Review

• Pinhole projection model
  • What are vanishing points and vanishing lines?
  • What is orthographic projection?
  • How can we approximate orthographic projection?

• Lenses
  • Why do we need lenses?
  • What is depth of field?
  • What controls depth of field?
  • What is field of view?
  • What controls field of view?
  • What are some kinds of lens aberrations?

• Digital cameras
  • What are the two major types of sensor technologies?
  • How can we capture color with a digital camera?
Assignment 1: Demosaicing
Historical context

- **Pinhole model:** Mozi (470-390 BCE), Aristotle (384-322 BCE)
- **Principles of optics (including lenses):** Alhacen (965-1039 CE)
- **Camera obscura:** Leonardo da Vinci (1452-1519), Johann Zahn (1631-1707)
- **First photo:** Joseph Nicephore Niepce (1822)
- **Daguerréotypes** (1839)
- **Photographic film:** Eastman (1889)
- **Cinema:** Lumière Brothers (1895)
- **Color Photography:** Lumière Brothers (1908)
- **Television:** Baird, Farnsworth, Zworykin (1920s)
- **First digitally scanned photograph:** Russell Kirsch, NIST (1957)
- **First consumer camera with CCD:** Sony Mavica (1981)
- **First fully digital camera:** Kodak DCS100 (1990)
10 Early Firsts In Photography

Early color photography

Sergey Prokudin-Gorsky (1863-1944)
Photographs of the Russian empire (1909-1916)

http://www.loc.gov/exhibits/empire/

Lantern projector
“Fake miniatures”

Create your own fake miniatures: http://tiltshiftmaker.com/

Idea for class participation: if you find interesting (and relevant) links, send them to me or (better yet) to the class mailing list (comp776@cs.unc.edu).
Today: Capturing light

Source: A. Efros
Radiometry

What determines the brightness of an image pixel?

Light source properties

Surface shape

Surface reflectance properties

Exposure

Optics

Sensor characteristics

Slide by L. Fei-Fei
Solid Angle

- By analogy with angle (in radians), the solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point.
- The solid angle $d\omega$ subtended by a patch of area $dA$ is given by:

$$d\omega = \frac{dA \cos \theta}{r^2}$$
**Radiometry**

- **Radiance ($L$):** energy carried by a ray
  - Power per unit area perpendicular to the direction of travel, per unit solid angle
  - Units: Watts per square meter per steradian ($W \text{ m}^{-2} \text{ sr}^{-1}$)

- **Irradiance ($E$):** energy arriving at a surface
  - Incident power in a given direction per unit area
  - Units: $W \text{ m}^{-2}$
  - For a surface receiving radiance $L(x, \theta, \phi)$ coming in from $d\omega$ the corresponding irradiance is

\[
E(\theta, \phi) = L(\theta, \phi) \cos \theta d\omega
\]
Radiometry of thin lenses

$L$: Radiance emitted from $P$ toward $P'$

$E$: Irradiance falling on $P'$ from the lens

What is the relationship between $E$ and $L$?

Forsyth & Ponce, Sec. 4.2.3
Example: Radiometry of thin lenses

The power \( \delta P \) received by the lens from \( P \) is
\[
\delta P = L \left( \frac{\pi d^2}{4} \right) \cos \alpha \delta \omega
\]

The radiance emitted from the lens towards \( dA' \) is
\[
\frac{\delta P}{\left( \frac{\pi d^2}{4} \right) \cos \alpha \delta \omega} = L
\]

The irradiance received at \( P' \) is
\[
E = L \cos \alpha \left( \frac{\pi d^2 \cos \alpha}{4 (z'/\cos \alpha)^2} \right) = \left[ \frac{\pi}{4} \left( \frac{d}{z'} \right)^2 \cos^4 \alpha \right] L
\]
Radiometry of thin lenses

- Image irradiance is linearly related to scene radiance
- Irradiance is proportional to the area of the lens and inversely proportional to the squared distance between the lens and the image plane
- The irradiance falls off as the angle between the viewing ray and the optical axis increases

\[ E = \left[ \frac{\pi}{4} \left( \frac{d}{z'} \right)^2 \cos^4 \alpha \right] L \]  

Forsyth & Ponce, Sec. 4.2.3
Radiometry of thin lenses

\[ E = \left[ \frac{\pi}{4} \left( \frac{d}{z'} \right)^2 \cos^4 \alpha \right] L \]

• Application:
The journey of the light ray

\[ X = E \cdot \Delta t \]

\[ E = \left[ \frac{\pi}{4} \left( \frac{d}{z'} \right)^2 \cos^4 \alpha \right] L \]

- Camera response function: the mapping \( f \) from irradiance to pixel values
  - Useful if we want to estimate material properties
  - Enables us to create high dynamic range images

Source: S. Seitz, P. Debevec
The journey of the light ray

\[ X = E \cdot \Delta t \]

- Camera response function: the mapping \( f \) from irradiance to pixel values

For more info

Source: S. Seitz, P. Debevec
The interaction of light and surfaces

What happens when a light ray hits a point on an object?

- Some of the light gets absorbed
  - converted to other forms of energy (e.g., heat)
- Some gets transmitted through the object
  - possibly bent, through “refraction”
- Some gets reflected
  - possibly in multiple directions at once
- Really complicated things can happen
  - fluorescence

Let’s consider the case of reflection in detail

- In the most general case, a single incoming ray could be reflected in all directions. How can we describe the amount of light reflected in each direction?
Bidirectional reflectance distribution function (BRDF)

- Model of local reflection that tells how bright a surface appears when viewed from one direction when light falls on it from another
- Definition: ratio of the radiance in the outgoing direction to irradiance in the incident direction

\[
\rho(\theta_i, \phi_i, \theta_e, \phi_e) = \frac{L_e(\theta_e, \phi_e)}{E_i(\theta_i, \phi_i)} = \frac{L_e(\theta_e, \phi_e)}{L_i(\theta_i, \phi_i) \cos \theta_i d\omega}
\]

- Radiance leaving a surface in a particular direction: add contributions from every incoming direction

\[
\int_{\Omega} \rho(\theta_i, \phi_i, \theta_e, \phi_e) L_i(\theta_i, \phi_i) \cos \theta_i d\omega_i
\]
BRDF’s can be incredibly complicated…
Diffuse reflection

- Light is reflected equally in all directions: BRDF is constant
- Dull, matte surfaces like chalk or latex paint
- Microfacets scatter incoming light randomly
- Albedo: fraction of incident irradiance reflected by the surface
- Radiosity: total power leaving the surface per unit area (regardless of direction)
Diffuse reflection: Lambert’s law

- Viewed brightness does not depend on viewing direction, but it does depend on direction of illumination

\[ B(x) = \rho_d(x)(N(x) \cdot S_d(x)) \]

- \( B \): radiosity
- \( \rho \): albedo
- \( N \): unit normal
- \( S \): source vector (magnitude proportional to intensity of the source)
Specular reflection

- Radiation arriving along a source direction leaves along the specular direction (source direction reflected about normal)
- Some fraction is absorbed, some reflected
- On real surfaces, energy usually goes into a lobe of directions
- Phong model: reflected energy falls of with $\cos^n(\delta \theta)$
- Lambertian + specular model: sum of diffuse and specular term
Specular reflection

Moving the light source

Changing the exponent
Photometric stereo

Assume:

- A Lambertian object
- A local shading model (each point on a surface receives light only from sources visible at that point)
- A set of known light source directions
- A set of pictures of an object, obtained in exactly the same camera/object configuration but using different sources
- Orthographic projection

Goal: reconstruct object shape and albedo

Forsyth & Ponce, Sec. 5.4
Surface model: Monge patch

Forsyth & Ponce, Sec. 5.4
Image model

• Known: source vectors $S_j$ and pixel values $I_j(x, y)$
• We also assume that the response function of the camera is a linear scaling by a factor of $k$
• Combine the unknown normal $N(x, y)$ and albedo $\rho(x, y)$ into one vector $g$, and the scaling constant $k$ and source vectors $S_j$ into another vector $V_j$:

$$I_j(x, y) = k \cdot B(x, y)$$
$$= k \rho(x, y) (N(x, y) \cdot S_j)$$
$$= (\rho(x, y) N(x, y)) \cdot (k S_j)$$
$$= g(x, y) \cdot V_j$$

Forsyth & Ponce, Sec. 5.4
Least squares problem

• For each pixel, we obtain a linear system:

\[
\begin{bmatrix}
I_1(x, y) \\
I_2(x, y) \\
\vdots \\
I_n(x, y)
\end{bmatrix}
= \begin{bmatrix}
V_1^T \\
V_2^T \\
\vdots \\
V_n^T
\end{bmatrix}
\begin{bmatrix}
g(x, y)
\end{bmatrix}
\]

\[
\begin{align*}
\text{known} & \quad \text{known} \\
(n \times 1) & \quad (n \times 3)
\end{align*}
\]

• Obtain least-squares solution for \(g(x,y)\)
• Since \(N(x,y)\) is the unit normal, \(\rho(x,y)\) is given by the magnitude of \(g(x,y)\) (and it should be less than 1)
• Finally, \(N(x,y) = g(x,y) / \rho(x,y)\)

Forsyth & Ponce, Sec. 5.4
Example

Revised albedo

Revised normal field

Forsyth & Ponce, Sec. 5.4
Recovering a surface from normals

Recall the surface is written as

\[(x, y, f(x, y))\]

This means the normal has the form:

\[N(x, y) = \left(\frac{1}{\sqrt{f_x^2 + f_y^2 + 1}}\right) \begin{pmatrix} -f_x \\ -f_y \\ 1 \end{pmatrix}\]

If we write the estimated vector \( g \) as

\[g(x, y) = \begin{pmatrix} g_1(x, y) \\ g_2(x, y) \\ g_3(x, y) \end{pmatrix}\]

Then we obtain values for the partial derivatives of the surface:

\[f_x(x, y) = \left(\frac{g_1(x, y)}{g_3(x, y)}\right)\]
\[f_y(x, y) = \left(\frac{g_2(x, y)}{g_3(x, y)}\right)\]

Forsyth & Ponce, Sec. 5.4
Recovering a surface from normals

*Integrability:* for the surface $f$ to exist, the mixed second partial derivatives must be equal:

\[
\frac{\partial (g_1(x,y)/g_3(x,y))}{\partial y} = \frac{\partial (g_2(x,y)/g_3(x,y))}{\partial x}
\]

(in practice, they should at least be similar)

We can now recover the surface height at any point by integration along some path, e.g.

\[
f(x,y) = \int_0^x f_x(s,y)ds + \int_0^y f_y(x,t)dt + c
\]

(for robustness, can take integrals over many different paths and average the results)

Forsyth & Ponce, Sec. 5.4
Surface recovered by integration

Forsyth & Ponce, Sec. 5.4
Limitations

• Orthographic camera model
• Simplistic reflectance and lighting model
• No shadows
• No interreflections
• No missing data
• Integration is tricky
Finding the direction of the light source

\[ I(x,y) = N(x,y) \cdot S(x,y) + A \]

Full 3D case:

\[
\begin{pmatrix}
N_x(x_1, y_1) & N_y(x_1, y_1) & N_z(x_1, y_1) & 1 \\
N_x(x_2, y_2) & N_y(x_2, y_2) & N_z(x_2, y_2) & 1 \\
\vdots & \vdots & \vdots & \vdots \\
N_x(x_n, y_n) & N_y(x_n, y_n) & N_z(x_n, y_n) & 1
\end{pmatrix}
\begin{pmatrix}
S_x \\
S_y \\
S_z \\
A
\end{pmatrix}
= 
\begin{pmatrix}
I(x_1, y_1) \\
I(x_2, y_2) \\
\vdots \\
I(x_n, y_n)
\end{pmatrix}
\]

For points on the *occluding contour*:

\[
\begin{pmatrix}
N_x(x_1, y_1) & N_y(x_1, y_1) & 1 \\
N_x(x_2, y_2) & N_y(x_2, y_2) & 1 \\
\vdots & \vdots & \vdots \\
N_x(x_n, y_n) & N_y(x_n, y_n) & 1
\end{pmatrix}
\begin{pmatrix}
S_x \\
S_y \\
A
\end{pmatrix}
= 
\begin{pmatrix}
I(x_1, y_1) \\
I(x_2, y_2) \\
\vdots \\
I(x_n, y_n)
\end{pmatrix}
\]

Finding the direction of the light source

Application: Detecting composite photos

Fake photo

Real photo
Next time: Color

Phillip Otto Runge (1777-1810)