Rules and Application of Polar Forms

- A polar value is symmetric with its values

- Given $\mathbf{P}(u_1,u_2,\ldots,u_{n-1},a)$ and $\mathbf{P}(u_1,u_2,\ldots,u_{n-1},b)$, the point represented by $\mathbf{P}(u_1,u_2,\ldots,u_{n-1},c)$ can be computed as $\mathbf{P}(u_1,u_2,\ldots,u_{n-1},a)$ =
$$\frac{(b-c)\mathbf{P}(u_1,u_2,\ldots,u_{n-1},a) + (c-a)\mathbf{P}(u_1,u_2,\ldots,u_{n-1},b)}{(b-a)}$$

- For degree n Bezier curves over a parametric interval $[a,b]$, the control points are relabeled $P_i = \mathbf{P}(u_1,u_2,\ldots,u_n)$ where $u_j=a$ if $j <= n-i$ otherwise $u_j=b$.

- For degree n B-Spline, with knot vector $(t_1, t_2, \ldots)$, the arguments of the polar values consist of groups of n adjacent knots from the knot vector, with the i-th polar value being $P(t_i, \ldots, t_{i+n-1})$.

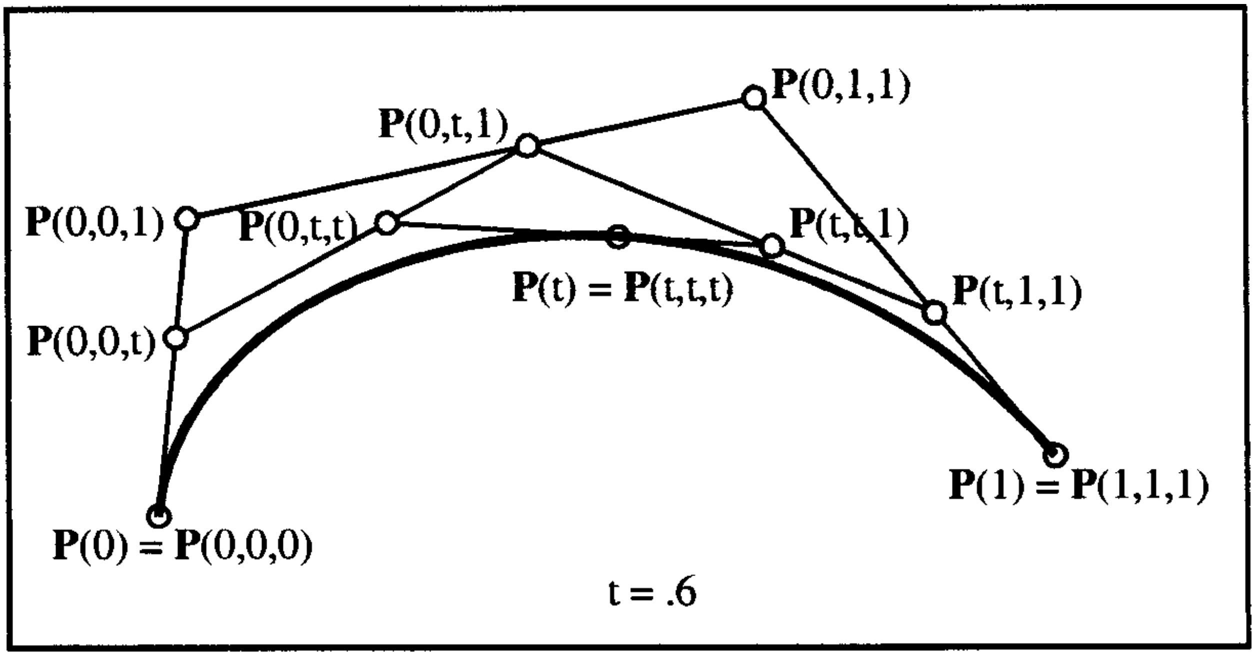

Figure 5.5: Subdividing a cubic Bézier curve.

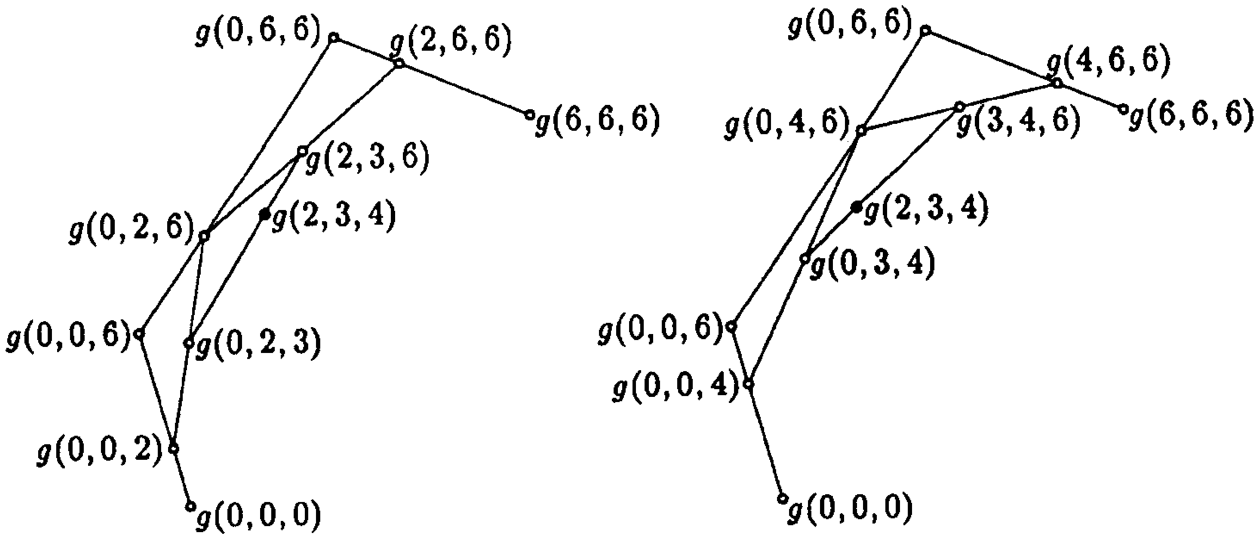

Fig. 2: Computing $g(2,3,4)$ in two different ways

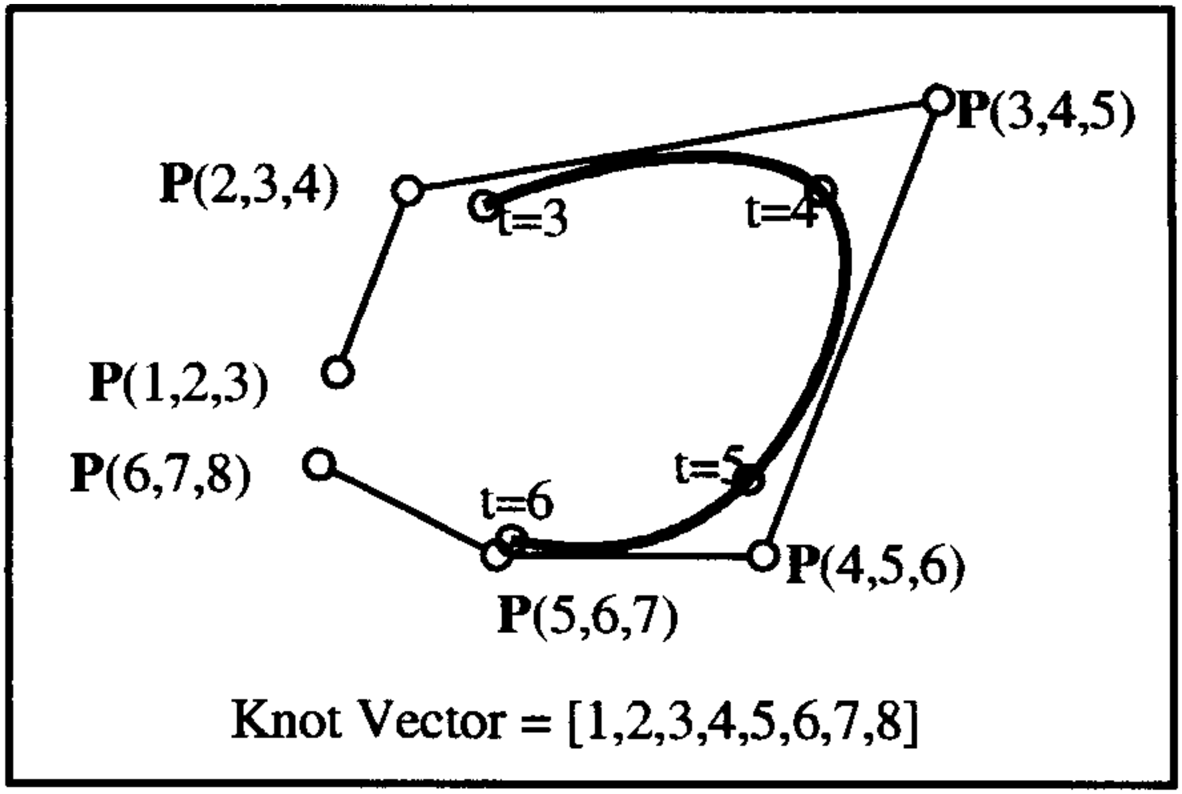

Figure 5.3: B-spline curve labeled using polar form.

	Initial	After Knot Insertion
Knot Vector:	[(0,0,0,1,3,4,4,4)]	[(0,0,0,1,2,3,4,4,4)]
Control Points:	**P**(0,0,0)	**P**(0,0,0)
	P(0,0,1)	**P**(0,0,1)
		P(0,1,2)
	P(0,1,3)	
		P(1,2,3)
	P(1,3,4)	
		P(2,3,4)
	P(3,4,4)	**P**(3,4,4)
	P(4,4,4)	**P**(4,4,4)

Figure 5.11: Before and after.

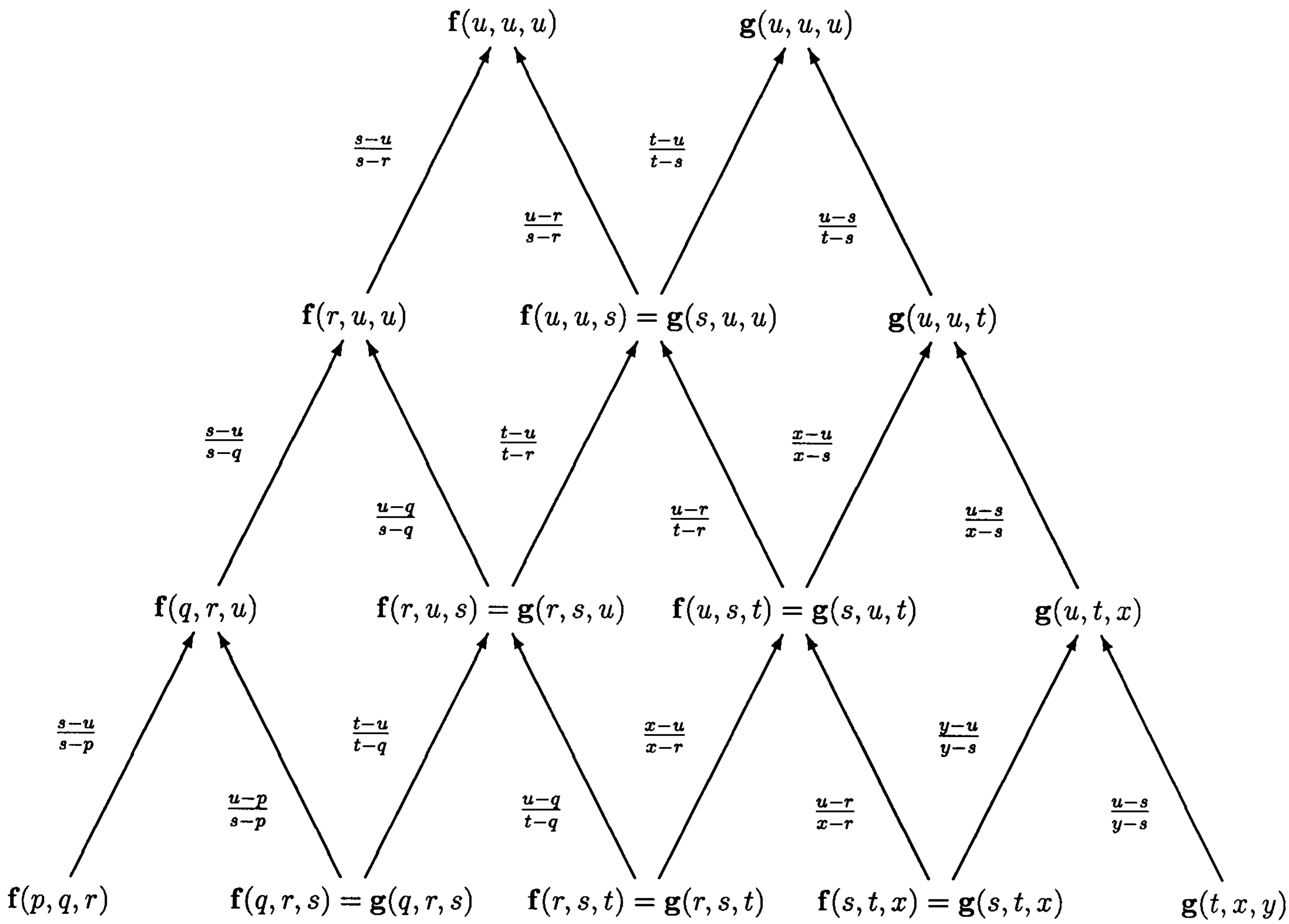

Figure 6: Overlapping de Boor schemes for two adjacent cubic B-spline segments $F : [r, s] \to \mathbb{R}^t$ and $G : [s, t] \to \mathbb{R}^t$ over the knot sequence $\ldots, p, q, r, s, t, x, y, \ldots$

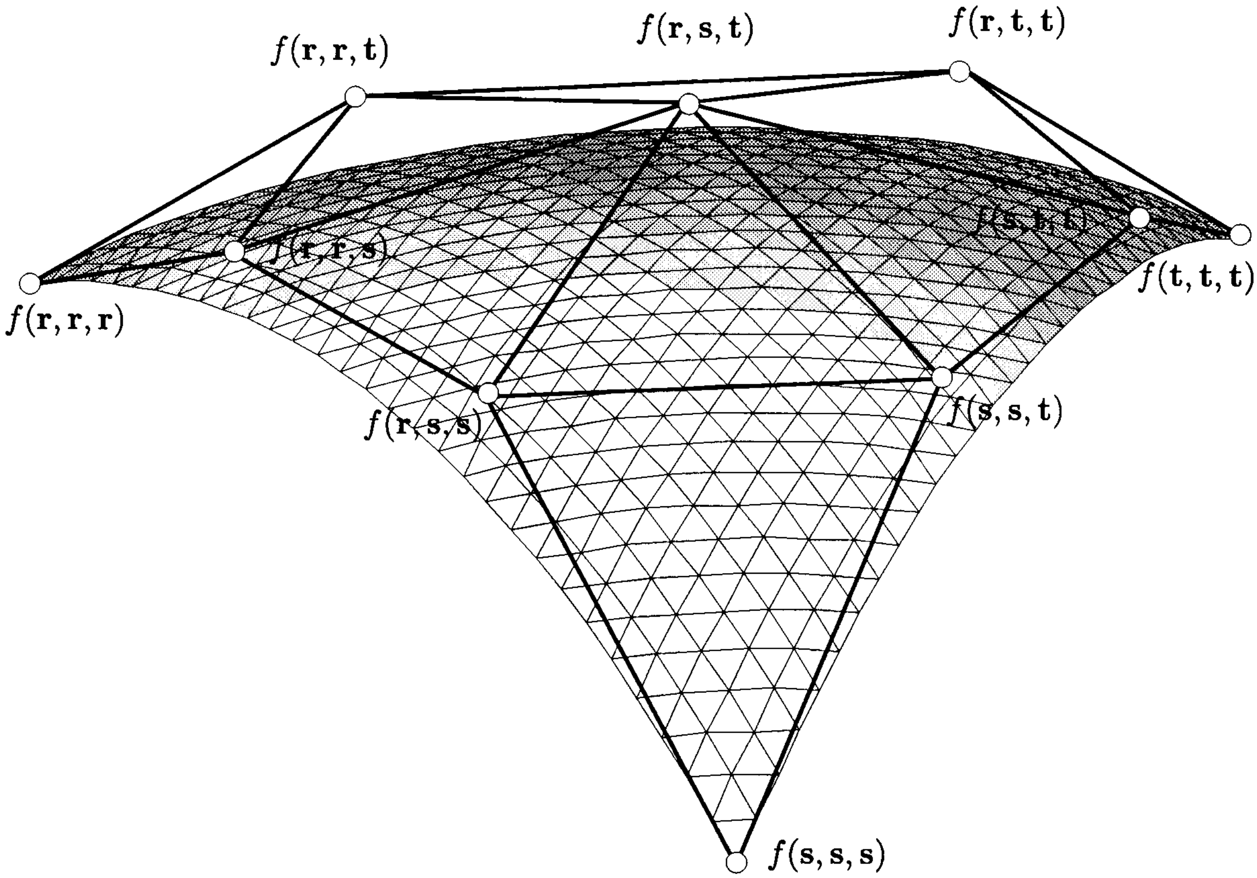

Figure 7: A cubic Bézier patch.

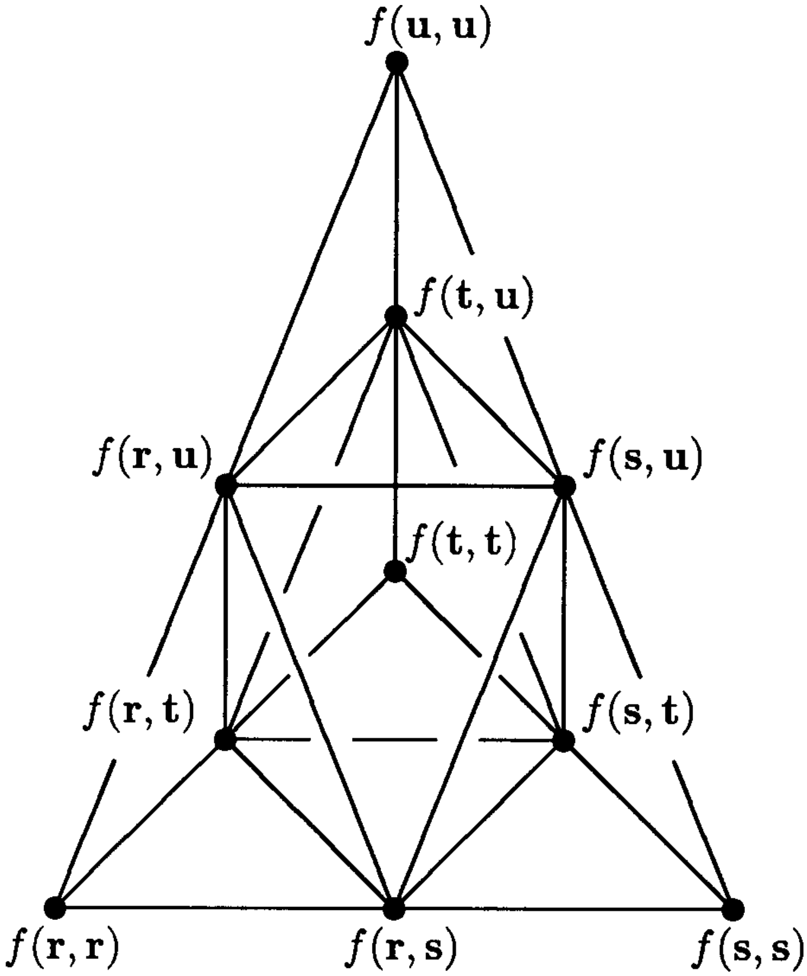

Figure 8: The de Casteljau Algorithm for a quadratic Bézier patch.

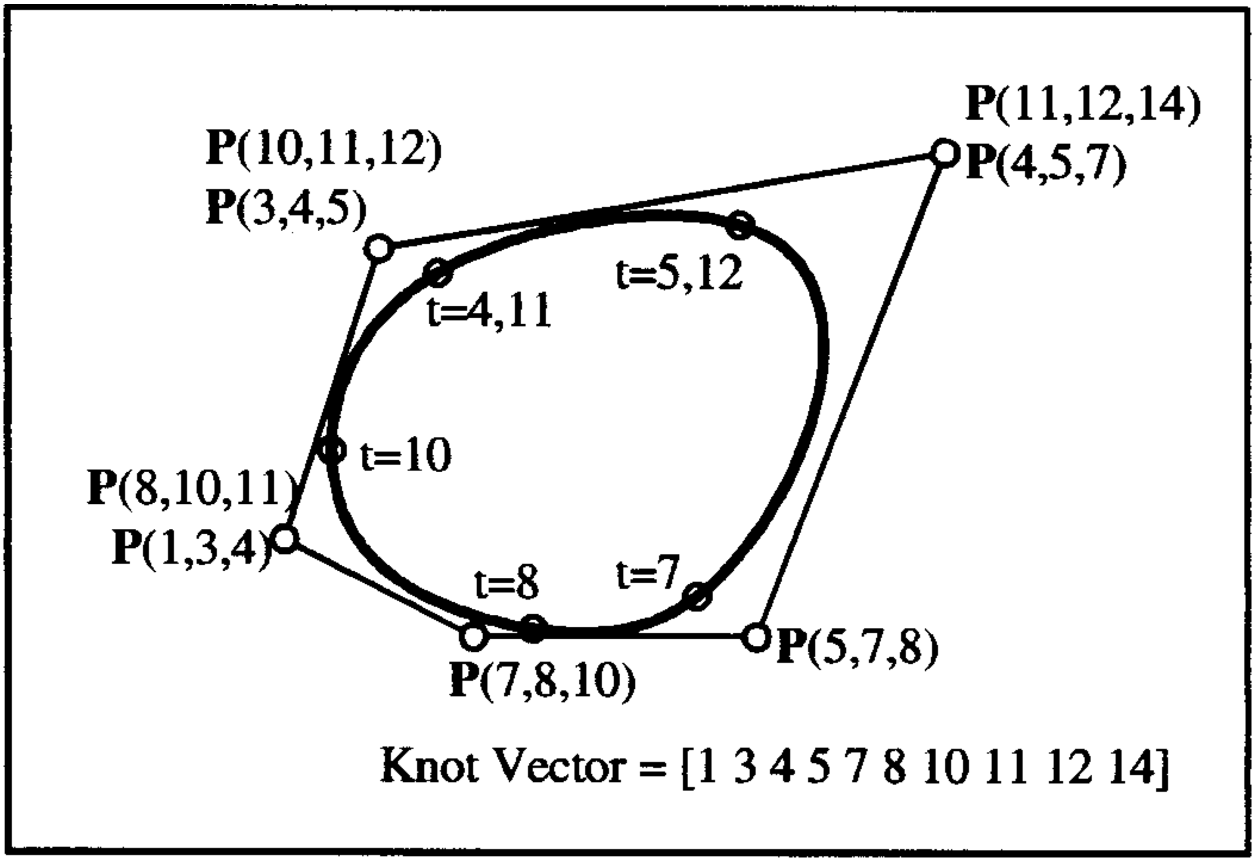

Figure 5.9: Periodic B-spline

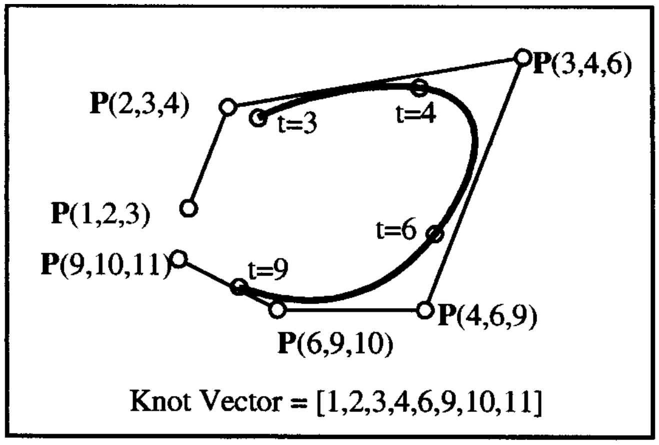

Figure 5.14: Sample B-spline

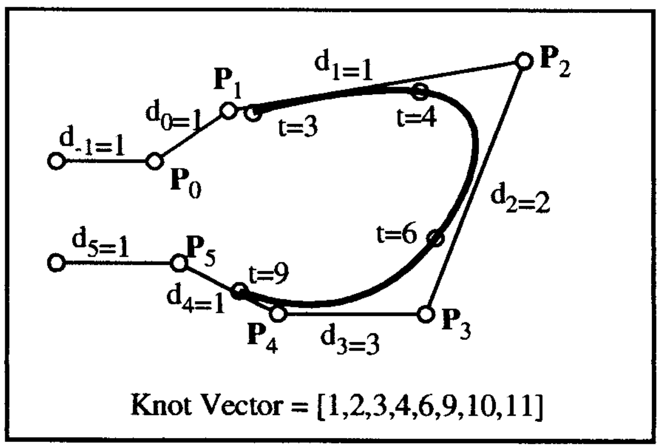

Figure 5.15: Sample B-spline with knot intervals

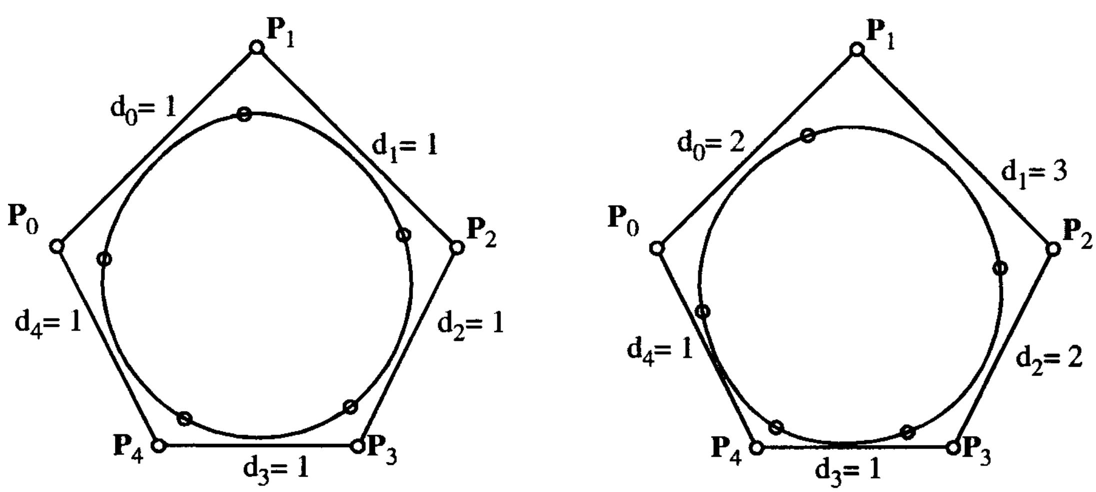

Figure 5.16: Periodic B-splines labelled with knot intervals

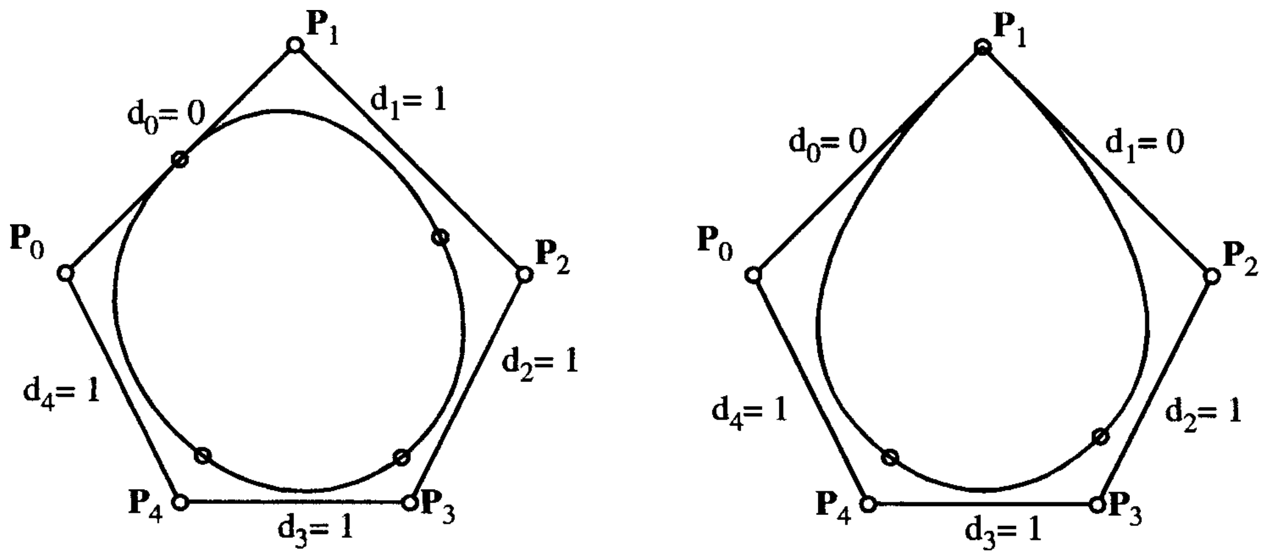

Figure 5.17: Periodic B-splines with double and triple knots.

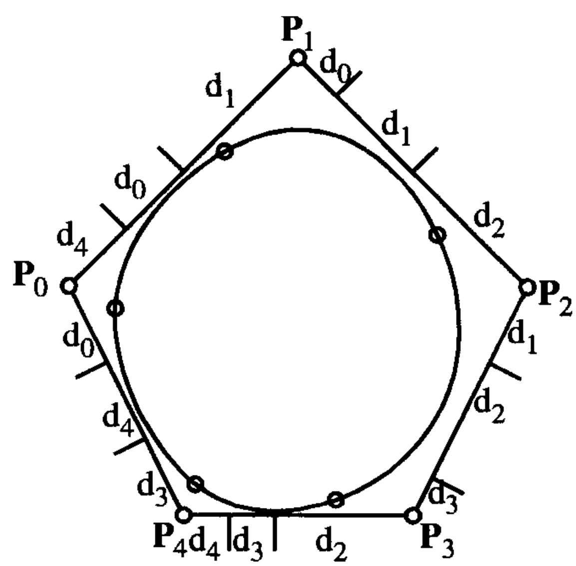

Figure 5.18: Cubic B-spline with control polygon marked.