QUATERNION-BASED SPARSE REPRESENTATION OF COLOR IMAGE

Licheng Yu, Yi Xu, Hongteng Xu, Hao Zhang

Department of Electronic Engineering, Shanghai Jiaotong University, Shanghai 200240, China
Shanghai Key Lab of Digital Media Processing and Communication
xuyi@sjtu.edu.cn

ABSTRACT

In this paper, we propose a quaternion-based sparse representation model for color images and its corresponding dictionary learning algorithm. Differing from traditional sparse image models, which represent RGB channels separately or process RGB channels as a concatenated real vector, the proposed model describes the color image as a quaternion vector matrix, where each color pixel is encoded as a quaternion unit and thus the inter-relationship among RGB channels is well preserved. Correspondingly, we propose a quaternion-based dictionary learning algorithm using a so-called K-QSVD method. It conducts the sparse basis selection in quaternion vector space, providing a kind of vectorial representation for the inherent color structures rather than a scalar representation via current sparse image models. The proposed sparse model is validated in the applications of color image denoising and inpainting. The experimental results demonstrate that our sparse image model avoids the hue bias phenomenon successfully and shows its potential as a powerful tool in color image analysis and processing domain.

***Index Terms*—** Quaternion, sparse representation, color images, denoising, inpainting, dictionary learning

1. INTRODUCTION

The theory of sparse representation has been proven as an effective model of image signal. Using a specific dictionary that contains a certain amount of atoms as its columns, we can represent an image signal as a sparse linear combination of these atoms. The dictionary can be pre-defined as a basis or frame, e.g, wavelets, curvelets, or a combination of several bases [1]. In recent years, the learning-based strategies for designing dictionary are proposed [2, 3]. The study of sparse representation has led to many applications in image processing and computer vision realm, such as compression [4], denoising [5], inpainting [1, 3], image classification [6] and object recognition [7].

There exists a range of empirical evidence for the significant impacts of sparse representation methods, however, few works discuss about the sparse representation model of color image. The works in [1, 3, 5] have achieved the state-of-the-art performance on gray-scale images. As for color images, they just treat RGB channels as three independent "gray-scale" images and process them in a monochrome way. It totally ignores the inter-relationship among the three channels, which is likely to produce hue distortions in the reconstruction results. Some works proposed to concatenate RGB channels to alleviate the hue distortion problem [8], where a dictionary that can jointly represent the channels is trained. However, the lack of explicit constraints on the correlations among RGB channels limits its development.

In this paper, we provide a view of expressing color images using quaternion-based sparse representation, where a color pixel is encoded as a quaternion unit and processed as vectors. In contrast, current sparse models consider a color pixel as a scalar quantity, losing the inter-relationship among the color channels during reconstruction. The use of quaternion in image processing has been proposed and studied in [9, 10, 11]. These works show the successful extension in color image filtering, cross-correlation, and compression, proving the advantages of quaternion-based model in describing color signals. However, the reason for these success is lack of investigation. The main contributions of our work exist in establishing a quaternion-based sparse representation model for color images and proposing the corresponding dictionary training method. Moreover, we give an interpretation for the effectiveness of the proposed model — the linear correlations among RGB channels are preserved consistently during dictionary training process. It is also validated in the experimental results of color image processing tasks of denoising and inpainting.

The rest of this paper is organized as follows. Section 2 introduces the quaternion-based sparse representation model. Section 3 shows the training method of the quaternion-based dictionary. Section 4 gives the experimental results for color image reconstruction, denoising and inpainting. Finally, it concludes in section 5 that the proposed sparse image model is a powerful tool in color image analysis and processing domain.

This work is supported in part by NSFC (60932006, 60902073), the 111 project (B07022), STCSM (12DZ2272600) and the High Technology Research and Development Program of China (2011AA01A107, 2012AA011702).

2. QUATERNION-BASED SPARSE REPRESENTATION MODEL FOR COLOR IMAGES

Given a color image $\mathbf{I}$, we denote the vector form of a patch in each channel as $\mathbf{y}_c \in \mathbb{R}^n$, where n is the dimension of the vector, while $c = r, g, b$ represents the RGB channels respectively. As aforementioned, current sparse models of color image are to deal with each single channel independently with possibly different dictionaries, which can be written as

$$\mathbf{y}_c = \mathbf{D}_c\alpha_c, \quad c = r, g, b \tag{1}$$

where $\mathbf{D}_c$ is the dictionary, and α_c is the coefficient vector which is assumed to be sparse. This monochrome processing strategy, however, fails to capture the inter-relationship among the RGB color channels.

A moderate improvement is to process the concatenation of the three channels [8]. The corresponding representation model can be expressed as

$$[\mathbf{y}_r^T, \mathbf{y}_g^T, \mathbf{y}_b^T]^T = [\mathbf{D}_r^T, \mathbf{D}_g^T, \mathbf{D}_b^T]^T\alpha. \tag{2}$$

The model in (2) adds a constraint on the coefficient—the coefficient vector should not only be sparse but also be shared by each channel. As a result, the correlations among channels are implied in the structure of dictionary. However, the results in [8] show that the dictionary $[\mathbf{D}_r^T, \mathbf{D}_g^T, \mathbf{D}_b^T]^T$ trained by K-SVD tends to be monochromatic. In other words, the interrelationship among color channels are not well preserved in each atom during training process.

The linear correlations among RGB channels are considered in [12, 13], which is proven to be useful in demosaicing [13] and color constancy [12]. This assumption suggests the sparse model for color image be

$$\mathbf{y}_c = [\mathbf{D}_r, \mathbf{D}_g, \mathbf{D}_b]\alpha_c, \quad c = r, g, b. \tag{3}$$

In such case, the final representation result is a combination of atoms from three dictionaries. Rather than simply increasing the dictionary dimension in model (1), this model implies that the three channels should be in a linear correlation. However, the difficulty of combining linear correlation constraint with the sparse model arises during training dictionary — getting $\mathbf{D}_r, \mathbf{D}_g, \mathbf{D}_b$ respectively is a clustering task [14], which depends on the distributions of nonzero coefficients of training samples. Introducing the clustering algorithm into sparse model would severely increase the complexity .

According to the analysis above, the implicit constraints of the color channels are not sufficient for the sparse model of color image. It is necessary to impose some explicit constraints on the sparse model for preserving color channels' correlation, especially during the dictionary training phase. Inspired by the theory of quaternion algebra, we find that the quaternion-based sparse model can provide us a series of constraints on the dictionaries and coefficients, well preserving the interrelationship among RGB channels.

The quaternion vector form of the patch of color image is denoted as $\dot{\mathbf{y}} = \mathbf{0} + \mathbf{y}_r i + \mathbf{y}_g j + \mathbf{y}_b k$. Here $\dot{\mathbf{y}} \in \mathbb{H}^n$ ($\mathbb{H}$ denotes the quaternion field), is a n-dimension quaternion vector, and each component has 4 parts, i.e., one real part and three imaginary parts. Pure quaternions are used to represent the red, green and blue channels[11], with the real part set to be zero [1]. Consequently, the dictionary and the corresponding coefficient can be represented as $\dot{\mathbf{D}} = \mathbf{0} + \mathbf{D}_r i + \mathbf{D}_g j + \mathbf{D}_b k$ and $\dot{\alpha} = \alpha_0 + \alpha_1 i + \alpha_2 j + \alpha_3 k$, respectively. So, the quaternion-based sparse representation model can be written as

$$\min_{\dot{\alpha}} \|\dot{\alpha}\|_0, \quad s.t. \quad \dot{\mathbf{y}} = \dot{\mathbf{D}}\dot{\alpha}, \tag{4}$$

where $\dot{\mathbf{D}} \in \mathbb{H}^{n\times K}$ is the quaternion dictionary, consisting of K pure quaternion atoms. And $\dot{\alpha} \in \mathbb{H}^K$ is a sparse quaternion vector. The objective function $\|\dot{\alpha}\|_0$ counts the number of nonzero components in the quaternion vector. Note all the algebra operations are under quaternion system, and some rules are listed in the supplementary file.

Besides the sparsity of coefficient vector, the main advantage of the model in (4) is highlighted in the fact that the coefficient vector obeys the following constraints,

$$\begin{cases} \mathbf{0} = -\mathbf{D}_r\alpha_1 - \mathbf{D}_g\alpha_2 - \mathbf{D}_b\alpha_3, \\ \mathbf{y}_r = \mathbf{D}_r\alpha_0 + \mathbf{D}_g\alpha_3 - \mathbf{D}_b\alpha_2, \\ \mathbf{y}_g = -\mathbf{D}_r\alpha_3 + \mathbf{D}_g\alpha_0 + \mathbf{D}_b\alpha_1, \\ \mathbf{y}_b = \mathbf{D}_r\alpha_2 - \mathbf{D}_g\alpha_1 + \mathbf{D}_b\alpha_0. \end{cases} \tag{5}$$

The equations above enforce explicit constraints on the correlations among RGB channels — each $\mathbf{y}_c$ is correlated linearly with three channel dictionaries and the coefficients should be in the null space of $[\mathbf{D}_r, \mathbf{D}_g, \mathbf{D}_b]$. By training the quaternion dictionary $\dot{\mathbf{D}}$ in a proper way, the interrelationship of the three channels for color patches $\mathbf{y}_c$ can be well preserved.

3. QUATERNION-BASED DICTIONARY TRAINING

The quaternion-based dictionary training process is an extension of the model in (4), in which both the dictionary and coefficients are unknown variables. This process can be formulated as

$$\{\hat{\dot{\mathbf{A}}}, \hat{\dot{\mathbf{D}}}\} = \arg\min_{\dot{\mathbf{D}}} \|\dot{\mathbf{Y}} - \dot{\mathbf{D}}\dot{\mathbf{A}}\|_2 + \lambda\|\dot{\mathbf{A}}\|_0, \tag{6}$$

where $\dot{\mathbf{Y}}^{n\times N} = \{\dot{\mathbf{y}}_i, 1 \le i \le N\}$ is the set of the sample image patches, $\dot{\mathbf{D}}^{n\times K} = \{\dot{\mathbf{d}}_i, 1 \le i \le K\}$ is the quaternion-based dictionary composed of K atoms, and $\dot{\mathbf{A}}^{K\times N} = \{\dot{\alpha}_i, 1 \le i \le N\}$ is the coefficient matrix which is supposed to be sparse. To achieve the optimized dictionary,

[1]Note the imaginary units i, j, k obey the quaternion rules that $i^2 = j^2 = k^2 = -1$ and $i \cdot j = -j \cdot i = k, j \cdot k = -k \cdot j = i, k \cdot i = -i \cdot k = j$.

we propose a training algorithm using the counterpart of K-SVD [3] in the quaternion algebra, which we call K-QSVD. It separates the training process into two steps, i.e, sparse coding stage and dictionary updating stage.

During the sparse coding stage, a sparse coefficient matrix $\dot{\mathbf{A}}$ is to be solved given the fixed $\dot{\mathbf{D}}$ in (6). Methods like matching pursuit (MP) [15], basis pursuit (BP) [16] and so on are suitable for this sparse coding problem. In this paper, we choose the orthogonal matching pursuit (OMP) [17] algorithm for quaternion extension, i.e, QOMP, because of its high efficiency. It can alleviate the complexity of solving the l_0-norm sparse coding problem by specifying the maximum number of non-zero coefficients per signal.

Given the sparse coding solution, the quaternion-based dictionary $\dot{\mathbf{D}}$ can be trained. Different from traditional ways which fix the coefficient matrix during dictionary updating, K-QSVD is highly efficient due to its simultaneous change in the coefficient values. For each atom $\dot{\mathbf{d}}_k$ and the coefficients corresponding to it, $\dot{\mathbf{A}}^k$—the kth row in $\dot{\mathbf{A}}$, we update both of them by decomposing the remaining expression error $\dot{\mathbf{E}}_k$ using QSVD (Quaternion Singular Value Decomposition), where $\dot{\mathbf{E}}_k = \dot{\mathbf{Y}} - \sum_{j \neq k} \dot{\mathbf{d}}_j \dot{\mathbf{A}}^j$. Here, we do not linger the details and only show the scheme of K-QSVD in Table 1. Note QSVD is more adaptable to color image than SVD, thus the update of each atom and its corresponding coefficient row can be more efficient in quaternion system. The advantages and details of QSVD are given in our supplementary file.

Figure 1 shows the dictionary training methods depicted in the model (2) and the quaternion-based model (4), as well as their corresponding trained dictionaries. Compared with our quaternion-based dictionary, the generated one by K-SVD algorithm appears monochromatic. As mentioned in [8], the dictionary trained by K-SVD is not rich enough to represent the diversity of colors, thus it tends to become gray so that it can at least well describe the basic spatial structures of the images. In contrast, the trained quaternion-based dictionary is rather colorful, which is competent in preserving the channel correlations, as well as the spatial coherency.

4. APPLICATIONS TO IMAGE PROCESSING AND EXPERIMENTAL RESULTS

For validation, we extend the quaternion-based sparse representation into real color image applications, such as reconstruction, denoising and inpainting.

4.1. Color Image Reconstruction

As for the sparse model in [8], the authors treat the concatenation of RGB channels as a single vector, then perform K-SVD to train the dictionary for reconstruction. Here, we compare it with the quaternion-based sparse model. The two models are shown in Figure 1(a).

Table 1. Quaternion-based dictionary learning

1. Initialization: Construct the training color data $\dot{\mathbf{Y}} = \{\dot{\mathbf{y}}_i, 1 \leq i \leq N\}$, and initialize the dictionary matrix $\dot{\mathbf{D}} = \{\dot{\mathbf{d}}_i, 1 \leq i \leq K\}$, where each atom $\dot{\mathbf{d}}_i \in \mathbb{H}^n$ and block patch $\dot{\mathbf{y}}_i \in \mathbb{H}^n$.

2. Repeat J times:

1) Sparse Coding Stage: Use QOMP to compute the coefficient matrix $\dot{\mathbf{A}} = \{\dot{\alpha}_i, 1 \leq i \leq N\}$, where each coefficient column $\dot{\alpha}_i \in \mathbb{H}^K$.

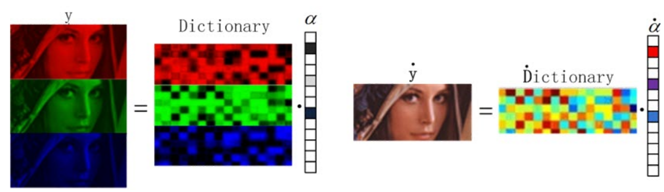
2) Codebook update Stage: Update each dictionary atom $\dot{\mathbf{d}}_k$ in $\dot{\mathbf{D}}^J$ through (i)-(iii) steps.

(i) Find the set of patches that use atom $\dot{\mathbf{d}}_k$, the index $\omega_k = \{i | 1 \leqslant i \leqslant N, \dot{\mathbf{A}}(k,i) \neq 0\}$, where $\dot{\mathbf{A}}(k,i)$ indicates the entry at k-th row and i-th column of the coefficient matrix $\dot{\mathbf{A}}$.

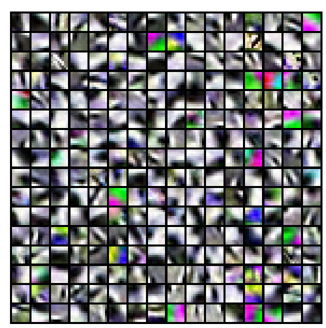
(ii) Compute the error $\dot{\mathbf{E}}_k = \dot{\mathbf{Y}} - \sum_{j \neq k} \dot{\mathbf{d}}_j \dot{\mathbf{A}}^j$ and select the columns corresponding to ω_k to form $\dot{\mathbf{E}}_k^R = \dot{\mathbf{E}}_k \omega_k$ for QSVD:

$$\dot{\mathbf{E}}_k^R = \dot{\mathbf{U}} \Lambda_r \dot{\mathbf{V}}^H.$$

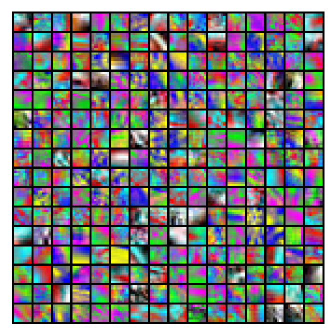
(iii) Update $\dot{\mathbf{d}}_k$ as the first column vector of $\dot{\mathbf{U}}$, and set its corresponding nonzero coefficient $\dot{\alpha}_R^k = \omega_k^T \dot{\alpha}^k$ to be the multiplication of the first column of $\dot{\mathbf{V}}^H$ and $\Lambda_r(1,1)$.



(a) Illustration for two types of sparse model



(b) K-SVD learned dictionary

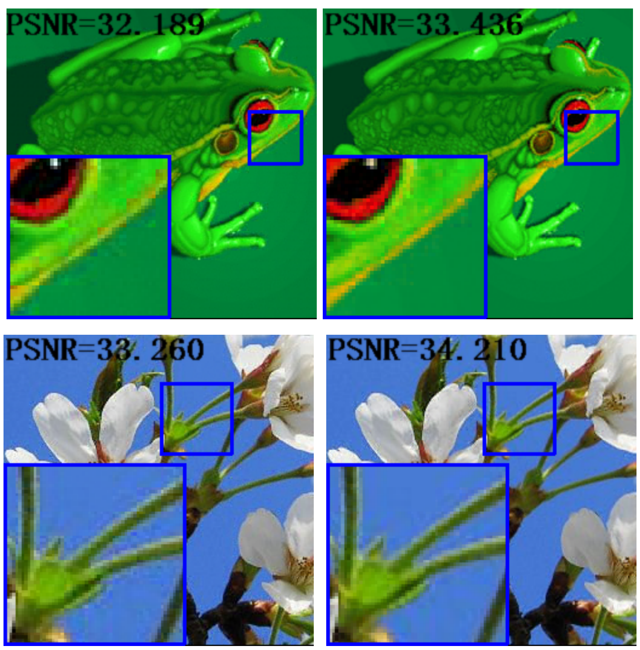


(c) K-QSVD learned dictionary

Fig. 1. Dictionaries with 256 atoms learned on a generic database of animal images. Each block is presented scaled and shifted to the [0,255] range per channel.

We firstly collect the training data as a set of 20000 examples taken from a database, whose size is 8*8 pixels and contents are similar to the represented image patches. Then we train the dictionaries using K-SVD and K-QSVD separately on the same training samples. In order to keep a reasonable computational complexity, both dictionaries consist of 256 atoms which are not particularly large. In the coding stage, we set the sparsity parameter L in OMP/QOMP for both methods, where L denotes the maximum number of non-zero coefficients allowed for representing each image block. Considering each coefficient column in our case is a quaternion vector that contains 4×256 real numbers, we fairly set the ratio to be 1:4 so that both ways use the same amount of non-zero real numbers.

Figure 2 gives some reconstructed images. It shows that the quaternion-based reconstruction presents more accurate color structures than model (2). Especially in the color junction parts, some hue distortions and artifacts would occur in the concatenation-based reconstruction due to lack of color variation in the monochromatic dictionary. In contrast, the quaternion-based dictionary is more competent in capturing the color structures, thus provides more accurate reconstruction.



(a) Reconstruction using model (2) (b) Reconstruction using our method

Fig. 2. Reconstruction using K-SVD and K-QSVD learned dictionaries. (a) 24 atoms to represent each RGB-concatenated block. (b) 6 atoms to represent each quaternion block.

4.2. Color Image Denoising

Another common application of sparse representation is denoising. Let $\dot{\mathbf{X}}_0$ be a clean image written in quaternion column of length N $(= \sqrt{N} \times \sqrt{N})$, then its noisy version could be:

$$\dot{\mathbf{Y}} = \dot{\mathbf{X}}_0 + \dot{\mathbf{w}}, \tag{7}$$

where $\dot{\mathbf{w}}$ is the white Gaussian noise in quaternion form with a spatially uniform deviation σ. We assume that all patches of size $\sqrt{n} \times \sqrt{n}$ in the clean image $\dot{\mathbf{X}}_0$ admit sparse representations. The denoising problem by quaternion-based sparse model can lead to the following energy minimization problem:

$$\{\hat{\dot{\alpha}}_{ij}, \hat{\dot{\mathbf{D}}}, \hat{\dot{\mathbf{X}}}\} = \underset{\dot{\mathbf{D}}, \dot{\alpha}_{ij}, \dot{\mathbf{X}}}{argmin}\, \lambda \left\| \dot{\mathbf{X}} - \dot{\mathbf{Y}} \right\|_2^2 + \sum_{i,j} \mu_{ij} \left\| \dot{\alpha}_{ij} \right\|_0 + \sum_{i,j} \left\| \dot{\mathbf{D}} \dot{\alpha}_{ij} - R_{ij} \dot{\mathbf{D}} \right\|_2^2, \tag{8}$$

where $\hat{\dot{\mathbf{X}}}$ is the estimation of $\dot{\mathbf{X}}_0$, and the dictionary $\hat{\dot{\mathbf{D}}}$ of size $n \times K$ is the estimation of the optimal dictionary which leads to the sparsest representation of the recovered image patches. The indices $[i, j]$ mark the location of the patch in the image, thus R_{ij} of size $n \times N$ is an operator extracting the square $\sqrt{n} \times \sqrt{n}$ patch of coordinates $[i, j]$ from the image $\dot{\mathbf{X}}$, and the vectors $\hat{\dot{\alpha}}_{ij}$ of size $K \times 1$ are the coefficient vectors for each patch. The first term in (8) enforces the likelihood that demands a proximity between $\dot{\mathbf{X}}$ and $\dot{\mathbf{Y}}$. The second and the third terms impose the image prior, assuming each quaternion patch can be sparsely represented without noise over $\hat{\dot{\mathbf{D}}}$. The solution of (8) can be found in [5], with all the algebra operations specified in quaternion system. During the implementation, the QOMP refers to,

$$\dot{\alpha}_{ij} = \underset{\dot{\alpha}_{ij}}{argmin} \left\| \dot{\alpha}_{ij} \right\|_0 \text{ s.t } \left\| \dot{\mathbf{D}} \dot{\alpha}_{ij} - R_{ij} \dot{\mathbf{D}} \right\|_2^2 \leq n(C\sigma)^2,$$

which stops when the approximation reaches the sphere of radius $\sqrt{n(C\sigma)^2}$ in each patch's quaternion space. Empirically, C is set to be 2.8 and λ to be 0.037.

Figure 3 gives the comparison of several denoising methods based on sparse representation. One is the method in [5] by applying the K-SVD denoising algorithm based on model (2), which might introduce color bias and washing effect. Later, an improved work [8] proposes to revise the OMP by adding a regularization so that the average value of each channel remains unchanged before and after sparse representation. This attempt does reduce some hue bias, but still lose the inter-relationship among channels; Moreover, such revision breaks the hypothesis of OMP that noise is in a sphere structure. It is noted in Figure 3, for both the two methods, color distortions appear in the tree and pedestrian parts, and the washing effects arise in the "gray" sky. In contrast, the quaternion-based method can present more true colors after denoising. Some PSNR and SSIM results are shown in Table 2. These quantitative evaluation results further prove the advantages of our method.

4.3. Color Image Inpainting and Denoising Pepper Noise

Image inpainting refers to filling the missing information in an image. Limited to the patch size, the learning-based method can only handle small missing holes. Thus, we temporarily focus on filling missing pixels like [3] [8] does. We choose one random full face image which is not used for training, and randomly delete a fraction r of the pixels, usually $r \in \{0.2, 0.9\}$. Our goal is to re-fill them.

Fig. 3. Columns from left to right: Original image, the image with noise of $\sigma = 25$, K-SVD denoising result [5], improved K-SVD denoising result [8], our K-QSVD denoising result.

Table 2. PSNR/SSIM values of different denoising methods. Each case is comprised of three parts: The top results are based on model (2) [5], the middle ones are obtained by its improved version [8], and the bottom ones are our results.

σ	barn	Athens	blueeye	bee
15	30.83/0.83	31.80/0.91	34.07/0.91	33.40/0.90
	30.79/0.82	32.33/0.91	34.30/0.92	33.42/0.91
	30.91/0.84	**32.40/0.93**	**34.42/0.92**	**33.47/0.91**
25	27.25/0.67	27.73/0.82	30.24/0.83	27.84/0.80
	27.30/0.65	28.14/0.80	30.47/0.83	28.12/**0.83**
	27.34/0.67	**28.26/0.83**	**30.69/0.84**	**28.24**/0.81

The coefficient matrix for each corrupted patch can be estimated only on the non-corrupted pixels using the pruned dictionary, which is generated by deleting the rows corresponding to those missing pixels in each patch. This coefficient matrix can be computed by OMP/QOMP, which is same as the one over the full dictionary for the reconstructed patch. The filling results of model (2) and our quaternion-based model are shown in Figure 4(a), with the mean reconstruction errors (RMS) provided. It can be observed that higher quality recovery with fewer artifacts is obtained using the quaternion-based method.

Such idea can also be extended to denoise salt and pepper noise. We can treat every noised pixel as a corrupted quaternion, with some imaginary parts being 0 or 255. Now for each block, the temporary dictionary is set by deleting the imaginary rows corresponding to those corrupted imaginary parts, and the rest procedures remain the same. Figure 4(b) shows that the method using model (2) makes the child face partially yellow. In comparison, our quaternion-based method has no such hue bias.

For images of size 256×256, the average time of denoising by K-SVD and the proposed method is 210s and 1500s, respectively. The average time of inpainting is 230s and 1440s respectively. (The test platform is MATLAB on Intel Core-i3 CPU with 4GB memory.) As each patch can be processed in a parallel way, the computation time can be significantly reduced using GPU execution.

5. CONCLUSION

In this paper, we address color image problems using the quaternion-based sparse representation, which treats the color pixel as a vector unit instead of a scalar quantity and thus overcomes the inaccuracy of describing color images and the lack of dictionary diversity. The experiments of reconstruction, denoising and inpainting on real color images prove its advantages in characterizing color structures and preserving the interrelationship among color channels. We would consider more extensive applications in the future work to show its potential as a powerful tool in color image classification and recognition realm.

6. REFERENCES

[1] M. Elad, J. L. Starck, P. Querreb, and D.L. Donoho, “Simultaneous cartoon and texture image inpainting using morphological component analysis (mca),” *Applied and Computational Harmonic Analysis*, vol. 19, no. 3, pp. 340–358, 2005.

[2] K. Engan, S.O. Aase, and J.H. Husøy, “Multi-frame compression: Theory and design,” *Signal Processing*, vol. 80, no. 10, pp. 2121–2140, 2000.

[3] M. Aharon, M. Elad, and A. Bruckstein, “K-svd: Design of dictionaries for sparse representation,” *Proceedings of SPARS*, vol. 5, pp. 9–12, 2005.

(a) Filling missing pixels

(b) Denoising salt and pepper noise

Fig. 4. The 2nd last column shows the results based on model (2) using the method in [3], and the last column is the quaternion-based results.

[4] R.M. Figueras i Ventura, P. Vandergheynst, P. Frossard, and A. Cavallaro, "Color image scalable coding with matching pursuit," in *Acoustics, Speech, and Signal Processing, 2004. Proceedings.(ICASSP'04). IEEE International Conference on*. IEEE, 2004, vol. 3, pp. iii–53.

[5] M. Elad and M. Aharon, "Image denoising via sparse and redundant representations over learned dictionaries," *Image Processing, IEEE Transactions on*, vol. 15, no. 12, pp. 3736–3745, 2006.

[6] E. Elhamifar and R. Vidal, "Robust classification using structured sparse representation," in *Computer Vision and Pattern Recognition (CVPR), 2011 IEEE Conference on*. IEEE, 2011, pp. 1873–1879.

[7] J. Wright, Y. Ma, J. Mairal, G. Sapiro, T.S. Huang, and S. Yan, "Sparse representation for computer vision and pattern recognition," *Proceedings of the IEEE*, vol. 98, no. 6, pp. 1031–1044, 2010.

[8] J. Mairal, M. Elad, and G. Sapiro, "Sparse representation for color image restoration," *Image Processing, IEEE Transactions on*, vol. 17, no. 1, pp. 53–69, 2008.

[9] C. Cai and S.K. Mitra, "A normalized color difference edge detector based on quaternion representation," in *Image Processing, 2000. Proceedings. 2000 International Conference on*. IEEE, 2000, vol. 2, pp. 816–819.

[10] S.C. Pei and C.M. Cheng, "Color image processing by using binary quaternion-moment-preserving thresholding technique," *Image Processing, IEEE Transactions on*, vol. 8, no. 5, pp. 614–628, 1999.

[11] N. Le Bihan and S.J. Sangwine, "Quaternion principal component analysis of color images," in *Image Processing, 2003. ICIP 2003. Proceedings. 2003 International Conference on*. IEEE, 2003, vol. 1, pp. I–809.

[12] G.D. Finlayson, S.D. Hordley, and P.M. Hubel, "Color by correlation: A simple, unifying framework for color constancy," *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, vol. 23, no. 11, pp. 1209–1221, 2001.

[13] H.S. Malvar, L. He, and R. Cutler, "High-quality linear interpolation for demosaicing of bayer-patterned color images," in *Acoustics, Speech, and Signal Processing, 2004. Proceedings.(ICASSP'04). IEEE International Conference on*. IEEE, 2004, vol. 3, pp. iii–485.

[14] E. Elhamifar and R. Vidal, "Sparse subspace clustering," in *Computer Vision and Pattern Recognition, 2009. CVPR 2009. IEEE Conference on*. IEEE, 2009, pp. 2790–2797.

[15] S.F. Cotter and B.D. Rao, "Sparse channel estimation via matching pursuit with application to equalization," *Communications, IEEE Transactions on*, vol. 50, no. 3, pp. 374–377, 2002.

[16] S.S. Chen, D.L. Donoho, and M.A. Saunders, "Atomic decomposition by basis pursuit," *SIAM journal on scientific computing*, vol. 20, no. 1, pp. 33–61, 1998.

[17] Y.C. Pati, R. Rezaiifar, and PS Krishnaprasad, "Orthogonal matching pursuit: Recursive function approximation with applications to wavelet decomposition," in *Signals, Systems and Computers, 1993. 1993 Conference Record of The Twenty-Seventh Asilomar Conference on*. IEEE, 1993, pp. 40–44.