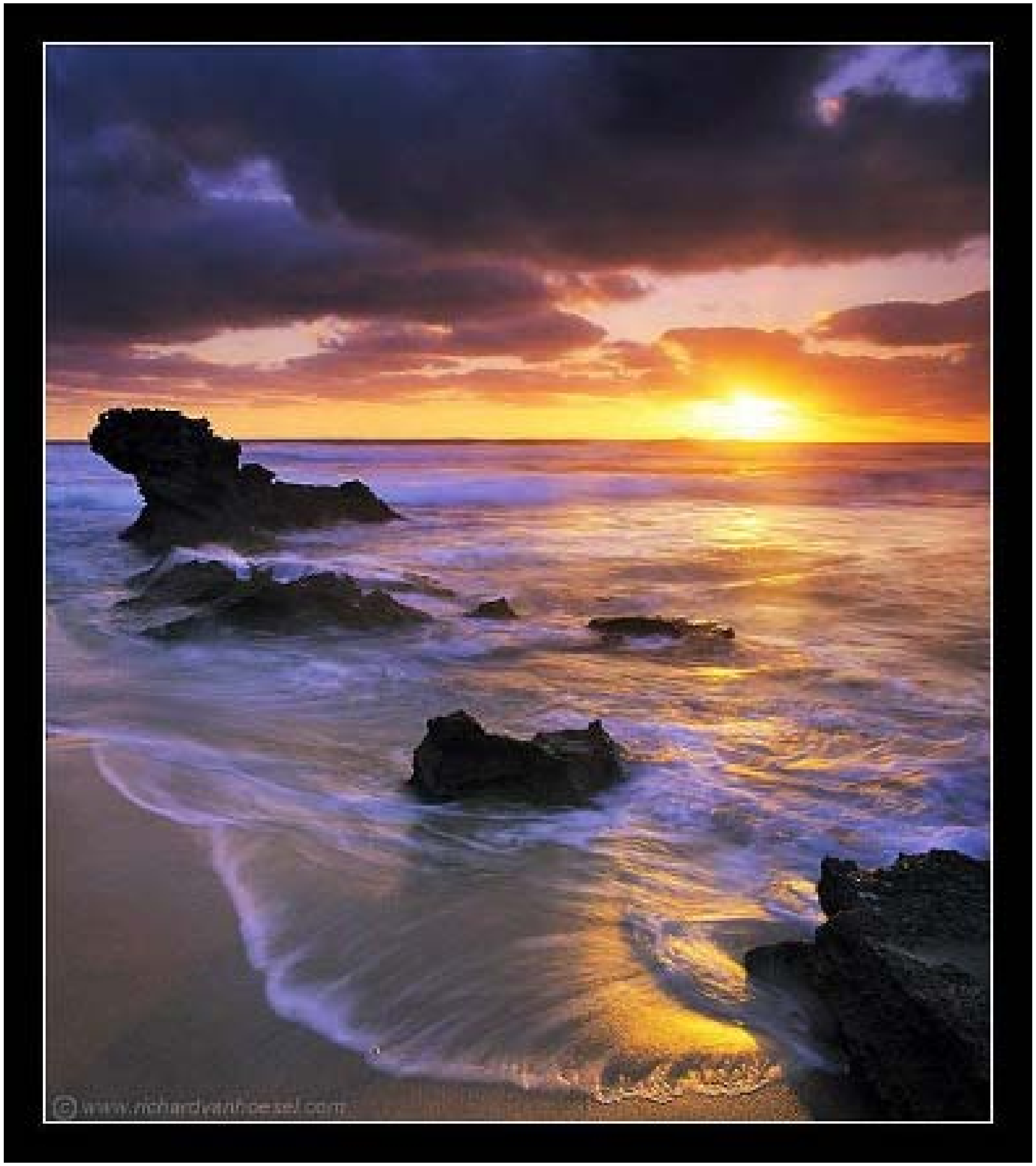
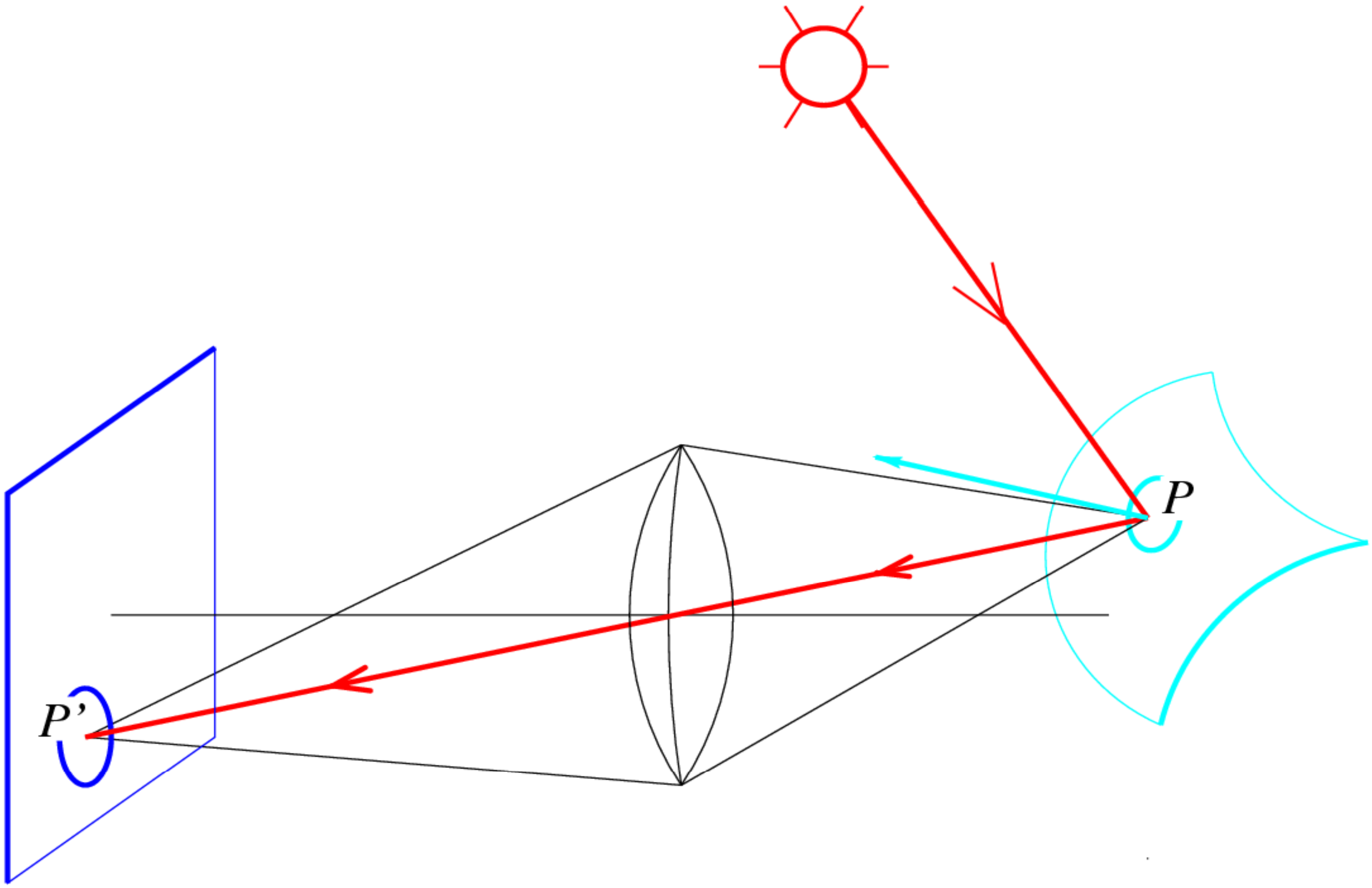
Capturing light



Source: A. Efros

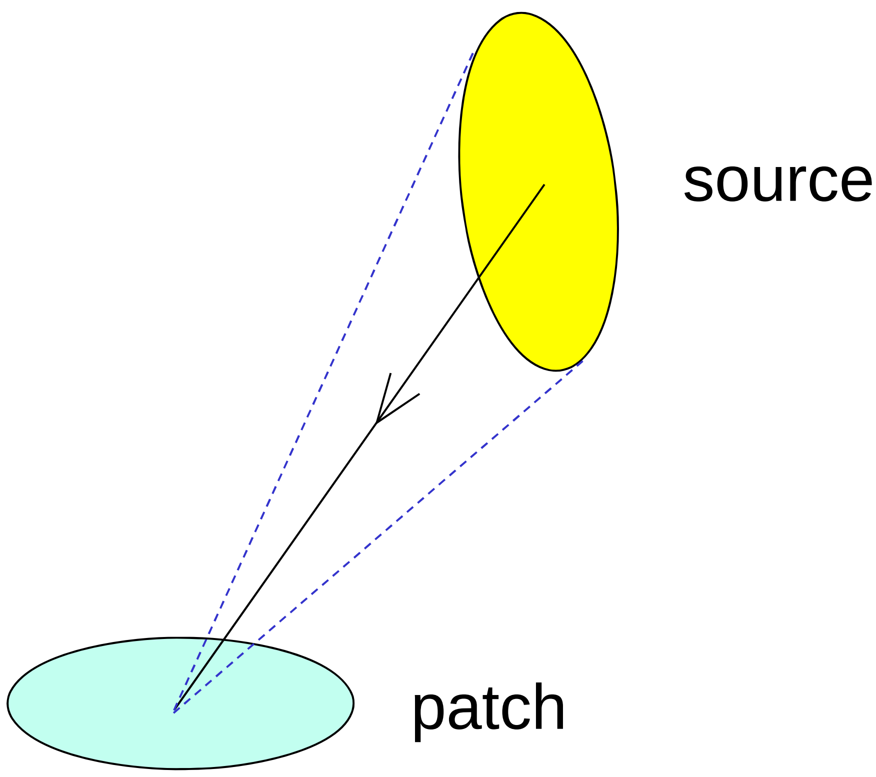
Image formation

How bright is the image of a scene point?



Radiometry: Measuring light

- The basic setup: a light source is sending radiation to a surface patch
- What matters:
  - How big the source and the patch “look” to each other

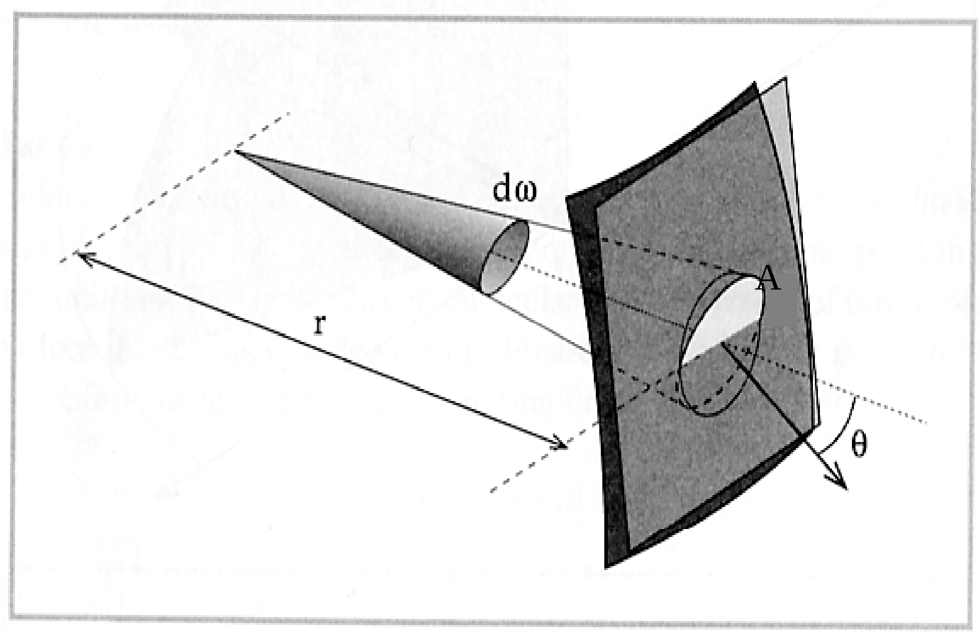


Solid Angle

- The solid angle subtended by a region at a point is the area projected on a unit sphere centered at that point
  - Units: steradians

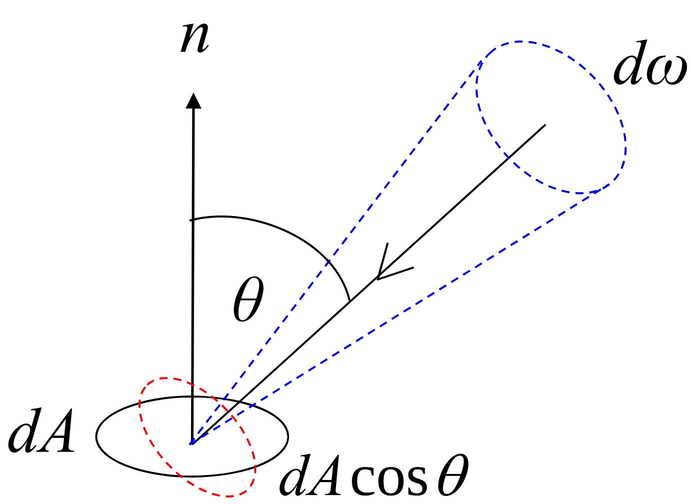
- The solid angle $d\omega$ subtended by a patch of area *dA* is given by:

$$d\omega = \frac{dA\cos\theta}{r^2}$$



Radiance

- Radiance (*L*): energy carried by a ray
  - Power per unit area perpendicular to the direction of travel, per unit solid angle
  - Units: Watts per square meter per steradian (W m^{-2} sr^{-1})

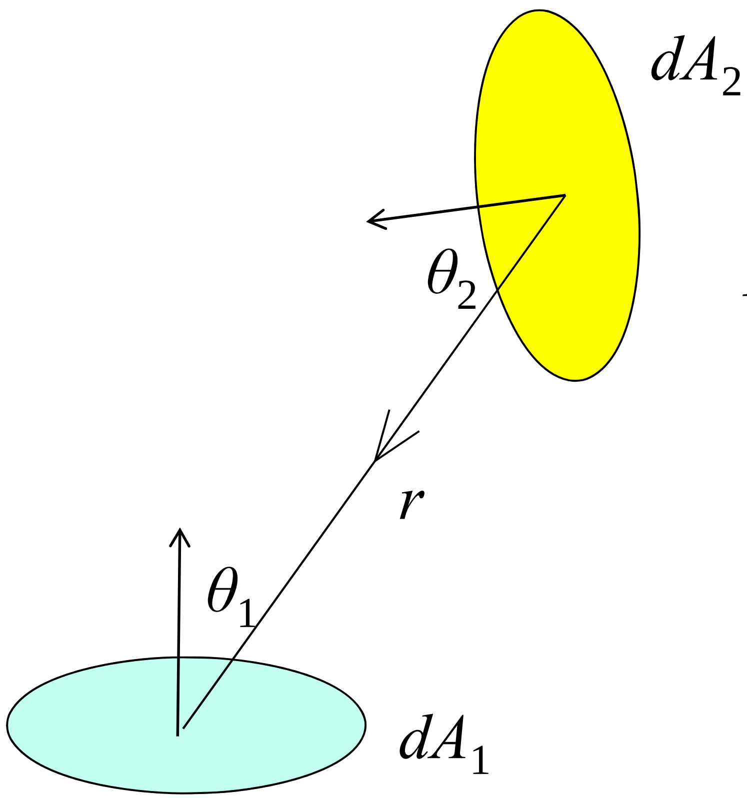


$$L = \frac{P}{dA \cos\theta \, d\omega}$$

$$P = L \, dA \cos\theta \, d\omega$$

Radiance

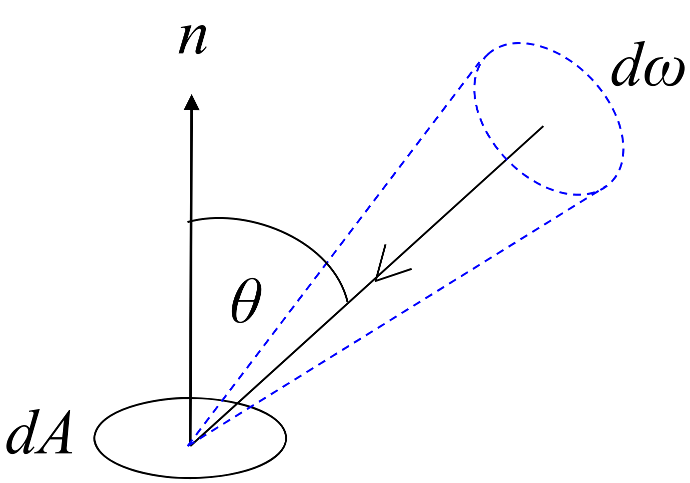
- The roles of the patch and the source are essentially symmetric



$$
\begin{aligned}
P &= L\, dA_1 \cos\theta_1\, d\omega_2 \\
&= L\, dA_2 \cos\theta_2\, d\omega_1 \\
&= \frac{L\, dA_1\, dA_2 \cos\theta_1 \cos\theta_2}{r^2}
\end{aligned}
$$

Irradiance

- Irradiance (*E*): energy arriving at a surface
  - Incident power per unit area *not foreshortened*
  - Units: W m^{-2}
  - For a surface receiving radiance *L* coming in from $d\omega$ the corresponding irradiance is



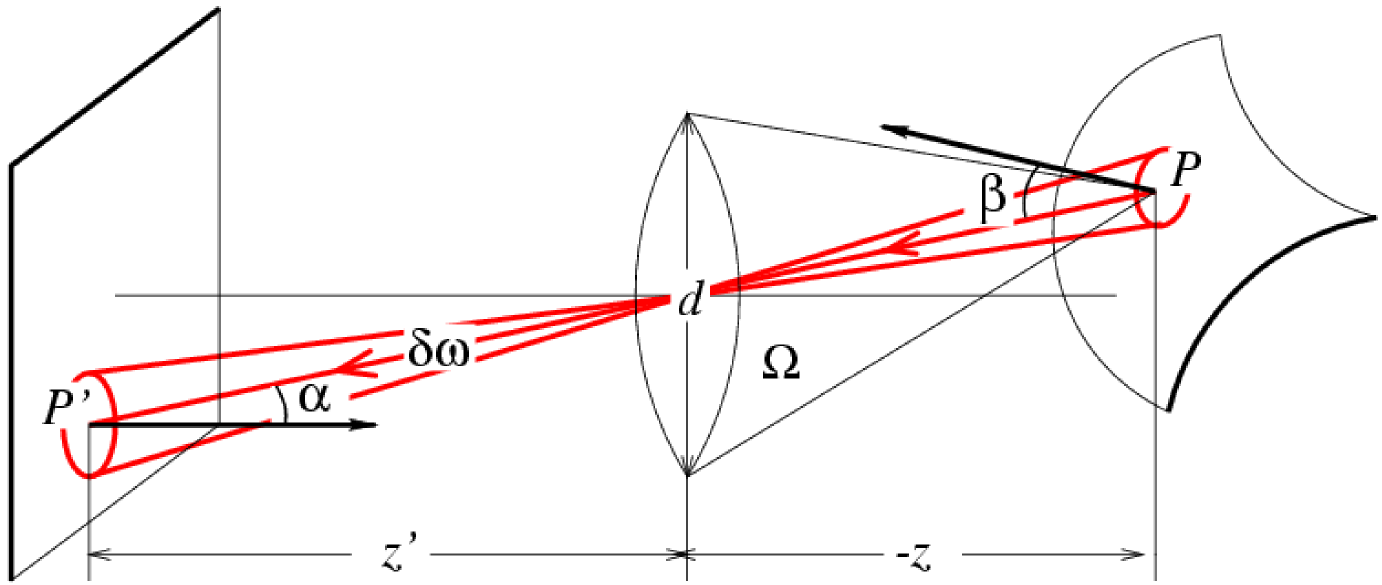
$$E = \frac{P}{dA} = \frac{L\, dA \cos\theta\, d\omega}{dA}$$

$$= L \cos\theta\, d\omega$$

Radiometry of thin lenses

L: Radiance emitted from *P* toward *P*’

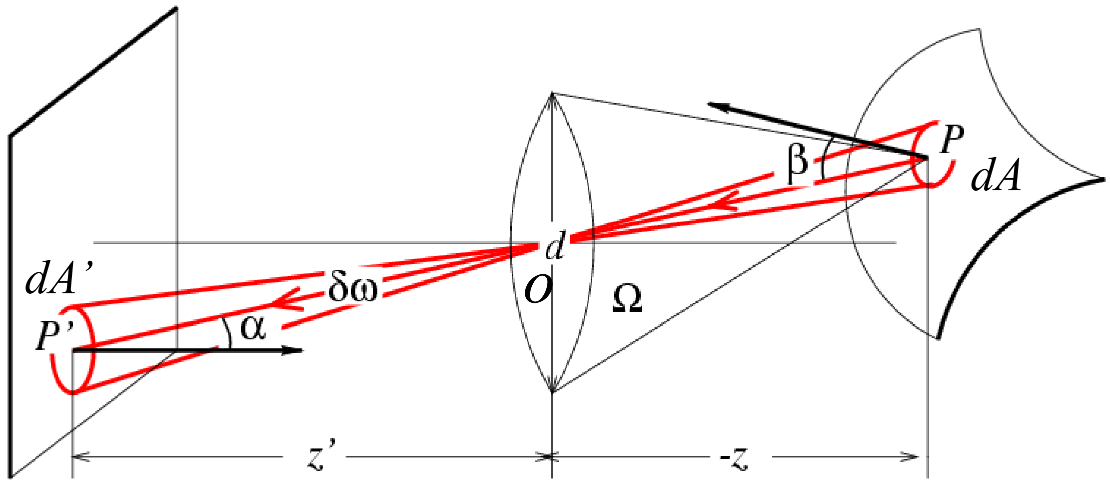
E: Irradiance falling on *P*’ from the lens



What is the relationship between *E* and *L*?

Forsyth & Ponce, Sec. 4.2.3

Radiometry of thin lenses



$$|OP| = \frac{z}{\cos\alpha}$$

$$|OP'| = \frac{z'}{\cos\alpha}$$

Area of the lens: $\frac{\pi d^2}{4}$

The power δP received by the lens from *P* is $\delta P = L\left(\frac{\pi d^2}{4}\right)\cos\alpha\,\delta\omega$

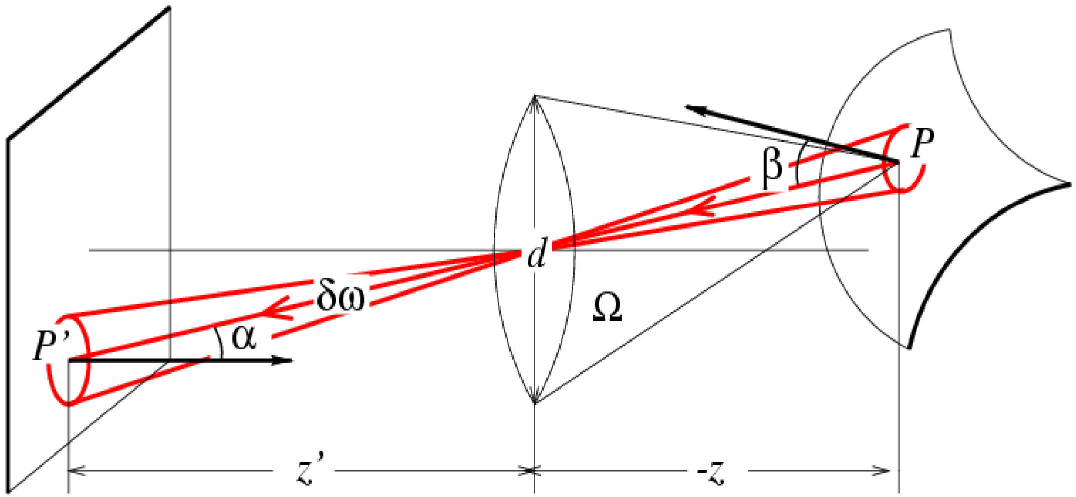
The radiance emitted from the lens towards P' is $\frac{\delta P}{\left(\frac{\pi d^2}{4}\right)\cos\alpha\,\delta\omega} = L$

The irradiance received at P' is

$$E = L\cos\alpha\underbrace{\left(\frac{\pi d^2\cos\alpha}{4(z'/\cos\alpha)^2}\right)}_{\text{Solid angle subtended by the lens at } P'} = \left[\frac{\pi}{4}\left(\frac{d}{z'}\right)^2\cos^4\alpha\right]L$$

Solid angle subtended by the lens at *P'*

Radiometry of thin lenses



$$E = \left[\frac{\pi}{4} \left(\frac{d}{z'} \right)^2 \cos^4 \alpha \right] L$$

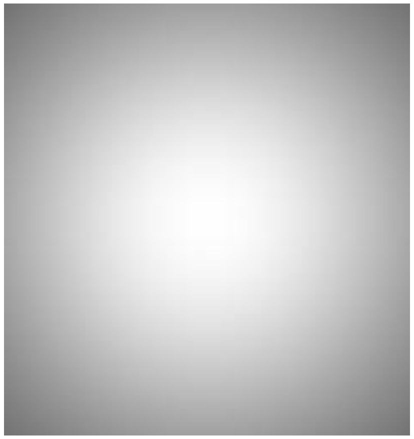
- Image irradiance is linearly related to scene radiance
- Irradiance is proportional to the area of the lens and inversely proportional to the squared distance between the lens and the image plane
- The irradiance falls off as the angle between the viewing ray and the optical axis increases

Forsyth & Ponce, Sec. 4.2.3

Radiometry of thin lenses

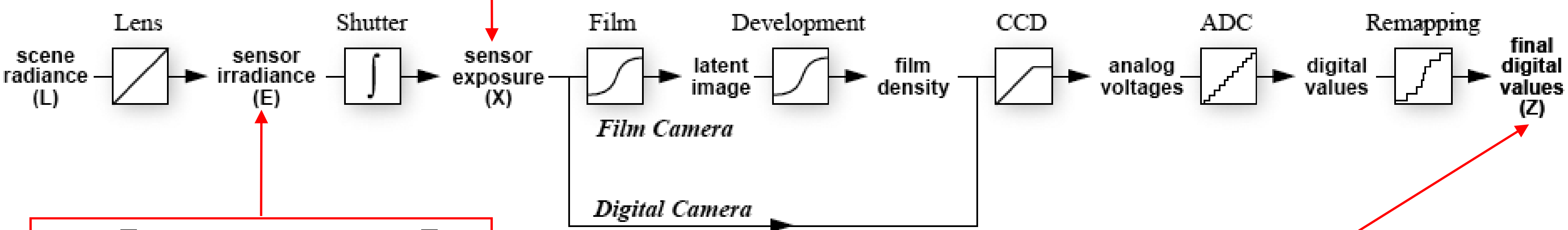
$$E = \left[\frac{\pi}{4} \left(\frac{d}{z'} \right)^2 \cos^4 \alpha \right] L$$

- Application:
  - S. B. Kang and R. Weiss, *Can we calibrate a camera using an image of a flat, textureless Lambertian surface?* ECCV 2000.



From light rays to pixel values

$$X = E \cdot \Delta t$$



$$E = \left[\frac{\pi}{4} \left(\frac{d}{z'} \right)^2 \cos^4 \alpha \right] L$$

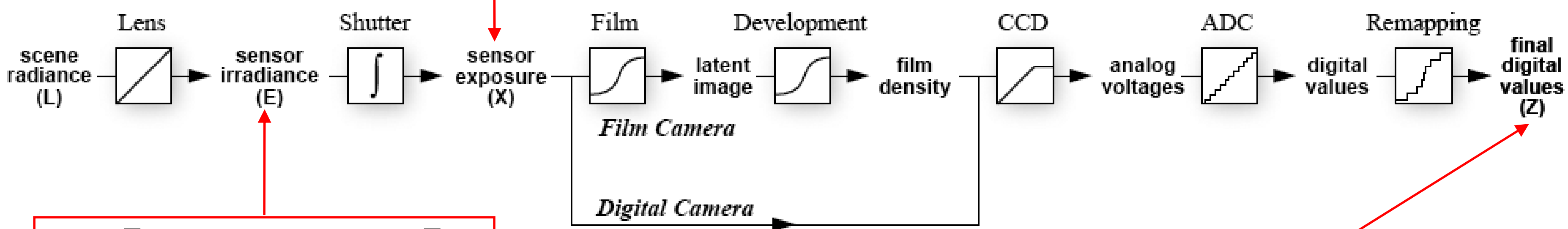
$$Z = f\left(E \cdot \Delta t\right)$$

- Camera response function: the mapping f from irradiance to pixel values
    - Useful if we want to estimate material properties
    - Enables us to create high dynamic range images

Source: S. Seitz, P. Debevec

From light rays to pixel values

$$X = E \cdot \Delta t$$



$$E = \left[\frac{\pi}{4} \left(\frac{d}{z'} \right)^2 \cos^4 \alpha \right] L$$

$$Z = f\left(E \cdot \Delta t\right)$$

- Camera response function: the mapping f from irradiance to pixel values

For more info

- P. E. Debevec and J. Malik. *Recovering High Dynamic Range Radiance Maps from Photographs*. In *SIGGRAPH 97*, August 1997

Source: S. Seitz, P. Debevec

The interaction of light and surfaces

What happens when a light ray hits a point on an object?

- Some of the light gets **absorbed**
  - converted to other forms of energy (e.g., heat)
- Some gets **transmitted** through the object
  - possibly bent, through "refraction"
  - Or scattered inside the object (subsurface scattering)
- Some gets **reflected**
  - possibly in multiple directions at once
- Really complicated things can happen
  - fluorescence

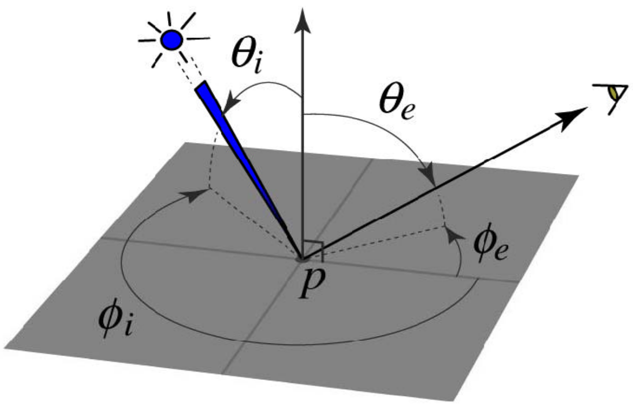
Let's consider the case of reflection in detail

- Light coming from a single direction could be reflected in all directions. How can we describe the amount of light reflected in each direction?

Slide by Steve Seitz

Bidirectional reflectance distribution function (BRDF)

- Model of local reflection that tells how bright a surface appears when viewed from one direction when light falls on it from another
- Definition: ratio of the **radiance** in the **emitted** direction to **irradiance** in the **incident** direction

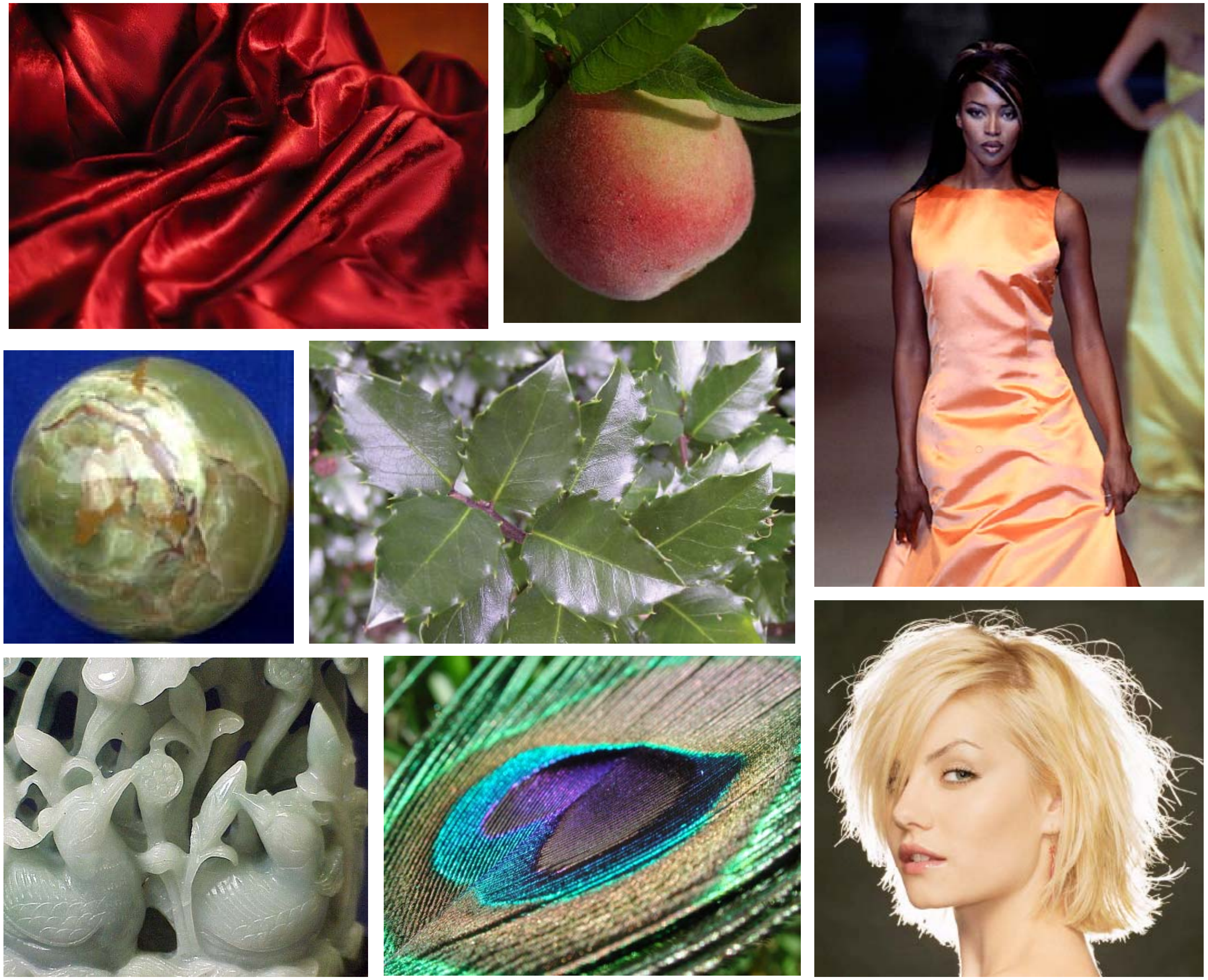


$$\rho(\theta_i, \phi_i, \theta_e, \phi_e) = \frac{L_e(\theta_e, \phi_e)}{E_i(\theta_i, \phi_i)} = \frac{L_e(\theta_e, \phi_e)}{L_i(\theta_i, \phi_i) \cos \theta_i d\omega}$$

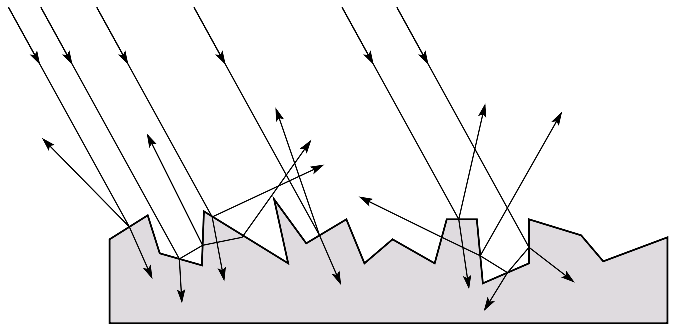
- Radiance leaving a surface in a particular direction: integrate radiances from every incoming direction scaled by BRDF:

$$\int_{\Omega} \rho(\theta_i, \phi_i, \theta_e, \phi_e,) L_i(\theta_i, \phi_i) \cos \theta_i d\omega_i$$

BRDFs can be incredibly complicated…



Diffuse reflection

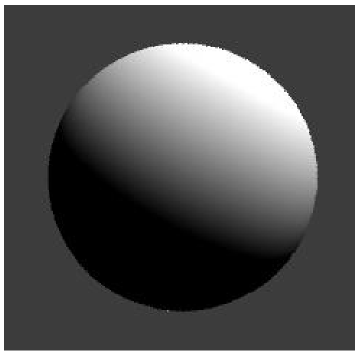


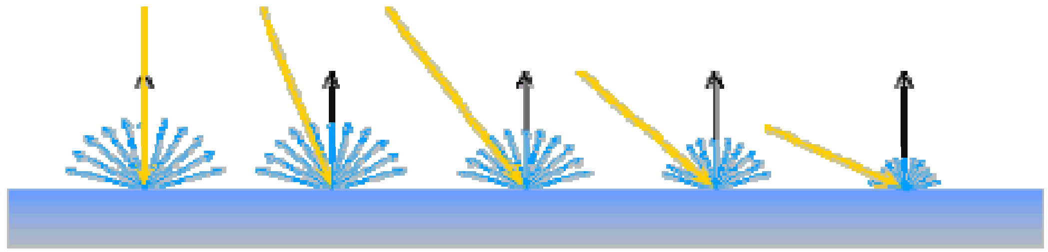
- Light is reflected equally in all directions
  - Dull, matte surfaces like chalk or latex paint
  - Microfacets scatter incoming light randomly

- **BRDF is constant**
  - *Albedo*: fraction of incident irradiance reflected by the surface
  - *Radiosity:* total power leaving the surface per unit area (regardless of direction)

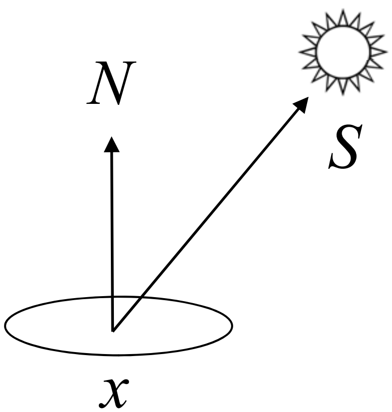
Diffuse reflection: Lambert's law

- Viewed brightness does not depend on viewing direction, but it *does* depend on direction of illumination





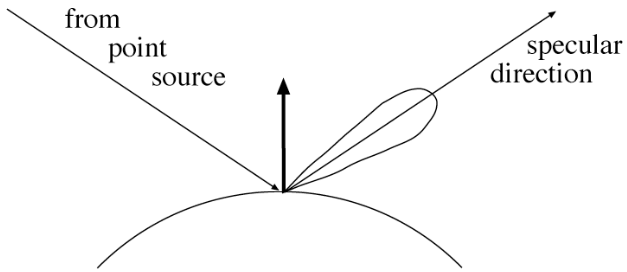
$$B(x) = \rho(x)\big(N(x)\cdot S(x)\big)$$

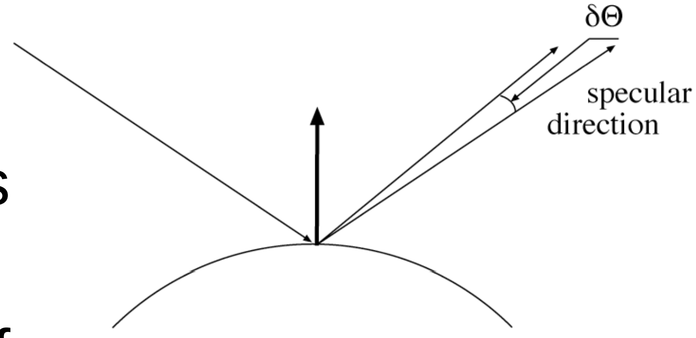


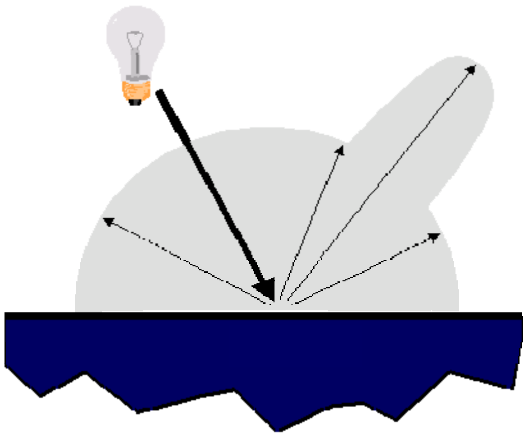
B: radiosity
ρ: albedo
N: unit normal
S: source vector (magnitude proportional to intensity of the source)

Specular reflection

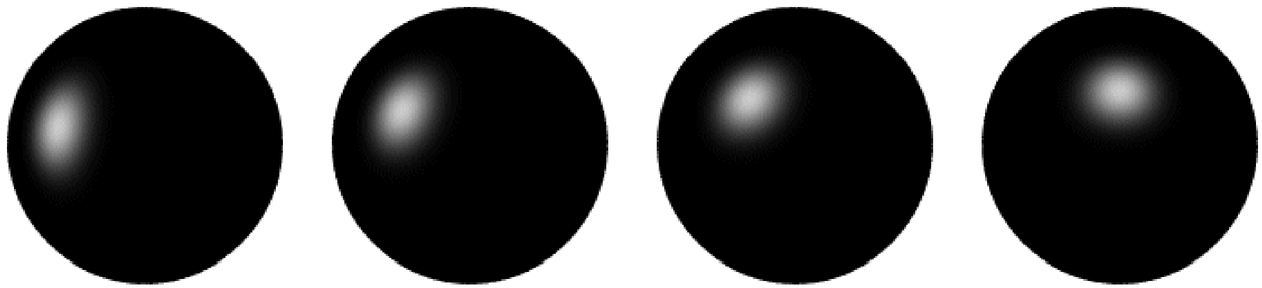
- Radiation arriving along a source direction leaves along the specular direction (source direction reflected about normal)
- Some fraction is absorbed, some reflected
- On real surfaces, energy usually goes into a lobe of directions
- Phong model: reflected energy falls of with $\cos^n(\delta\theta)$
- Lambertian + specular model: sum of diffuse and specular term



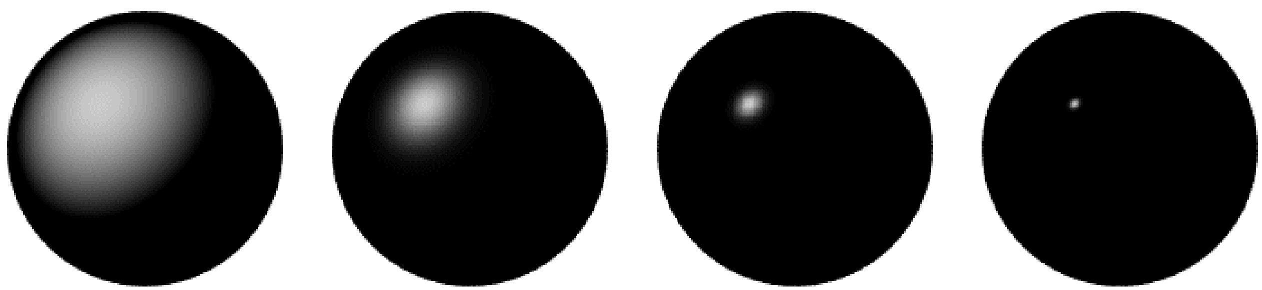




Specular reflection



Moving the light source



Changing the exponent

Photometric stereo (shape from shading)

- Can we reconstruct the shape of an object based on shading cues?

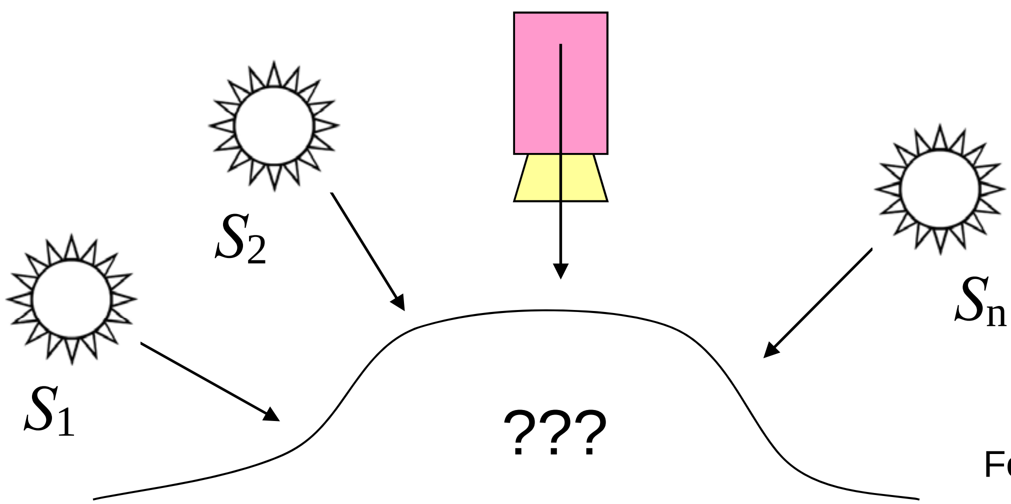
Luca della Robbia,
Cantoria, 1438

Photometric stereo

Assume:

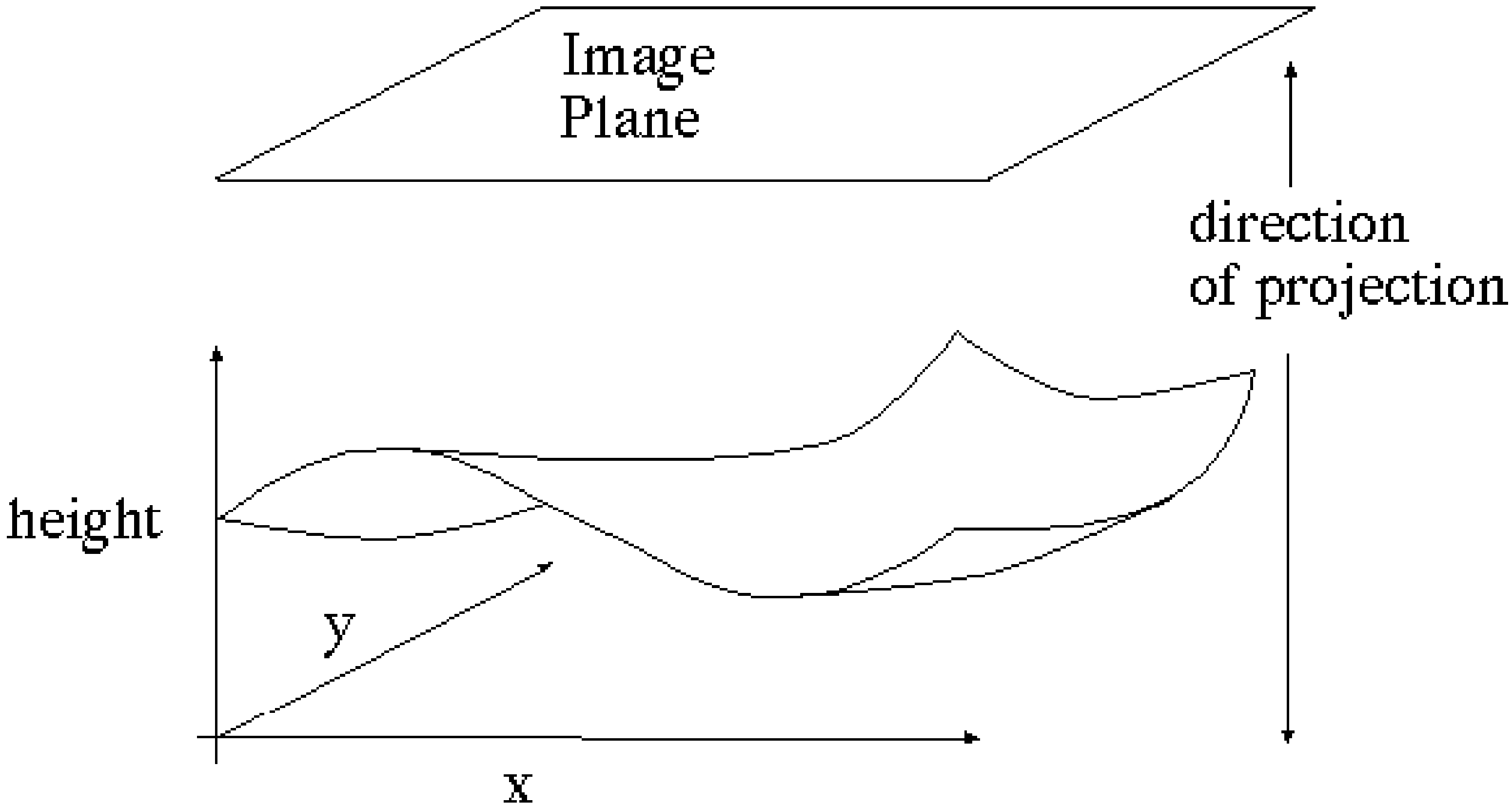
- A Lambertian object
- A *local shading model* (each point on a surface receives light only from sources visible at that point)
- A set of *known* light source directions
- A set of pictures of an object, obtained in exactly the same camera/object configuration but using different sources
- Orthographic projection

Goal: reconstruct object shape and albedo



Forsyth & Ponce, Sec. 5.4

Surface model: Monge patch



Forsyth & Ponce, Sec. 5.4

Image model

- Known: source vectors S_j and pixel values $I_j(x,y)$
- We also assume that the response function of the camera is a linear scaling by a factor of k
- Combine the unknown normal $N(x,y)$ and albedo $\rho(x,y)$ into one vector g, and the scaling constant k and source vectors S_j into another vector V_j:

$$
\begin{aligned}
I_j(x,y) &= k\,B(x,y) \\
&= k\,\rho(x,y)\left(N(x,y)\cdot S_j\right) \\
&= \left(\rho(x,y)N(x,y)\right)\cdot(k\,S_j) \\
&= g(x,y)\cdot V_j
\end{aligned}
$$

Forsyth & Ponce, Sec. 5.4

Least squares problem

- For each pixel, we obtain a linear system:

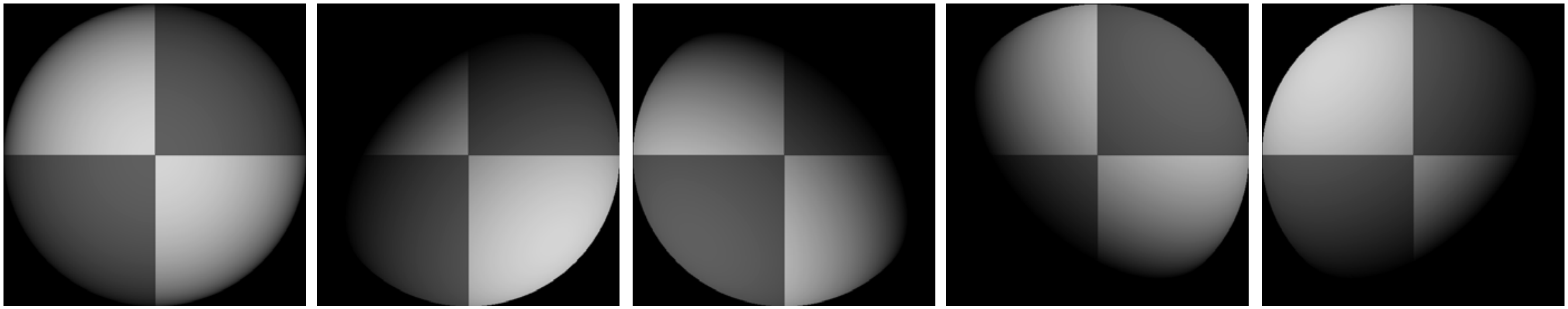
$$\begin{bmatrix} I_1(x,y) \\ I_2(x,y) \\ \vdots \\ I_n(x,y) \end{bmatrix} = \begin{bmatrix} V_1^T \\ V_2^T \\ \vdots \\ V_n^T \end{bmatrix} g(x,y)$$

($n \times 1$) known — ($n \times 3$) known — (3×1) unknown

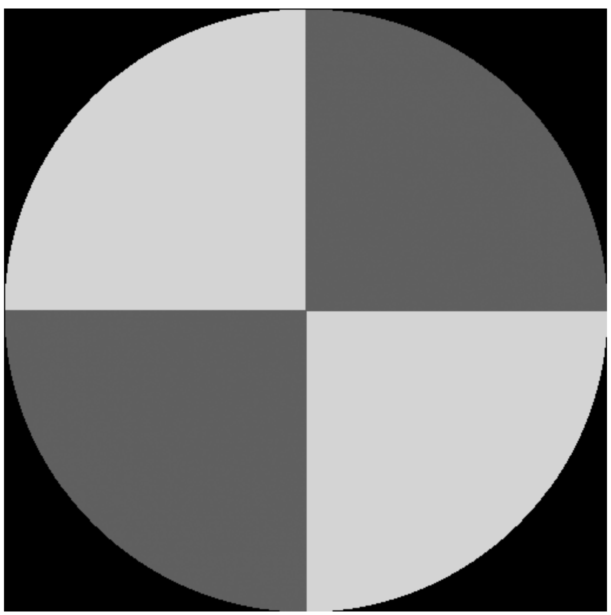
- Obtain least-squares solution for $g(x,y)$
- Since $N(x,y)$ is the unit normal, $\rho(x,y)$ is given by the magnitude of $g(x,y)$ (and it should be less than 1)
- Finally, $N(x,y) = g(x,y) / \rho(x,y)$

Forsyth & Ponce, Sec. 5.4

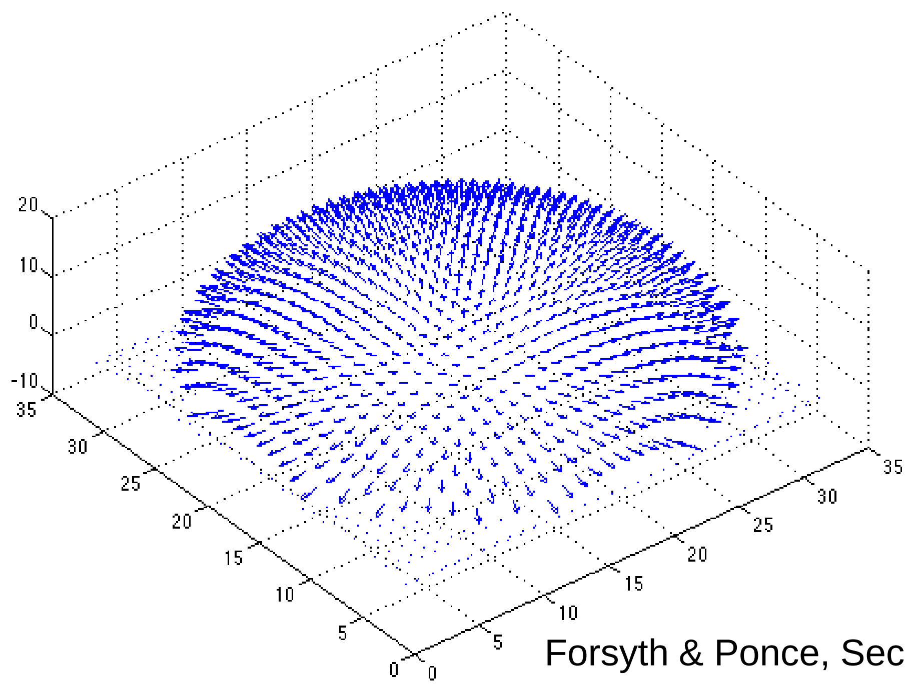
Example



Recovered albedo



Recovered normal field



Forsyth & Ponce, Sec. 5.4

Recovering a surface from normals

Recall the surface is written as

$$(x, y, f(x, y))$$

This means the normal has the form:

$$N(x,y) = \left(\frac{1}{\sqrt{f_x^2 + f_y^2 + 1}} \right) \begin{pmatrix} -f_x \\ -f_y \\ 1 \end{pmatrix}$$

If we write the estimated vector *g* as

$$\mathbf{g}(x,y) = \begin{pmatrix} g_1(x,y) \\ g_2(x,y) \\ g_3(x,y) \end{pmatrix}$$

Then we obtain values for the partial derivatives of the surface:

$$f_x(x,y) = \left(g_1(x,y) / g_3(x,y) \right)$$

$$f_y(x,y) = \left(g_2(x,y) / g_3(x,y) \right)$$

Forsyth & Ponce, Sec. 5.4

Recovering a surface from normals

Integrability: for the surface f to exist, the mixed second partial derivatives must be equal:

$$\frac{\partial\left(g_1(x,y)/g_3(x,y)\right)}{\partial y} =$$

$$\frac{\partial\left(g_2(x,y)/g_3(x,y)\right)}{\partial x}$$

(in practice, they should at least be similar)

We can now recover the surface height at any point by integration along some path, e.g.

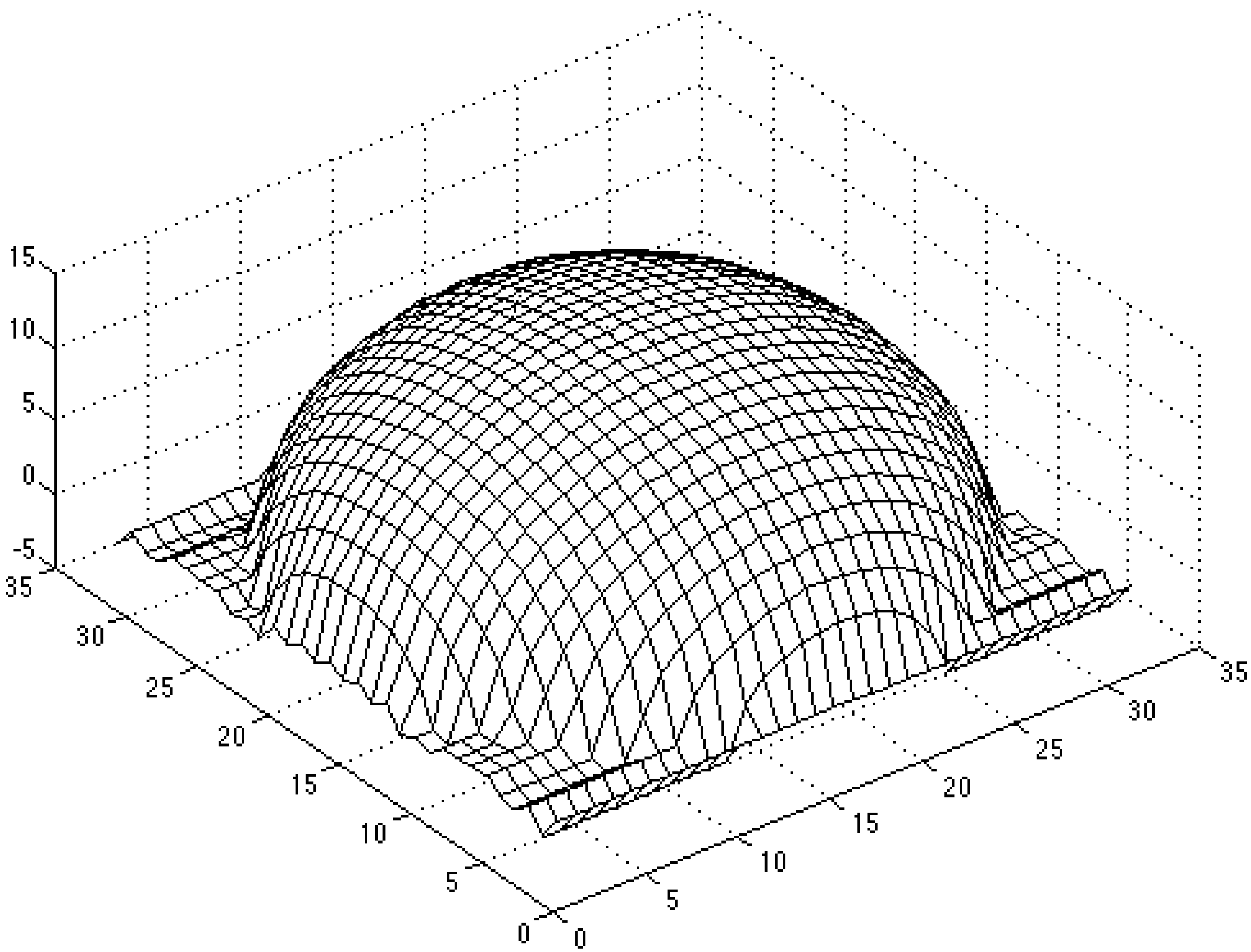
$$f(x,y) = \int_0^x f_x(s,y)\,ds +$$

$$\int_0^y f_y(x,t)\,dt + c$$

(for robustness, can take integrals over many different paths and average the results)

Forsyth & Ponce, Sec. 5.4

Surface recovered by integration



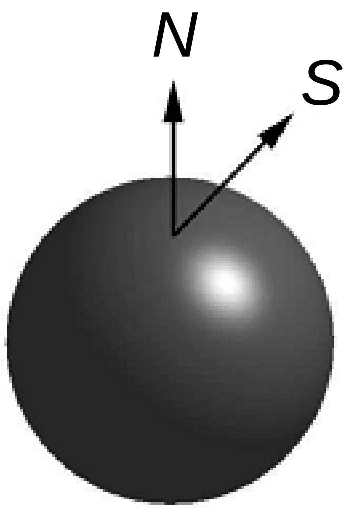
Forsyth & Ponce, Sec. 5.4

Limitations

- Orthographic camera model
- Simplistic reflectance and lighting model
- No shadows
- No interreflections
- No missing data
- Integration is tricky

Finding the direction of the light source

$$I(x,y) = N(x,y) \cdot S(x,y) + A$$



Full 3D case:

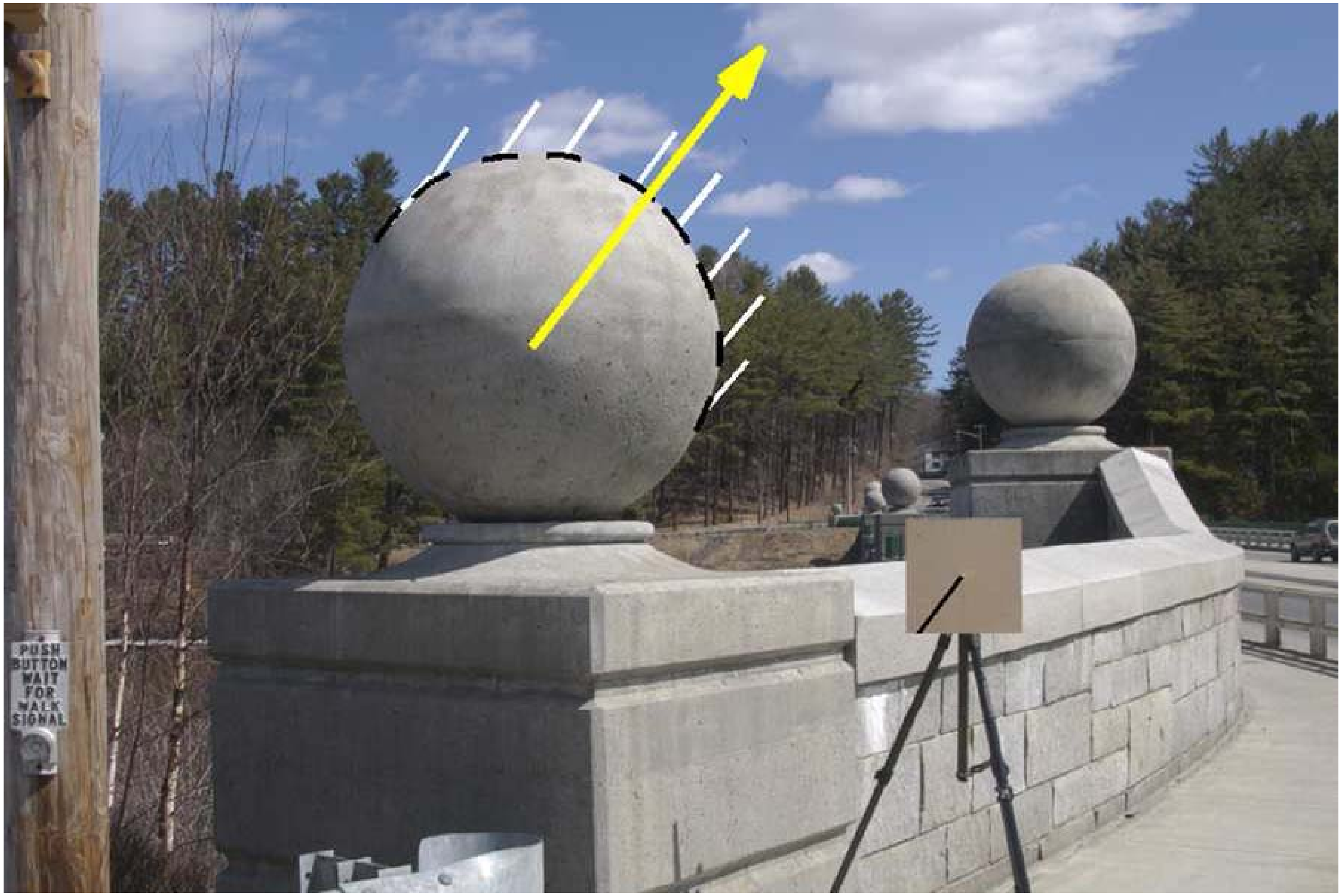
$$\begin{pmatrix} N_x(x_1,y_1) & N_y(x_1,y_1) & N_z(x_1,y_1) & 1 \\ N_x(x_2,y_2) & N_y(x_2,y_2) & N_z(x_2,y_2) & 1 \\ \vdots & \vdots & \vdots & \vdots \\ N_x(x_n,y_n) & N_y(x_n,y_n) & N_z(x_n,y_n) & 1 \end{pmatrix} \begin{pmatrix} S_x \\ S_y \\ S_z \\ A \end{pmatrix} = \begin{pmatrix} I(x_1,y_1) \\ I(x_2,y_2) \\ \vdots \\ I(x_n,y_n) \end{pmatrix}$$

For points on the *occluding contour*:

$$\begin{pmatrix} N_x(x_1,y_1) & N_y(x_1,y_1) & 1 \\ N_x(x_2,y_2) & N_y(x_2,y_2) & 1 \\ \vdots & \vdots & \vdots \\ N_x(x_n,y_n) & N_y(x_n,y_n) & 1 \end{pmatrix} \begin{pmatrix} S_x \\ S_y \\ A \end{pmatrix} = \begin{pmatrix} I(x_1,y_1) \\ I(x_2,y_2) \\ \vdots \\ I(x_n,y_n) \end{pmatrix}$$

P. Nillius and J.-O. Eklundh, “Automatic estimation of the projected light source direction,” CVPR 2001

Finding the direction of the light source



P. Nillius and J.-O. Eklundh, “Automatic estimation of the projected light source direction,” CVPR 2001

Application: Detecting composite photos

Fake photo

Real photo