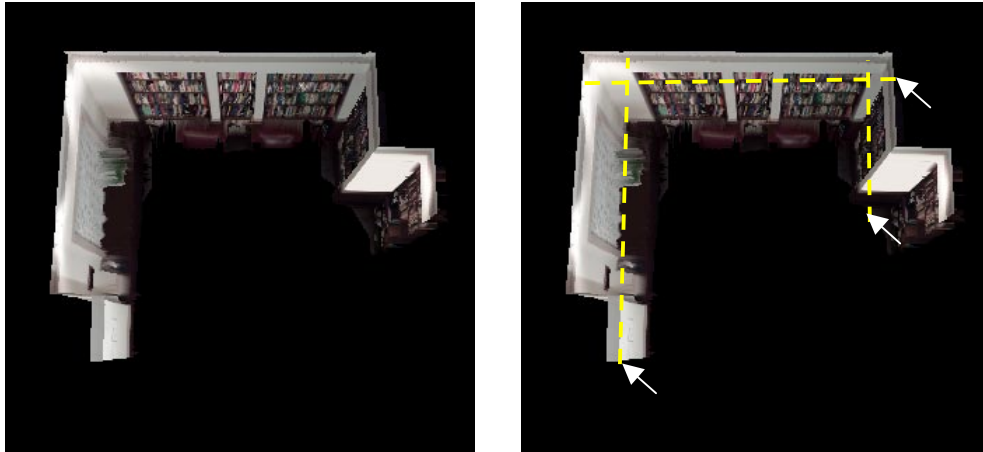


Figure 5-23. Sitterson Hall’s reading room (partial model seen from above). Spatial relationship among the walls (left). Dashed lines represent the planes used for relief texture acquisition (right).

The modeling of an environment is obtained by simply instantiating the relief textures at the same relative positions used for acquisition (Figure 5-25). The associated depth information guarantees the correct registration among the various textured surfaces. For the example of the reading room, two extra polygons, represented in Figure 5-25 by dashed lines, were used to capture outliers. Figure 5-26 shows two views of the reading room rendered using relief texture mapping. In this example, an extra polygon was used to represent the ceiling. Notice the parallax effect that allows the plaque to become visible in the image to the right.



Figure 5-24. Relief textures used in the reading room representation. Black regions correspond to clipped or unavailable samples in the original data set.

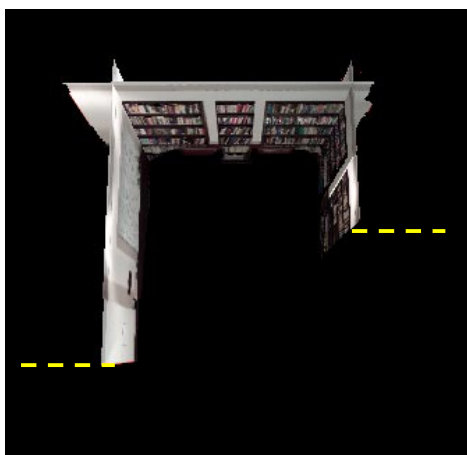


Figure 5-25. Modeling of an immersive environment. The relief textures are positioned in the same relative positions used for acquisition (see Figure 5-23(right)). Dashed lines represent extra polygons used to capture outliers.

The basic strategy of using planes parallel to walls can be generalized to handle more complex environments. First, the geometric model is examined, searching for flat areas that could be represented using conventionally texture-mapped polygons. A geometry simplification algorithm is then applied to the remaining portions of the model, producing a low-count polygonal representation. A least-square algorithm can be used to produce best-fit planes for the simplified model. Such planes are then used for the construction of relief textures. User intervention is required to instantiate extra polygons responsible for handling outliers and to improve the least-squares solution.



Figure 5-26. Room rendered using relief texture mapping. Notice the parallax effect that allows the plaque to become visible in the image to the right.

5.7 Discussion

The mapping of displacement values into column indices for perpendicular faces introduces a truncation error whose magnitude is bounded by 0.5 texel^{16} . This may, theoretically, cause minor misregistration problems for isolated pixels along the edge shared by two perpendicular polygons. The effect of such a truncation is further minimized by the texture's spatial coherence, by the existence of other kinds of discretizations inherent to digital images and by filtering, as discussed in Chapter 4. Truncation errors seem not to be a concern for the case of relief texture-mapped objects, since one relief texture containing actual displacement values is always warped to each of the visible faces of the object's bounding box. The occurrence of skins may, however, cause color discontinuities along shared edges in object representations. Although subtle, such effects can be distracting in animated sequences due to the human's visual sensitivity to them. Once skins have been satisfactorily removed, these effects disappear.

Shaded-relief maps have been used in cartography for many years to convey the illusion of three-dimensionality [Batson75]. More recent, terrain rendering has become an important component of flight simulators, games and topographic applications, with real-time visualization of large-scale surfaces attracting the attention of several researchers [Hoppe98] [Pajarola98]. Although terrain data is usually presented as irregular grids matching the high-scale details of surfaces, relief texture representation for such data sets can be easily constructed. Figure 5-27 shows a view of Northern Vancouver, British Columbia, rendered using relief texture mapping. Such a view corresponds to an area of approximately $26,000 \text{ km}^2$ and was created by patching 25 256×256 -texel relief textures [Hillesland99]. Hillesland used digital orthophotos¹⁷ covering Vancouver and the Fraser Valley in British Columbia [Triathlon95]. Since the elevation data was scattered, a 2-D Delaunay triangulation was computed and the height associated with each texel was interpolated [Hillesland99]. Registration between the color images and the elevation data was performed using Universal Transverse Mercator (UTM) coordinates available for

¹⁶ Measured in the texture space associated with the perpendicular face.

¹⁷ Image maps produced by removing distortions inherent in aerial photography [Triathlon95].



Figure 5-27. Northern Vancouver rendered using a mosaic of 5 by 5 relief textures created from aerial photographs and elevation data [Hillesland99].

both data sets. A close-up of one of the relief textures used to create Figure 5-27, illustrating the silhouettes of some mountains, is shown in Figure 5-28.

An interesting property of relief texture mapping is its support for dynamic changes in surface shape. Surface attributes are represented using a linear array and their values can be easily changed over time. This suggests the use of relief textures to animate surfaces undergoing changes of both shape and color. The UNC nanoManipulator [Taylor94] is a virtual-environment interface to a scanned-probe microscope that allows chemists, biologist and physicists to explore the surface of a material at nanometer scale. The height values output by the microscope are sent through a network to a 3-D graphics workstation for visualization. Researchers from the nanoManipulator project have suggested, and are exploring, visualization strategies based on dynamic relief textures, which are updated as new data become available from the microscope. Figure 5-29 shows



Figure 5-28. Close-up on one of the relief textures used to create Figure 5-27. Notice the silhouettes of the mountains.

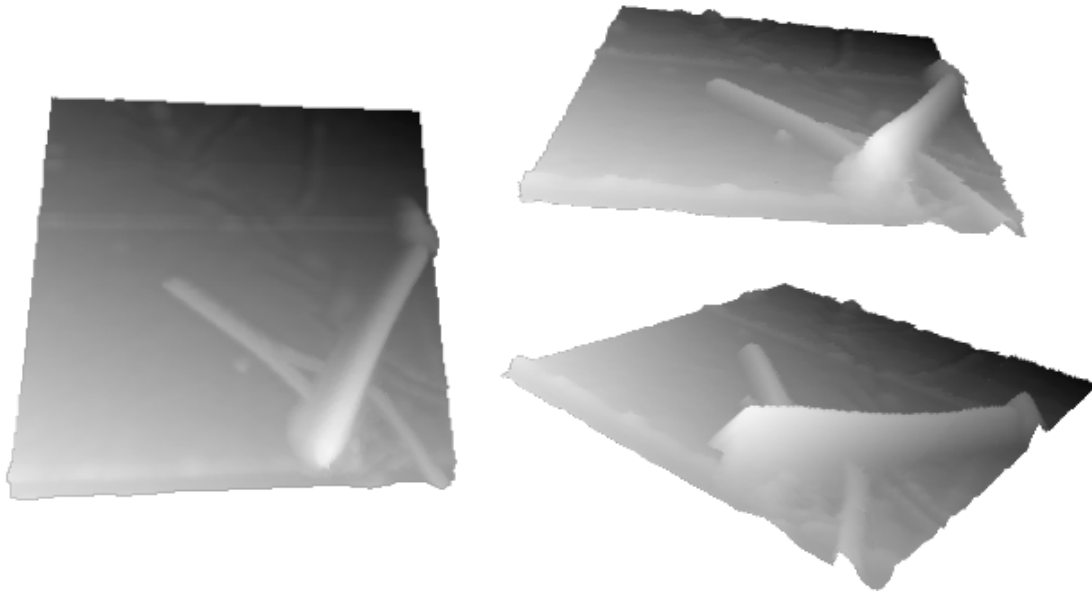


Figure 5-29. Three views of a surface scanned with UNC nanoManipulator scanned-probe microscope and rendered in gray scale using relief texture mapping.

three views of a surface scanned with the probe microscope and rendered using relief texture mapping.

Yang has explored a combination of post-rendering warp [Mark99] and relief texture mapping to produce remote rendering of live action [Yang99]. In this approach, a server acquires and sends color and compressed depth data to remote locations. The scene is modeled as a set of relief textures whose updates are conditioned to changes in the original scene. Figure 5-30 illustrates the idea using the DECface model [Waters94]. The

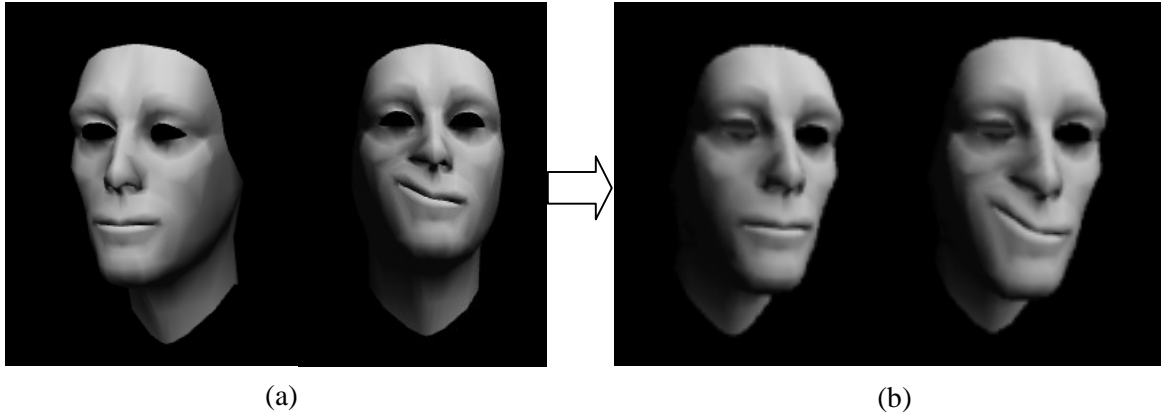


Figure 5-30. Relief texture created locally (a) and sent to remote sites for visualization from arbitrary viewpoints (b).

images on the left are views of a deformable polygonal model at the server site while the images on the right correspond to relief texture-mapping reconstructions from new viewpoints at the client side. As the facial expression changes (Figure 5-30(a)), updated data are sent to clients (Figure 5-30(b)).

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Chapter 6 – Multiresolution, Inverse Pre-Warping, Clipping and Shading

Image pyramids have long been used in computer graphics to produce antialiased texture mapped images at fixed per-pixel cost [Williams83]. This chapter discusses the construction and use of relief texture pyramids for representing levels of detail and for antialiasing. Representing each texture using fixed resolution implies a constant amount of work during the pre-warp, independently of the number of pixels it covers on the screen. Relief texture pyramids can be used to keep the warping cost proportional to its contribution to the final image and to reduce the effects of aliasing. The chapter also describes an inverse pre-warping strategy for relief textures and presents an efficient incremental algorithm for performing clipping in 1-D. It discusses the use of normal maps for shading relief textures and the use of the shadow map algorithm [Williams78] to cast shadows both onto and from relief-texture mapped representations.



Figure 6-1. Relief texture pyramid showing five levels for the front face of the statue. From left to right the dimensions of the textures are respectively 256x256, 128x128, 64x64, 32x32 and 16x16 texels.

6.1 Relief Texture Pyramids

A relief texture pyramid is a set $P = \{T_{2^{m-l}} : l \leq m; l, m \in Z^+\}$, where $T_{2^{m-l}}$ is a relief texture with dimensions $2^{m-l} \times 2^{m-l}$ associated with level l of the pyramid and m is its maximum level (Figure 6-1). $T_{2^{m-l}}$ is obtained by averaging color and depth data of groups of $2^l \times 2^l$ adjacent texels in the highest resolution relief texture (level zero). Only texels representing actual surface samples¹⁸ are considered during the averaging process. The lengths of vectors \vec{a} and \vec{b} (camera parameters representing the horizontal and vertical sample spacing, respectively) are doubled from level l to level $l+1$ of the pyramid, *i.e.*, $\vec{a}_l = 2^l \vec{a}_0$, $\vec{b}_l = 2^l \vec{b}_0$, where subscripts identify the levels. This compensates for the halving of the number of texels in each dimension between levels l and $l+1$, thus keeping the spatial coverage of the relief texture unchanged.

Relief texture pyramids provide a unified representation for implementing both levels of detail and antialiasing. Levels of detail (LODs) are pre-computed simplified object representations used to reduce the number of rendered primitives at a given time. During the rendering, a representation is selected based on the object's estimated contribution to the final image. Relief texture pyramids can be regarded as realizations of image-based LODs¹⁹ and can also be used for antialiasing, exploiting the filtering capabilities available in texture mapping hardware.

A multiresolution object representation consists of six relief-texture pyramids, one for each face of the object's bounding box. Figures 6-2 shows the first four levels of such an object representation rendered using the same algorithms described in previous chapters. Examples of image-based LODs mapped onto two quadrilaterals are shown in Figures 6-3 and 6-4. 128x128-texel relief textures (level 1) were used to create Figure 6-3. Figure 6-4 was produced using 64x64-texel relief textures (level 2) and shows the same object at three different distances. A level 0 rendering of the same view is shown in Figure 5-11.

¹⁸ Transparent samples are ignored.

¹⁹ Mip-map pyramids [Williams83] are the simplest example of (static) image-based LODs.



Figure 6-2. Image-based LODs rendered using the first four levels of an object's multiresolution representation, whose front images are shown in Figure 6-1.

Selection of geometric LODs is usually based on a user specified pixel error requirement [Erikson99]. By estimating the maximum error (in object space) introduced by the simplification algorithm and projecting such an error onto the view plane, a screen space error is obtained [Erikson99]. Although automatic level selection for textured LODs has not been implemented, it can be based on the projected areas of the associated quadrilaterals. Figure 6-5 shows a distant object rendered using texture LODs produced at different levels of a relief texture pyramid. From left to right, the source textures used for warping consist of 256x256, 128x128, 64x64, 32x32 and 16x16 texels, respectively.



Figure 6-3. Textured LOD produced with 128x128-texel relief textures. The corresponding pre-warped textures are shown on the right.

For this example, the renderings produced with levels 0 and 1 (first and second images from left to right) are unnecessarily expensive and more prone to aliasing. Levels 2 and 3 produce acceptable results with significantly less work.



Figure 6-4. Views of a textured LOD rendered from different distances using 64x64-textel relief textures.



Figure 6-5. A distant object rendered using textured LODs. From left to right: 256x256, 128x128, 64x64, 32x32 and 16x16-textel relief textures were used for the warping. Image resampling was performed using bilinear interpolation.

6.1.1 Bilinear versus trilinear filtering

Higher-order filters are usually regarded as producing better results than lower-order filters when used for image reconstruction and resampling. In texture mapping applications, trilinear interpolation is often preferred to bilinear interpolation for image resampling despite being computationally more expensive and usually introducing excessive blurring (Figure 6-6). For relief texture mapping, however, bilinear interpolation is the preferred resampling method.

Although mip maps [Williams83] are very effective for reducing aliasing artifacts due to texture minification' such an issue is not crucial in the texture map stage of the relief texture mapping algorithm. Also, the distinction between the apparent distance of a



Figure 6-6. Checkerboard texture mapped using bilinear (left) and trilinear (mip mapped) resampling (right).

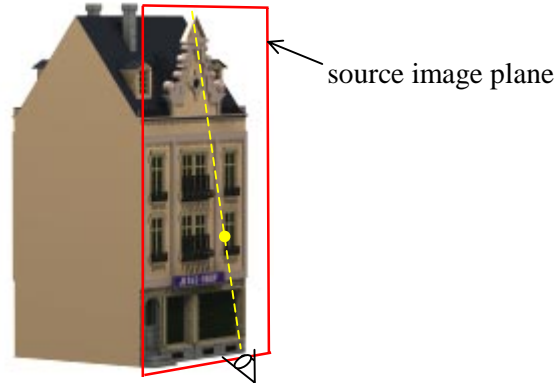


Figure 6-7. Façade observed from a grazing angle. The perspective effect causes the tip of the roof ornament to project closer to the viewer onto the source image plane (dot).

rendered surface and the actual distance of the texture-mapped polygon is an important aspect to be considered. Both topics are discussed next.

Consider the situation depicted in Figure 6-7, where a building façade is observed at a grazing angle through a viewing plane. The pre-warp causes samples whose projections are adjacent on the viewing plane to also be adjacent in the pre-warped texture. Thus, samples that need to be averaged together for antialiasing purposes are always close to each other and bilinear interpolation suffices to produce satisfactory resampling. Figure 6-8 shows the renderings of the view represented in Figure 6-7 using both bilinear (left) and trilinear filtering (right). The results are very similar except that the image resampled using bilinear interpolation is sharper. Trilinear filtering, on the other hand, produced better antialiased contours (Figure 6-8).

The second argument against the use of trilinear interpolation for relief texture mapping has to do with the difference between the apparent distance of a surface and the actual distance of the associated texture-mapped polygon(s). While an observer perceives the apparent distance of the surface, conventional mip map level estimation algorithms try to keep the ratio between pixel and texel sizes equal to one. For this reason, they are not suitable for projective-based representations such as the perpendicular plane representation used for objects. For example, consider the rendering shown in Figure 6-9. The difference between the projected areas of the two quadrilaterals leads to the selection of different mip map levels for rendering each polygon. This causes regions perceived as



Figure 6-8. Building façade pre-warped and then texture mapped using bilinear (left) and trilinear (mip mapping) resampling (right).

spatially close to each other to exhibit different amounts of blurring. In the case of Figure 6-9, the forehead and nose are blurrier than other parts of the statue.

An approximation to trilinear filtering capable of reducing both rendering costs and aliasing artifacts can be achieved by combining LOD selection with bilinear resampling. Such a strategy was used to render the views of the statue in Figure 6-5.



Figure 6-9. Conventional mip map level estimation algorithms are not suited for projection-based representations. The image containing the forehead and nose is blurrier than the other one.

6.1.2 Cost considerations

The use of bilinear interpolation for texture resampling also has some computational advantages over the use of mip map filtering. Thus, let T be a $n \times n$ relief texture whose projection, from a viewpoint V , covers p pixels on the screen. Assuming that the texture mapping resampling is performed using bilinear interpolation, the worst–

case rendering cost for T adds up to n^2 warps and p bilinear filtering operations. If, however, trilinear resampling is used, the worst-case rendering cost amounts to n^2 warps, $f = \frac{n^2}{4} + \frac{n^2}{4^2} + \dots + 1 \cong \frac{n^2}{3}$ averaging operations applied to groups of four adjacent texels (for the construction of the mip map) and p trilinear filtering operations. Trilinear interpolation also requires twice as much bandwidth between the texture memory and the rasterizer as bilinear interpolation. Moreover, the pre-warp needs to be performed every time the viewer moves and, in this case, a new mip map needs to be created.

6.2 Inverse Pre-Warping

Ideally, only texels effectively contributing to some screen fragment should be pre-warped. Unfortunately, such an optimal strategy is not practical for interactive applications. In the case of the pre-warp, Equations (3-13a) and (3-13b) define many-to-one mappings and, therefore, are not invertible (Figure 6-10). Inverting them would be equivalent to solving the following indeterminate system

$$u_s = u_i(1 + k_3 \text{displ}(u_s, v_s)) - k_1 \text{displ}(u_s, v_s)$$

$$v_s = v_i(1 + k_3 \text{displ}(u_s, v_s)) - k_2 \text{displ}(u_s, v_s).$$

Without knowing (or introducing additional restrictions to) the value of $\text{displ}(u_s, v_s)$, the system accepts multiple solutions. Thus, any inverse approach has to search for the

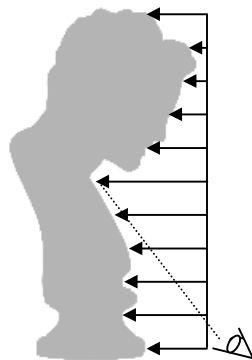


Figure 6-10. Two source texels are mapped to the same location in the pre-warped image. The pre-warping equations define many-to-one mappings and cannot be directly inverted.

closest sample from the viewpoint along each target ray²⁰.

6.2.1 Searching along epipolar lines

Although inverse mapping solutions ultimately require a search to be performed, this search can be performed along segments of epipolar lines. The approach described here is similar to the inverse warping methods of McMillan and Bishop [McMillan97]. In this case, however, only the pre-warping is being inverted, allowing the algorithm to be efficiently implemented using simple incremental computations. Nevertheless, it has no significant advantages over the forward approach described in the previous chapters and is presented here for completeness.

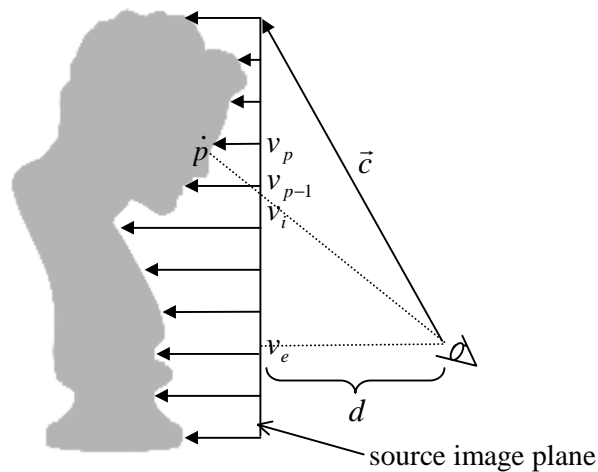


Figure 6-11. Two-dimensional representation of the search strategy used for an inverse pre-warper.

Consider the situation illustrated in Figure 6-11, where $d = \vec{c} \cdot \vec{f}$ is the distance between the viewer to the source image plane, v_e is the row coordinate of the epipole and \vec{c} is the vector from the target COP to the origin of the source image plane. \vec{f} is the unit vector normal to the source image plane (Figure 3-17). Let v_i be the row coordinate of the intersection of the source image plane with the viewing ray through \hat{p} (Figure 6-11). From similar triangles,

²⁰ This assertion also applies to the “inverse” rendering of all kinds of range images.

$$\frac{d}{(v_i - v_e)} = \frac{displ_p}{(v_p - v_i)}, \quad (6-1)$$

where $displ_p$ is the displacement value associated with \dot{p} . Equation (6-1) can be rewritten as

$$displ_p = c_{vi} \Delta v_p,$$

where $\Delta v_p = v_p - v_i$.

The search for the visible sample through v_i should then start at v_i and proceed in the opposite direction to the epipole²¹, ending when Inequality (6-2) is satisfied for a given source texel with row coordinate v_s , displacement $displ_s$ and $\Delta v_s = v_s - v_i$.

$$displ_s \leq c_{vi} \Delta v_s \quad (6-2)$$

In 3-D, the search should stop when the condition

$$displ_s \leq c_{ui} \Delta u_s \text{ and } displ_s \leq c_{vi} \Delta v_s$$

is satisfied. In this case, c_{ui} and Δu_s are the column analogs of c_{vi} and Δv_s , respectively. For each pre-warped texel (u_i, v_i) , this search proceeds along the epipolar line corresponding to the parallel projection of the ray passing through the target COP and (u_i, v_i) onto the source image plane. Since the source and target image planes do not coincide in general, the resulting image still needs to be texture mapped onto the source image plane.

6.3 One-dimensional Clipping

Skipping the pre-warp for samples having zero displacement speeds up the rendering of relief textures, as discussed in Chapter 3. Another way to accelerate the rendering is to avoid transforming samples whose projections fall outside of the limits of the source image plane. This section presents a 1-D clipping algorithm for use during the pre-warp. It will be described in the context of relief textures, but the same ideas also apply to perspective projection images with depth.

²¹ Assuming that all samples are behind the source image plane, such as the case shown in Figure 6-11, no texels closer to the epipole than v_i can possibly map to v_i .

Consider the two-dimensional representation depicted in Figure 6-12, where P is the plane defined by the target COP and by the two vertices of the source image plane associated with u_m , the extremum column closest to the COP. According to Figure 6-12, all samples outside of the shaded region project beyond the limits of the source image plane. Thus, let t_s be a source texel at column u_s whose corresponding sample s_s lies on P (Figure 6-12). Also, let r_u be the ratio

$$r_u = \frac{displ_s}{\Delta u_s},$$

where $\Delta u_s = |u_m - u_s|$, $displ_s$ is the displacement value associated with t_s and r_u is the tangent of the angle between P and the source image plane. Thus, for any given source texel t_j , t_j projects onto the source image plane if and only if

$$displ_j \leq r_u \Delta u_j. \quad (6-3)$$

Notice that if splatting is used for reconstruction, all samples not satisfying Inequality (6-3) can be automatically clipped. Extra care must be taken if linear interpolation is used for reconstruction, such as the case of the two-pass 1-D approach. For instance, consider the situation of sample s_{k+1} shown in Figure 6-12. Although its projection falls outside the limits of the source image plane, the column coordinate of its projection is still required to appropriately interpolate colors between s_{k+1} and its visible

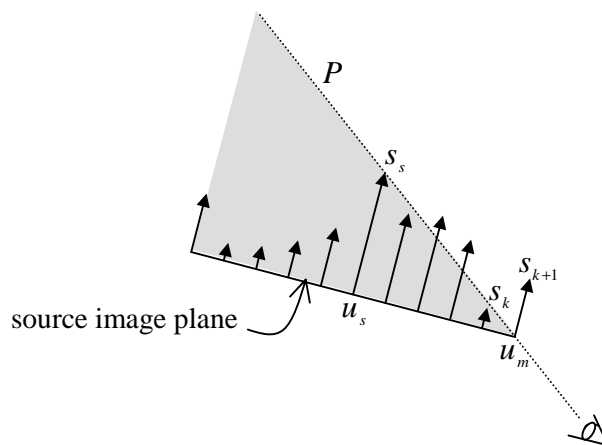


Figure 6-12. Clipping (top view). All samples whose outside the shaded triangle project beyond the limits of the source image plane.

neighbor s_k . The necessary condition for performing clipping in 1-D follows: *given three consecutive texels t_{k-1} , t_k and t_{k+1} on the same row, such that none of them satisfies Inequality (6-3), the warp of t_k can be safely skipped.* A similar property applies to columns. In this case, Inequality (6-3) becomes

$$displ_j \leq r_v \Delta v_j, \quad (6-4)$$

where $r_v = \tan \beta$ and $\Delta v_s = |v_m - v_j|$ have analogous definitions to r_u and Δu_s , respectively. Thus, clipping can be efficiently implemented during both the horizontal and the vertical passes of the pre-warp by checking the conditions defined by Inequalities (6-3) and (6-4), respectively, for groups of three consecutive texels.

6.4 Shading

Image-based rendering techniques can create photo-realistic pictures from images, thus avoiding expensive simulations of surface-light interactions. Such a property not only leads to considerable computational savings but also provides a simple way to represent and render complex physical processes. Unfortunately, the photometric properties of many materials cannot be represented using just a few pictures. For instance, the appearance of non-diffuse surfaces changes with viewpoint and illumination direction. For these surfaces, correct rendering would require the use of huge image databases to store their appearance from arbitrary viewpoints. The Light Field [Levoy96] and Lumigraph [Gortler96] approaches approximate such a solution. If the scene illumination is allowed to vary, storage requirements become even bigger [Wong97].

A simple approach capable of producing local illumination solutions²² consists of extending image-based representations with surface normals. Normals are geometric properties of surfaces and therefore do not change with viewpoint. They can be interpolated during the pre-warp and used by a deferred shading strategy to compute view-dependent effects. *Normal maps* are special kinds of texture maps that store normal vector components instead of color data [Fournier92]. Multiresolution representations for

²² Local illumination solutions only account for direct illumination, not considering light contributions resulting from multiple inter-reflections.

normal maps have been used in conjunction with geometric models [Cohen98] [Fournier92] and their use with image-based representations is straightforward. The alpha channel allows normal vectors to be assigned to portions of a relief texture corresponding to non-diffuse surfaces.

The shadow map algorithm [Williams78] computes shadows by rendering the scene both from the desired viewpoint and from the point of view of each light source. The camera and depth buffer information associated with the desired viewpoint are used to reproject each of its pixels onto the image planes of the light sources. Depth comparisons are then performed to decide which light sources illuminate each sample. When presented with modulated depth buffers computed for relief texture mapping, the same algorithm casts shadows from and onto relief texture-mapped objects, including self-shadows.

6.5 Discussion

The averaging of displacement values across the levels of a relief texture pyramid makes the threshold-based skin-detection approach less effective at the low resolution levels of the pyramid. Also, as a smaller number of texels is used to represent the same image, it becomes impractical to mark discontinuities by discarding texels. Fortunately, low-resolution representations should only be used when their projections cover small areas on the screen, in which case, skin-related artifacts are hidden by blurring. Figure 6-13 highlights a skin-related artifact in a textured LOD. It is characterized by a sudden change in shading across a vertical line. In this example, the frontal relief texture contains a skin connecting the jaw to the chest of the statue, which is lightly shaded (Figure 6-14 (left)). The side view of the jaw, on the other hand, is considerably darker (Figure 6-14 (right)). The kind of artifact shown in Figure 6-13 occurs because texels from each image have priority when rendered to their own face of the box, according to the algorithm presented in Figure 5-8. In this case, the skin overwrites correct samples of the jaw and neck mapped from the side view onto the frontal plane. By removing the skin, the dark portion of the jaw is no longer overwritten and the artifact disappears.



Figure 6-13. Skin-related artifact caused by large difference in the shading of a region around the chin as viewed by two adjacent relief textures (Figure 6-14).



Figure 6-14. Front and right relief textures of the statue representation (level 0). The region responsible for the skin-related artifact is highlighted.

Image re-shading is a difficult problem. Whereas evaluating shading expressions for each sample of a range image is straightforward, removing light, deleting shadows, and maintaining consistent illumination among several image-based representations in a scene are not trivial tasks [Fournier93] [Oliveira98]. All these operations require knowledge about the properties of the light sources involved. Reference images may also record the result of light interaction with objects not represented in the pictures, such as

the case of caustics and shadows cast by objects clipped from the image's field of view. In general, a substantial amount of information is needed to change the illumination in a picture consistently.

6.6 Summary

This chapter has covered a variety of topics including multiresolution representations for relief textures, inverse pre-warping, clipping and shading. Multiresolution representations are constructed by averaging color and displacement values of groups of adjacent texels and are similar to mip map pyramids. The resulting LOD representations are rendered using the same algorithms described in previous chapters.

The use of bilinear interpolation for the texture resampling presents some advantages over trilinear filtering when used for relief texture mapping. While mip mapping is very effective for reducing aliasing artifacts caused by texture minification, pre-warp images tend to require little amounts of minification. Thus, bilinear interpolation produces sharper images, is less prone to undesirable blurring due to polygonal orientation with respect to the screen, and is computationally less expensive than trilinear interpolation. Combining LOD selection and bilinear resampling can approximate trilinear filtering.

The discussion about scene rendering was extended to include shadows. Once the depth buffer has been modulated to account for relief occlusions, the shadow map algorithm [Williams78] can be used to cast shadows both onto and from relief texture-mapped representations. This chapter has also discussed an inverse pre-warping strategy and presented an efficient incremental algorithm for performing 1-D clipping on the fly.

Chapter 7 – Conclusions and Future Work

The idea of texture mapping, *i.e.*, the ability to assign properties to surfaces, is one of the most powerful concepts in computer graphics. It has been explored and extended in many ways to produce realistic images, create impressive effects and accelerate rendering. This research has presented one more extension: the ability to represent three-dimensional surface details and view-motion parallax. This closing chapter returns to the issue of one-dimensional warp and reconstruction and explains why it works. It also provides additional discussions on view-dependent texture mapping and image-based representations for dynamic environments. The chapter ends with a summary of the major points presented in this dissertation and with a list of suggested areas for future exploration.

7.1 Why One-Dimensional Warp and Reconstruction Works

The surprisingly good results produced by 1-D warp and reconstruction strategies follow from spatial-coherence preservation under projective mappings: groups of spatially adjacent samples in 3-D are projected onto nearby pixels of an image plane²³. Remember that the pre-warp is just a reprojection of a range image from a new COP. Together with the separability of perspective projection into orthogonal components, spatial-coherence preservation guarantees that although samples are shifted along independent rows (columns), their movements are consistent with the movements of their original neighbors. Thus, linear interpolation produces good reconstruction for continuous surfaces in both passes of the pre-warp. This argument supports the use of two-pass 1-D interpolation for all view-independent surface attributes, such as normal fields, and for camera-space Z values as discussed in Section 5.5. The resulting normal

²³ The converse, though, is not true, mainly because of view-motion parallax, *i.e.*, adjacent pixels in one image may come from distant samples in 3-D.

and depth maps are likely to be as good as the reconstructed intensity image for diffuse surfaces.

The occurrence of depth discontinuities causes regions of a source image to be expanded as a result of a parallax effect and are the major cause of source image topology disruption. Such expansions correspond to the exposure of surfaces not represented in the original image (skins). Even in the presence of depth discontinuities, the results produced by one-dimensional resampling are satisfactory for most practical situations, as discussed in Section 4.7.

7.2 Discussion

7.2.1 View-Dependent Texture Mapping

View-dependent texture mapping [Debevec98] consists of selecting and blending textures pre-acquired from directions that approximate the desired view. In this case, an underlying geometric model determines the shape of the surfaces. Relief texture mapping, on the other hand, creates an illusion of three-dimensionality by projecting textures, specifically warped for the desired view, onto planar or box-shaped models. It is, therefore, a *projective* view-dependent texture-mapping algorithm. Unlike the blending strategies used in conventional approaches, relief texture mapping is based on a texture reconstruction strategy that uses a small set of source images.

7.2.2 Dynamic Environments

Virtual reality applications, games and animations often need to continuously update position, orientation and shape of objects in a scene. By treating each object as a separate entity, geometry-based approaches naturally support such changes. Relief textures are hybrid representations and can be used to construct dynamic environments. Moreover, data associated with relief textures can be changed incrementally over time to produce morphing effects, such as the case of the face shown in Figure 5-30.

7.2.3. Depth Complexity Considerations

A relief texture is a single-layer image representation. Under the surface continuity assumption, the entire scene has a depth complexity of one and there is no need for multiple samples along each ray. Objects, on the other hand, are represented with six perpendicular relief textures, which is equivalent to a multi-layer representation. Thus, for instance, in Figure 6-14, the combined samples from the left and right images contain information about the front part of the jaw and from the neck, which are not visible in any of the individual images simultaneously.

If one wants to remove skins from individual relief textures, such as the walls of the reading room shown in Figure 5-24, holes may appear. In this case, alternative representations such as layered-depth images (LDIs) [Shade98] could be used. However, reconstruction of LDIs is limited to splatting. The existence of multiple samples along each ray introduces ambiguity about which samples should be connected, making 1-D interpolation and mesh-based reconstruction impractical. Alternatively, creating and rendering several layers, each consisting of individual relief textures can be used to produce similar results. In the supporting animation entitled “Town Exploration Using Relief Texture Mapping”, the skin between the façade ornament and the roof of one of the buildings (*Jeans Store*) was removed and the resulting hole was seamlessly filled by adding an extra conventionally texture-mapped polygon.

7.2.4. 3-D Photography

In the near future, one can expect the development and popularization of new devices for simultaneous acquisition of color and depth data. 3-D photography should then become a standard part of the computer graphics modeling repertoire, and relief texture mapping offers a suitable strategy for rendering such primitives.

7.3 Synopsis

This dissertation has presented an extension to texture mapping that supports the representation of three-dimensional surface details and view-motion parallax. This approach results from the factorization of the 3-D image warping equation of McMillan

and Bishop [McMillan97] into a pre-warp followed by standard texture mapping. The pre-warp is applied to images with per-texel displacements and handles only the parallax effects resulting from the direction of view and the displacements of texture elements. The subsequent texture-mapping operation handles the transformation from texture to screen coordinates. By doing so, relief texture mapping takes advantage of the efficiency of forward transforms to solve visibility issues, while exploring the superior filtering capabilities of inverse mappings.

Pre-warping equations for both parallel and perspective-projection images with depth have been derived and 1-D algorithms for performing clipping, hidden surface removal, image reconstruction and anti-aliasing have been presented. The use of a depth buffer modulation strategy for achieving correct occlusions and allowing shadow computation was described.

The one-dimensional nature of the pre-warping equations follows from the separability of the perspective projection into orthogonal components. The effectiveness of 1-D warp and reconstruction strategies results from the spatial coherence preservation under projective mappings and from the separability of perspective projection.

I have presented algorithms for modeling and rendering three-dimensional objects and immersive environments using relief textures. The use of quantized displacement values makes the storage requirements of relief textures equivalent to the requirements of regular textures, without apparent image degradation. Quantized displacement values can be used in combination with lookup tables to reduce the computational cost associated with the pre-warping of one texel to essentially two additions and two multiplications.

I have shown how to construct and render multiresolution representations for range images. These can be used to reduce aliasing artifacts and to keep the rendering cost of relief textures proportional to their contribution to the final image. Bilinear interpolation is the preferred image resampling strategy during the texture mapping stage of the relief texture-mapping algorithm. It produces sharper images, is less prone to undesirable blurring due to polygon orientation and is computationally less expensive than trilinear interpolation.

Relief texture mapping naturally integrates itself with current graphics APIs and provides a framework for combining the photo-realistic promise of image-based modeling and rendering techniques with the advantages of polygonal rendering.

7.4 Future Work

Some possible areas for further exploration include hardware design for relief texture mapping, automatic or semi-automatic extraction of relief textures from scenes, use of relief textures for geometry simplification, and representation of non-diffuse surfaces.

7.4.1 Hardware implementation

One important area for future investigation is the design of efficient hardware implementations for relief texture mapping using the derived pre-warping equations. Adding this pre-warping capability to the texture memory of a graphics accelerator may allow relief texture mapping to become as commonly used as conventional texture mapping.

7.4.2 Extraction of relief textures and geometric simplification

Automatic or semi-automatic extraction of relief textures from 3-D environments and geometry simplification are two important related areas for exploration. A semi-automatic procedure that combines geometry simplification and least-squares solutions has been outlined in Section 5-6. Such a strategy serves both purposes at once, as it helps to identify areas that could benefit most from relief representations, while replacing flat regions with conventionally texture-mapped polygons.

7.4.3 Representations for non-diffuse surfaces

The striking realism obtained with the use of image-based rendering techniques is attenuated as the viewer perceives incorrect shading clues, such as fixed highlights, on non-diffuse surfaces. Highlights produced with Phong shading are unconvincing for most

materials, and new algorithms for reshading photographs, accounting for more believable view-dependent effects, are required.

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