

COMP 768

Spring 2009

Paper Presentation:
Overview of Cloth Modeling and
Simulation

Main References

- Hing N. Ng, Richard L. Grimsdale, "Computer Graphics Techniques for Modeling Cloth," *IEEE Computer Graphics and Applications*, vol. 16, no. 5, pp. 28-41, Sept. 1996 [doi:10.1109/38.536273]
- X. Provot. "Deformation constraints in a mass-spring model to describe rigid cloth behavior," *Proceedings of Graphic Interface '95*, pages 147-- 154, Qu'ebec, Canada, May 1995.
- David Baraff , Andrew Witkin, "Large steps in cloth simulation," *Proceedings of the 25th annual conference on Computer graphics and interactive techniques*, p.43-54, July 1998 [doi:10.1145/280814.280821]

Outline

- Motivations and Challenges of cloth sim
- Example commercial cloth simulators
- Survey of techniques in cloth sim research
- A bit more detailed examination of techniques of two papers
 - X. Provot, "Deformation constraints in a mass-spring model to describe rigid cloth behavior," *Proceedings of Graphic Interface '95*
 - D. Baraff and A. Witkin, "Large Steps in Cloth Simulation," SIGGRAPH 98

Motivations

- Entertainment
- Fashion design
- Textile engineering

Challenges of Cloth Simulation

- Sheer size:
As a flexible object, a cloth model is usually represented by many small elements, so the computation time is very long.
- Cloth simulation as stiff systems because cloth is
 - strongly resistant to stretching motions
 - permissive in bending and shearing motions
- Collision:
 - Low tolerance because of the thin layer
(Once it interpenetrates, it is very discernible.)
 - Self-intersection

Commercial Cloth Simulators

Commerical Cloth Simulators

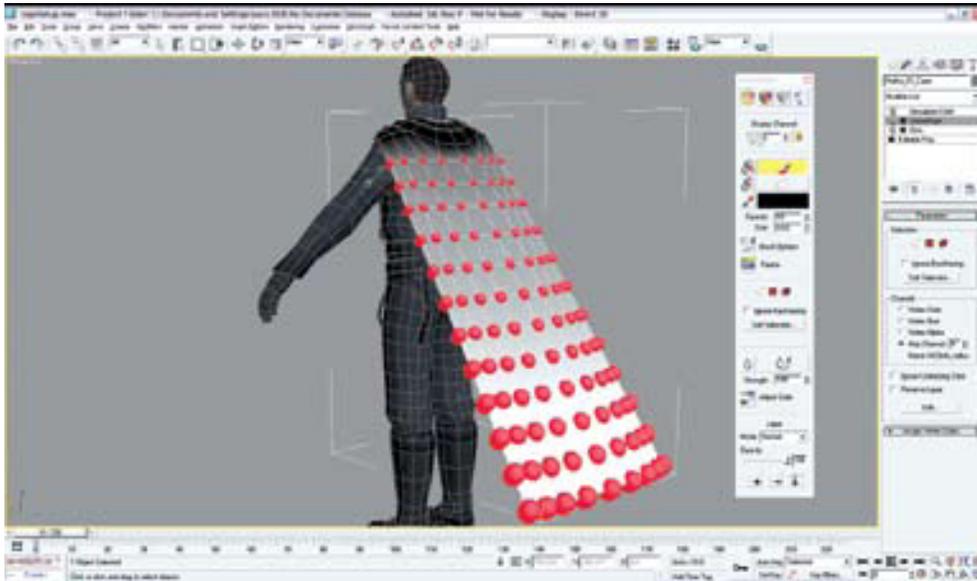
- Havok Cloth
- Maya Cloth and nCloth

Features:

- Physically-based simulation of cloth
- Multithreaded and platform-optimized (including PLAYSTATION®3)
- Artist-driven control of cloth behavioral properties, for example, stretching, damping and bending
- Artist-friendly, modeller-based, cloth setup tools

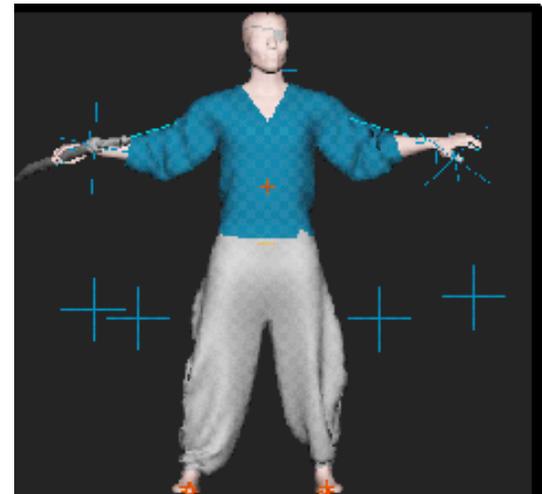
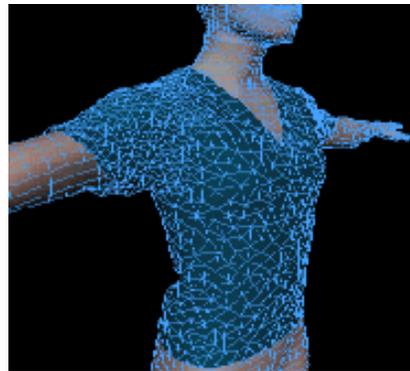
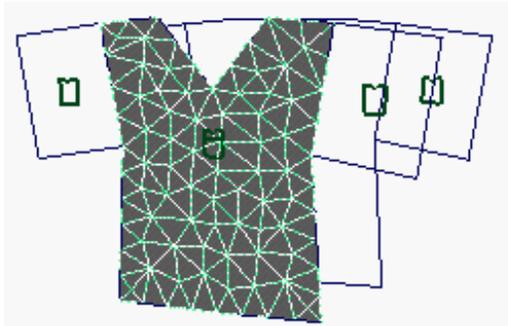
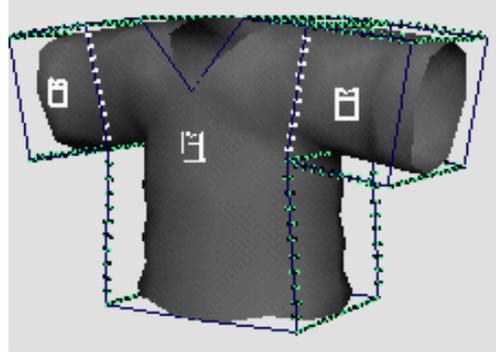
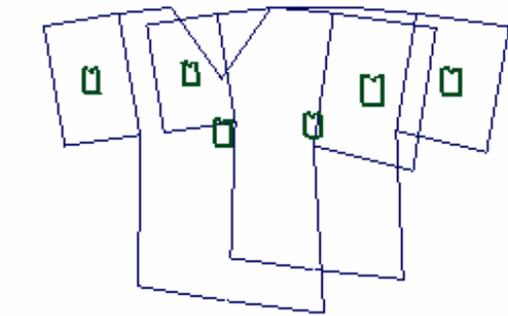
<http://www.havok.com/content/view/584/96/>

Commerical Cloth Sim :: Havok Cloth



<http://www.havok.com/content/view/584/96/>

Commerical Cloth Sim :: **Maya Cloth**



Commerical Cloth Sim :: **Maya nCloth**

Some of the features:

- Create cloth-on-cloth simulations—such as a cape over a jacket or a shirt over pants—with believable influences and collisions
- Interpenetration with the collision object is rare—in such instances Maya nCloth quickly recovers and continues the simulation
- Simulations can have force fields
- Support tear Maya nCloth fabrics

<http://usa.autodesk.com/adsk/servlet/item?siteID=123112&id=7639477>

Video: http://images.autodesk.com/adsk/files/6_obert.wmv

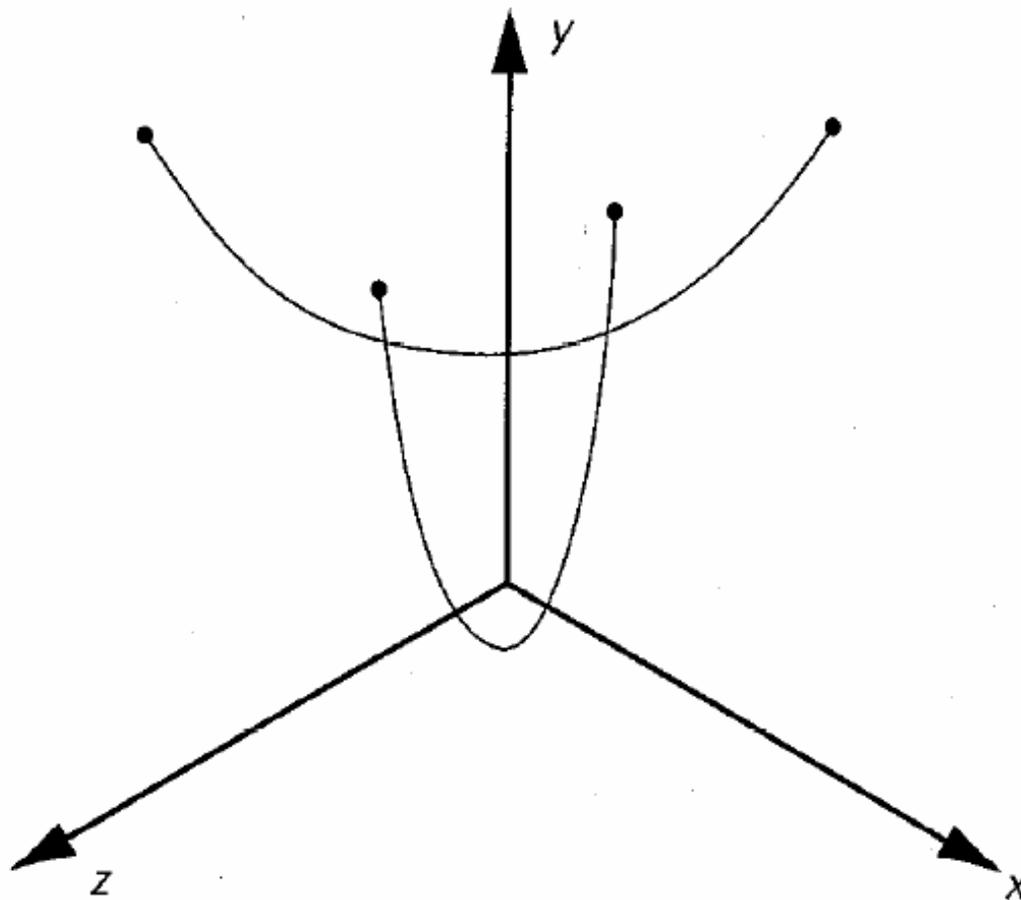
Categories of Techniques

- Geometrical
- Physical
- Hybrid

Geometrical Techniques

- To represent folds and creases by geometrical equations
- Emphasize on the appearance of the cloth, especially the folds and creases, but not so much on the physical aspects of the cloth

- Key idea: Use of catenary curves



2 Two catenary curves.

- A catenary curve has the natural shape of a hanging heavy cable
- Equation:
$$y = a \cdot \cosh(x / a)$$
- Limitation: Only for hanging cloth

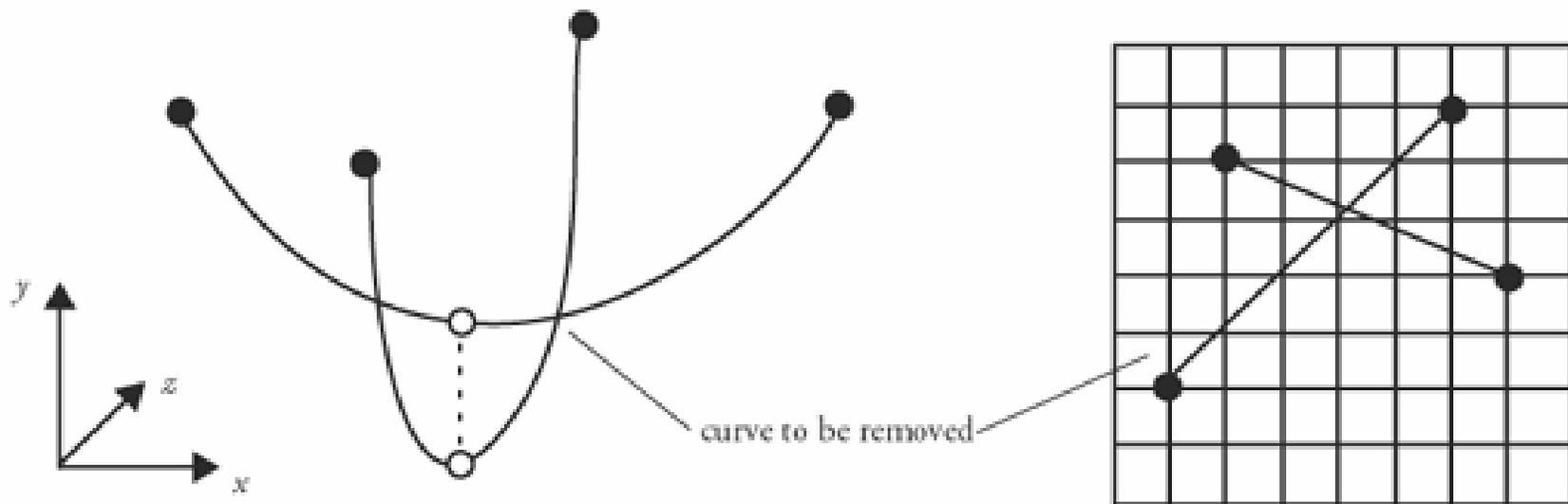
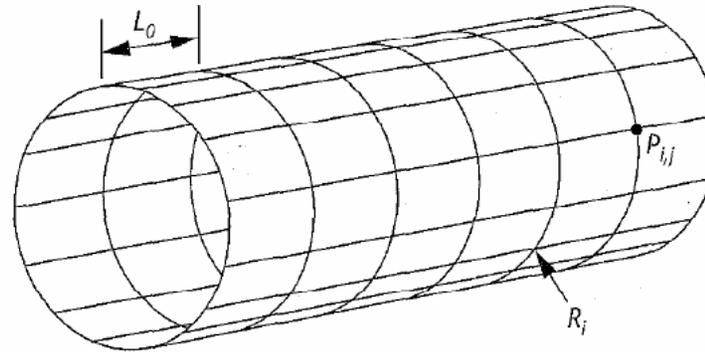


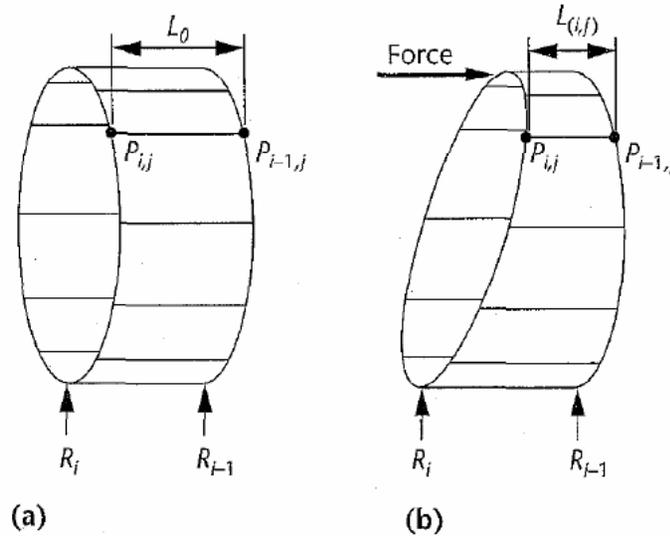
Figure 6.44 Two catenary curves supporting the same point

Categories of Techniques :: Geometrical :: **Example 2: Agui et. al. 1990**

4 Cloth sleeve represented as a hollow cylinder.

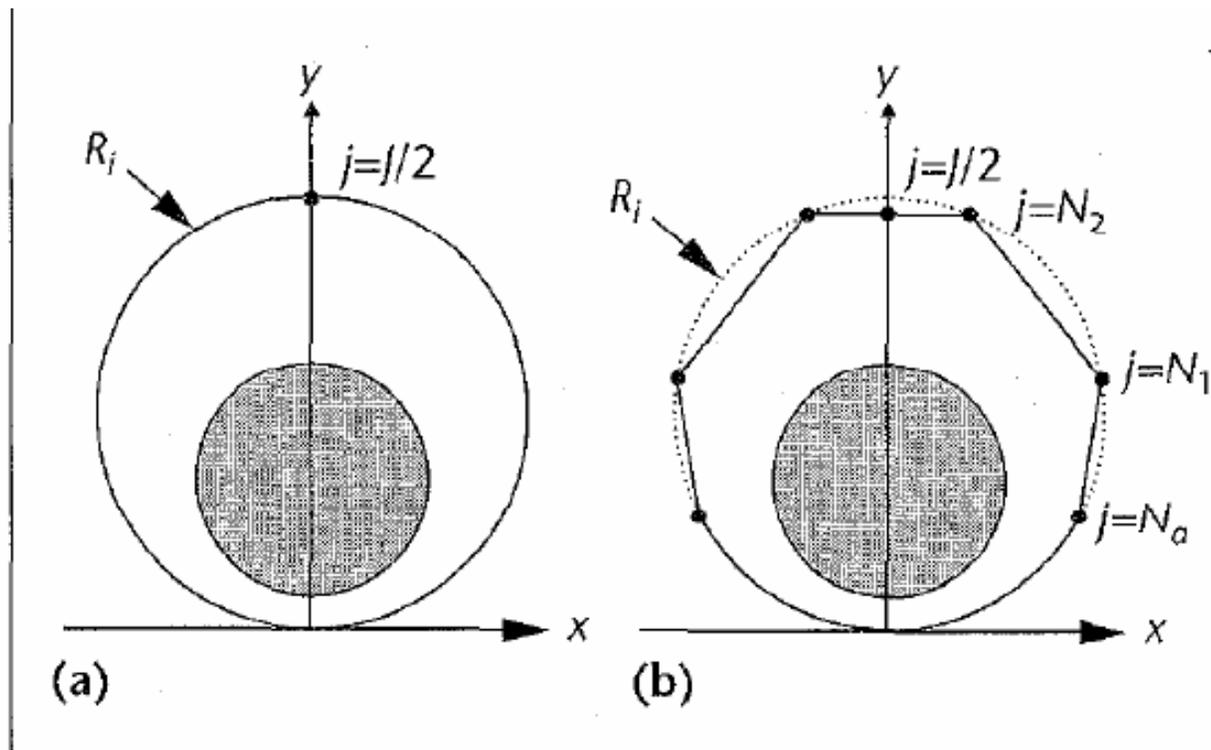


5 A sleeve section.
(a) before bending and
(b) after bending.



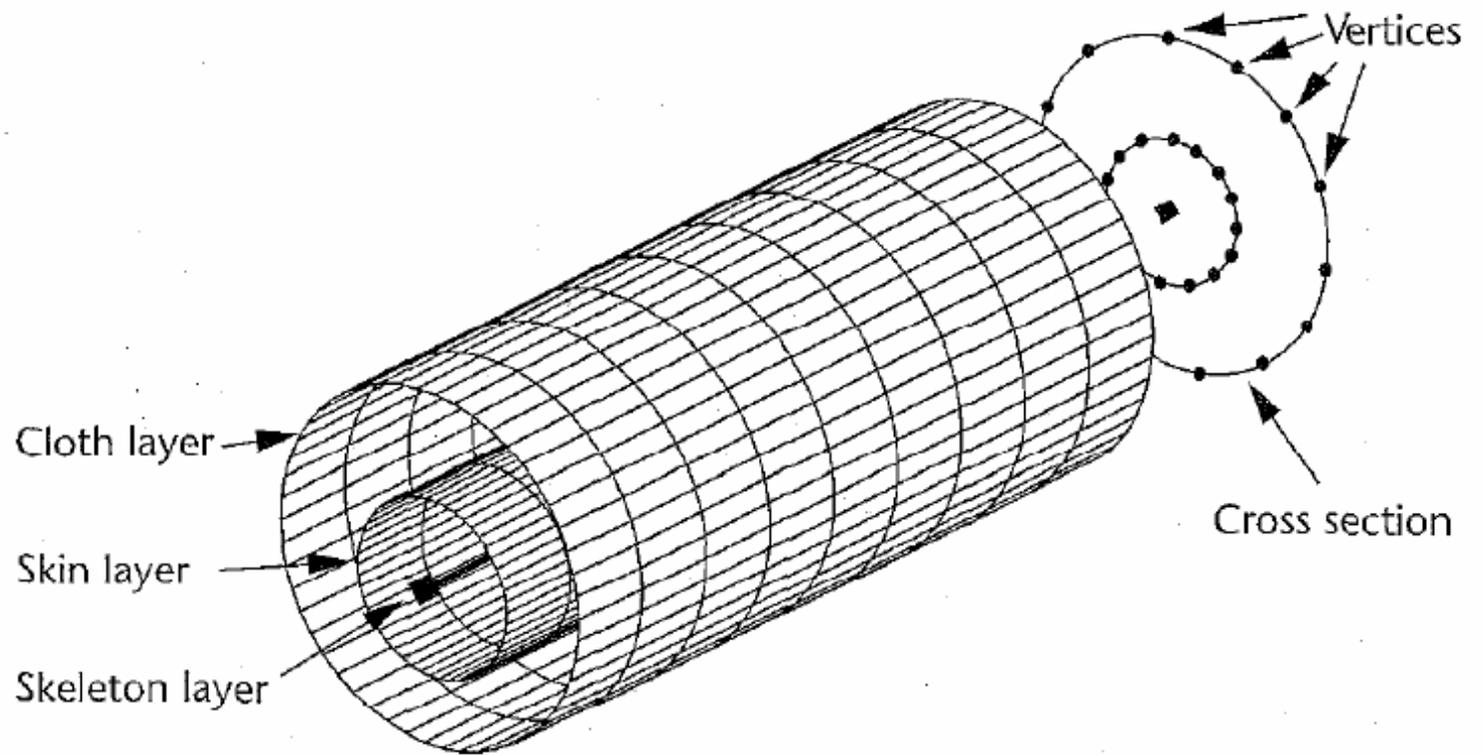
- Modeled sleeve on a bending arm
- Key ideas:
 - Represent cloth as a hollow cylinder consisting of a series of circular rings, converted into polygons
 - Assume the top is flat and the cylinder is symmetrical about its y-axis
 - The corners are then smoothed

6 The process of forming a polygon:
(a) undeformed ring R_i
(b) transformed to compute a sleeve fold.



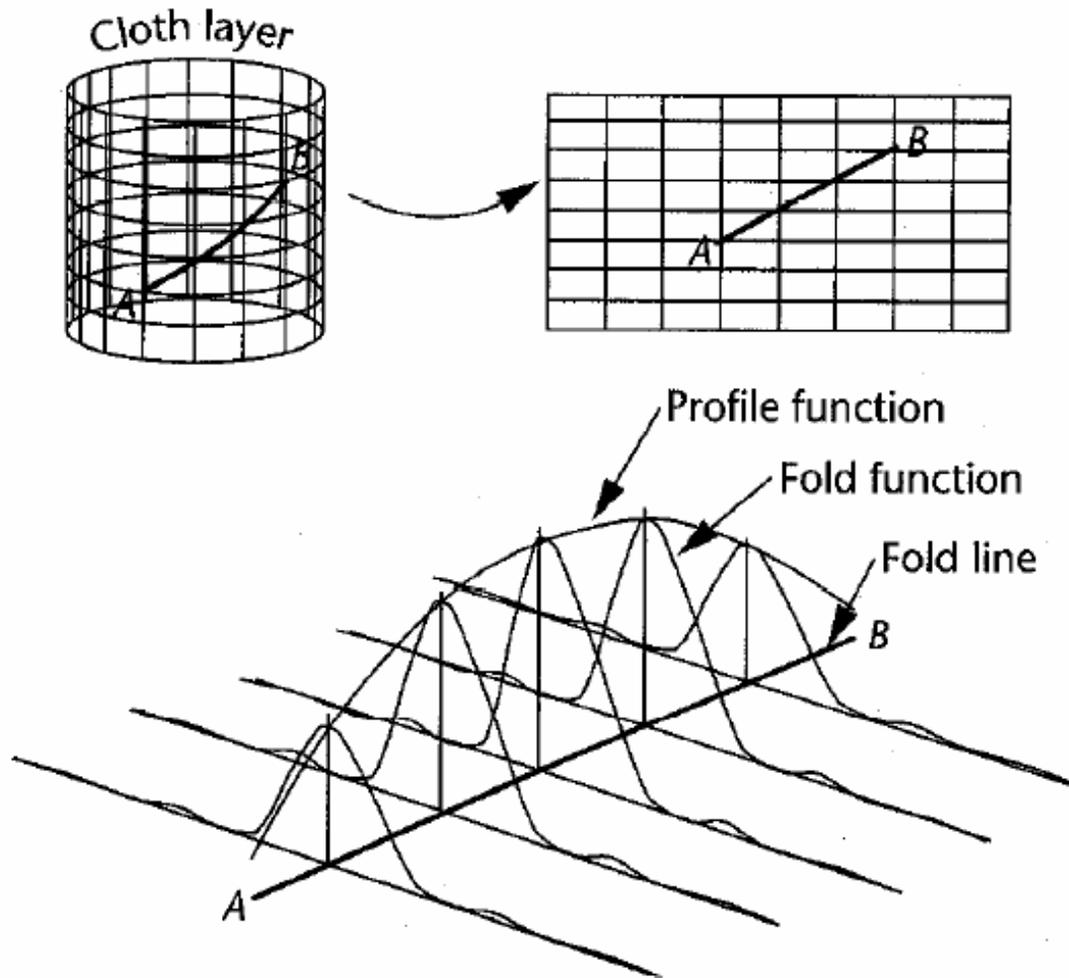
- Mimic garment manufacturing
- Garment is represented by a number of 3D surface panels
- These panels can be positioned interactively on a digitized upper torso of a mannequin
- The panels are "sewn" together using interpolation with C^0 continuity
- Designer can add fold and texture. Folds are introduced as a harmonic function superimposed on the panels.

7 Layered structure for associating cloth to skin and skeleton for animation.



- Define layers: cloth, skin
- The algorithm is based on the following ideas:
 - Folds are likely to occur where the cloth is slack
 - Folds occur as result of underlying structural formation, e.g. arm bending
- Fold generation:
 - Allow users to control fold generation. When a fold line is generated, sinusoidal functions are mapped along the line to mimic natural folds
 - Also developed rules to generate the fold lines automatically

Categories of Techniques :: Geometrical :: **Example 4: Ng et. al. 1995, 1996**



8 Mapping process to determine fold lines and generate folds along them.

Physical Techniques

- Represent cloth model as a triangular or rectangular grid
- Points of finite mass as vertices
- Forces or energies of points on the grid are calculated in relations to other points

- Energy-based Techniques
- Force-based Techniques

- Finding Governing Equations
- Solving the Equations
- Collision Handling

- Calculate the energy of the whole cloth from a set of equations
- Determine the shape of the cloth by positioning the points to achieve a minimum energy state
- Often for static simulations

- **Energy-based**

$$\ddot{x} = M^{-1} \left(-\frac{\partial E}{\partial x} + F \right)$$

x = geometric state of the cloth

M = mass distribution of the cloth

E = scalar function of x ; the cloth's internal energy

F = a function of x and \ddot{x} ; other forces

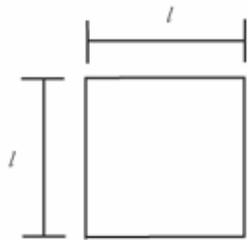
(air drag, contact, constraint, internal damping) acting on the cloth

- **General Types of Internal Energy (potential energy), E**

- stretch
- shear
- bending

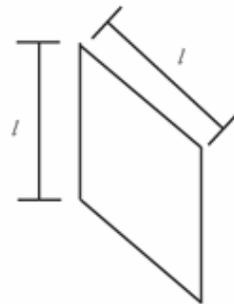
Categories of Techniques :: Physical :: The Ideas :: **Energy-based Techniques**

Stretch



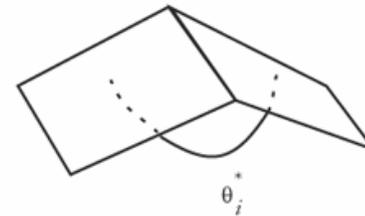
(a) Original quadrilateral of mesh

Shear

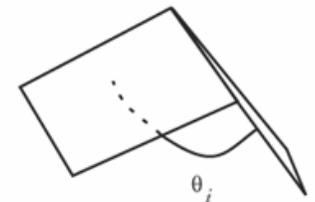


(b) Skew of original quadrilateral without changing the length of edges

Bending

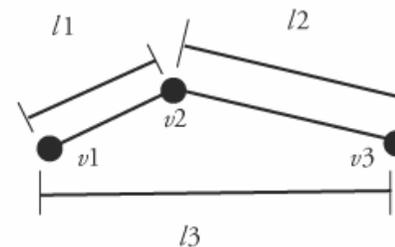


Original dihedral angle



Bending along the edge that changes dihedral angle

Figure 6.48 Control of bending by dihedral angle



$$l1 = |v2 - v1|$$

$$l2 = |v3 - v2|$$

$$l3 = |v3 - v1|$$

Figure 6.49 Control of bending by separation of adjacent vertices

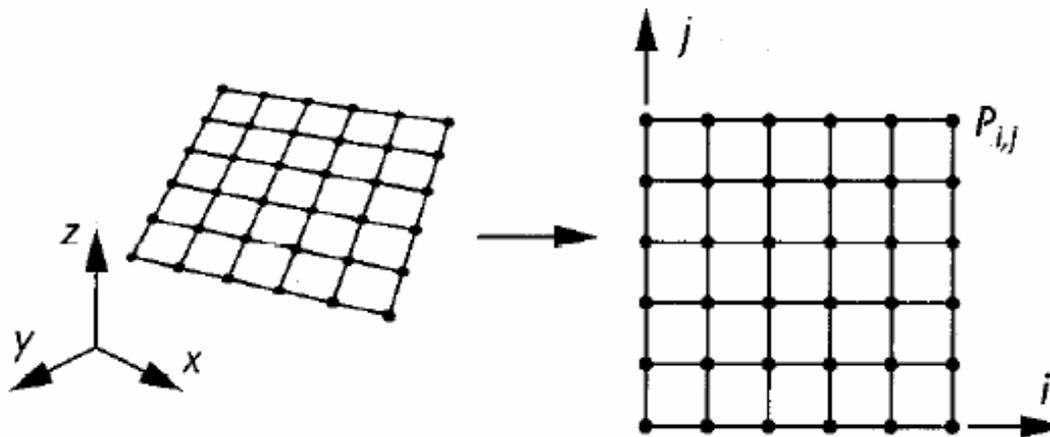
- Numerically solve

$$\ddot{x}(t) = M^{-1} \left(-\frac{\partial E}{\partial x} + F \right) (t)$$

```
while(1)
{
    compute the force part at t;
    compute accel, vel, pos;
    t = t + Δt;
}
```

- Represent the forces among points as differential equations
- Perform numerical integration to obtain the point positions at each time step
- Usually used in dynamic simulations

- Model draped cloth
- Represent cloth in 3D space by a 2D grid



9 A 3D cloth represented as a 2D grid.

- Energy-based
- The energy at a point is calculated in relation to its 8 surrounding points
- Find the equilibrium shape of the draped cloth by using steepest decent method to determine the energy minimum
- Demonstrated examples: hanging clothes, cloth draped over a sphere

- Developed deformable model for general flexible objects but also used for cloth simulation
- Used Lagrange's equations of motion, involving mass density, damping, internal elastic force
- Discretized by using a finite-difference or finite-element method resulting in a large system of simultaneous ODE, numerically solved at each time step
- Demonstrated examples: waving flag, a carpet draped over various objects, a piece of cloth being torn apart

Categories of Techniques :: Physical :: **Example 3: Sakaguchi 1991**

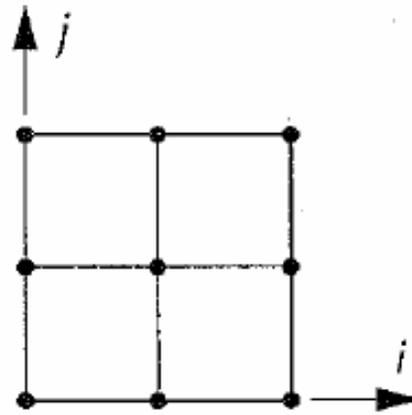
- Use a rectangular grid to represent cloth
- Use of Newton's law of dynamics

$$\frac{d^2x}{dt^2} = \frac{1}{m}(F_{ext} + F_{int})$$

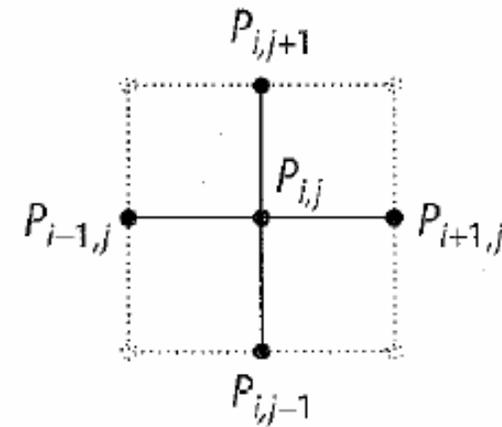
$$\text{where } F_{int} = F_{spring} + F_{viscosity} + F_{plasticity}$$

- Determine the position and velocity using Euler method
- Consider the interaction of cloth with underlying human body
- Collision response:
 - Consider conservation of momentum and friction
- Demonstrated examples: cloth hanging from a pole, blowing in the wind, a dress on a model of a human body

11 (a) Cloth represented as a grid and
(b) point $P_{i,j}$ with its four neighboring points.



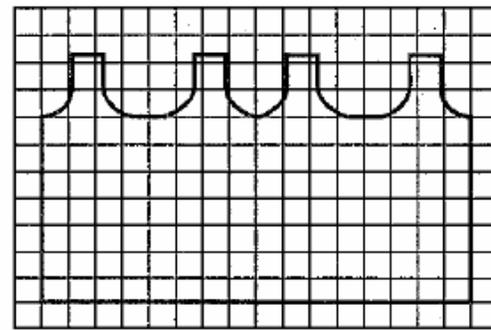
(a)



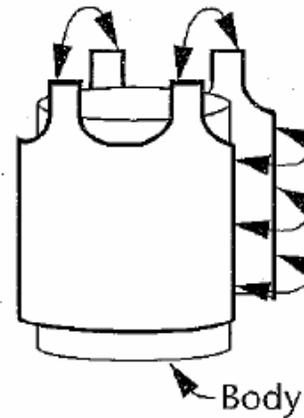
(b)

- Approach as garment production by a tailor: panels being "sewn" together

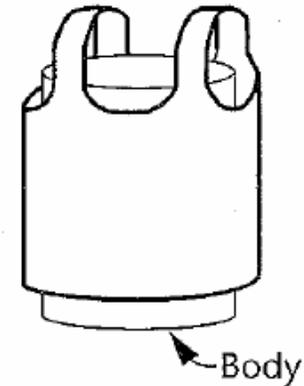
12 Three main parts of the Thalmann system: (a) 2D panel edit, (b) 3D seaming, and (c) 3D animation.



(a)

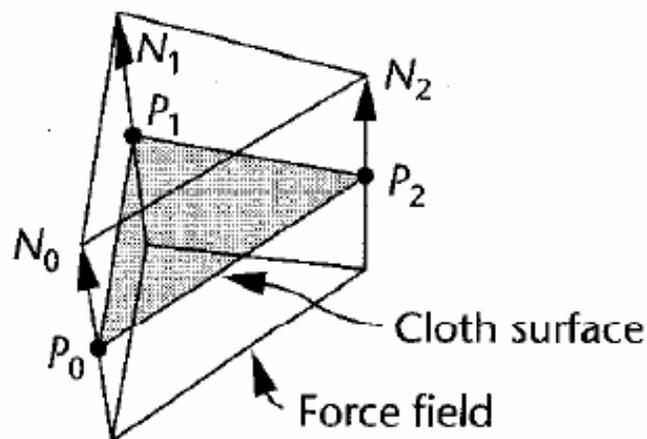


(b)



(c)

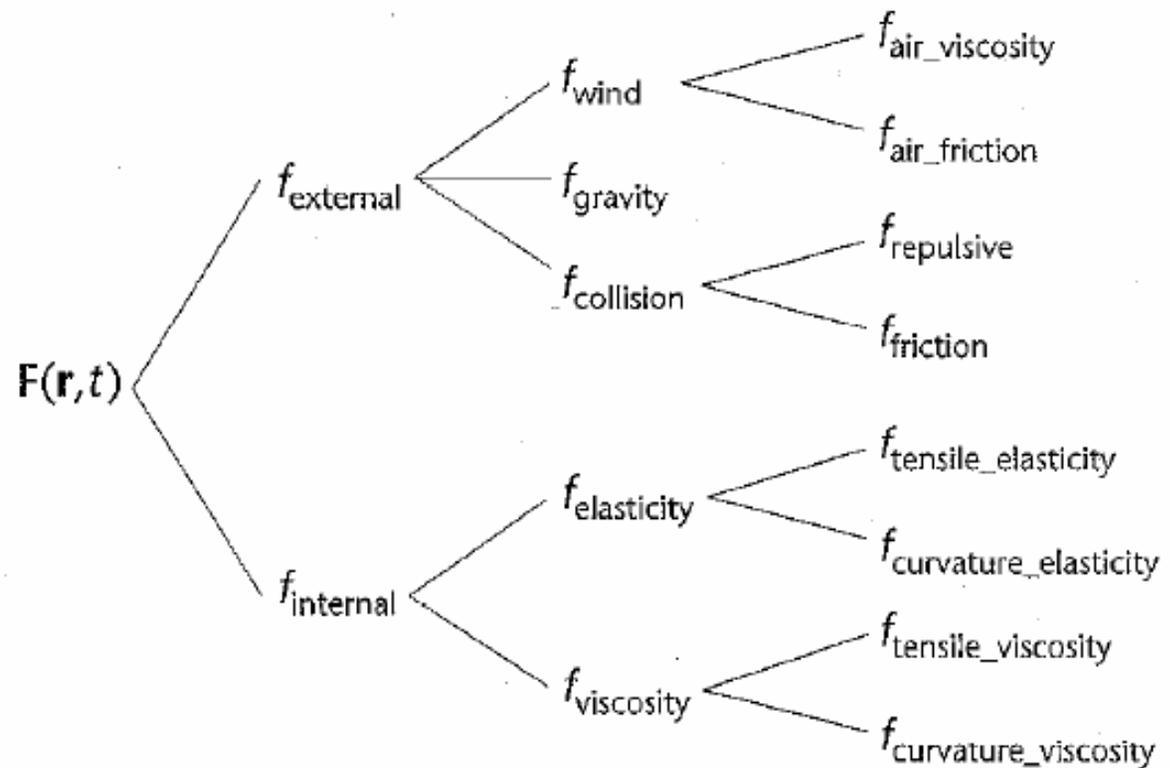
- Investigated collision between garment and the body
- Proposed a method for creating a volume force field around the cloth surface:
 - The normal of each vertex of the triangle form a force field
 - When a point enters this volume force field, a repulsive force is applied in the next time step to make the point move away from the cloth triangle



13 Force field around a cloth surface.

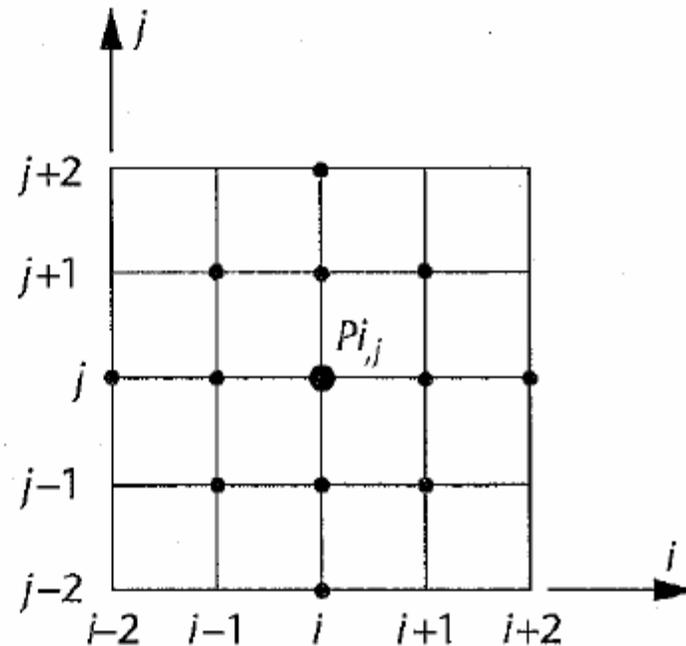
- Use Newton's law of motion, and include extensive components of forces
- Use Euler method

14 Components of $F(\mathbf{r}, t)$.



- Consider interaction with 12 neighboring points

15 $P_{i,j}$ and its neighbors.

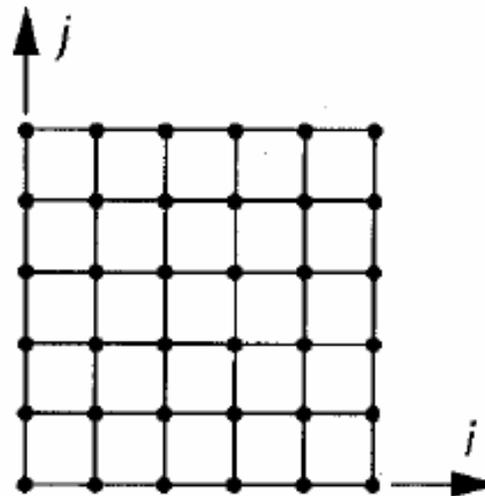
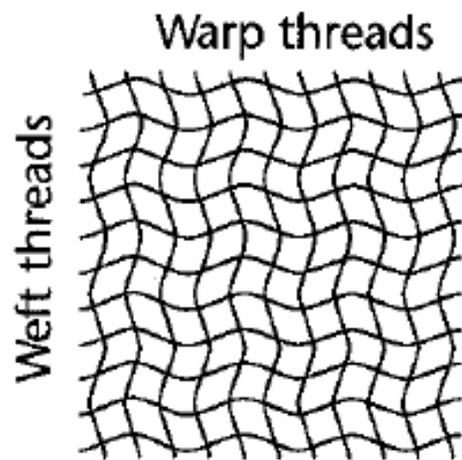


Categories of Techniques :: Physical :: **Example 6: Yang et al (Thalmanns group) 1994**

- Collision response was calculated using inverse dynamics and conservation of momentum
- To increase the speed of collision detection between and body:
 - use a bounding box around a facet
 - use octree structure to search for intersection
- Instead of using normal constraints for rigid bodies, he developed special constraints for attaching and seaming the cloth and smoothing the seams
- Demonstrated examples: 25x40 grid took 20 minutes on a SGI Indy; 40% computation time is on handling collision

- Represent cloth using particles forming triangles
- Use elasticity theory and Newtonian dynamics for deformable objects, which include cloth
- The model takes 2-pass approach:
 - 1st pass:
Interparticle constraint and external constraints, e.g. gravity and wind
 - 2nd pass:
determine collision by using the law of conservation of momentum

- Model draping cloth
- Particle-based
- Treat the intersecting points of the warp and weft threads of woven cloth as particles

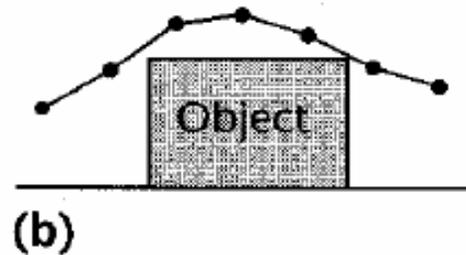
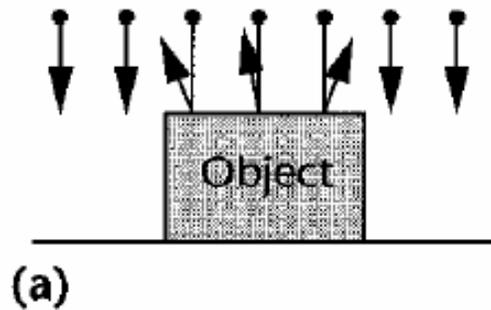


17 A woven cloth as particles.

- Simulation was done in 2 stages:
 - Stage 1: To get a rough shape of the draped cloth.
 - Particles are allowed to fall freely.
 - The collision with object or ground is determined.
 - Determine the particles' positions using the equation:
$$m a + c v = m g$$
where c is air friction
 - Stage 2: To generate fine detail in the cloth shape
 - Perform energy minimization to the interparticle energy functions.
 - Their total energy of a particle P_{ij} is defined as:

$$E_{total_{ij}} = E_{repel_{ij}} + E_{stretch_{ij}} + E_{bend_{ij}} + E_{trellis_{ij}} + E_{gravity_{ij}}$$

Categories of Techniques :: Physical :: **Example 8: Breen et al 1992, 1994**



18 The two stages of the Breen et al. simulation.

- To search for the energy minima, Breen developed an algorithm called Stochastic Global Descent (SGD) to determine global minima of a system
- Demonstrated examples: simulation time of their examples took from 3 days to a week on an IBM RS6000 workstation

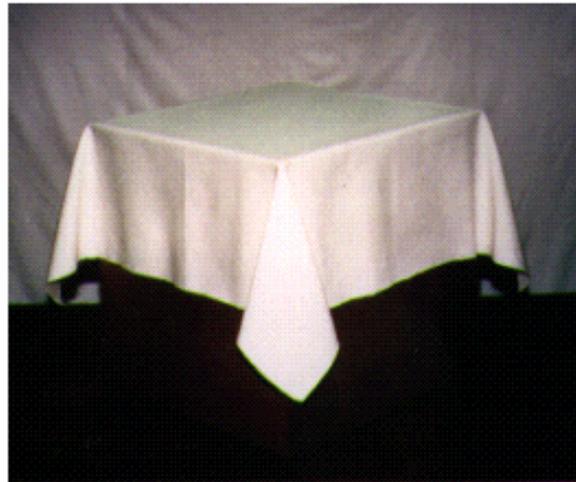
- Derived the bending energy and trellis energy terms from empirical textile measurement data
- Demonstrated examples: cloth draped over an armchair, a square table, and a circular table

Categories of Techniques :: Physical :: **Example 9: Breen et al 1994**



Categories of Techniques :: Physical :: **Example 9: Breen et al 1994**

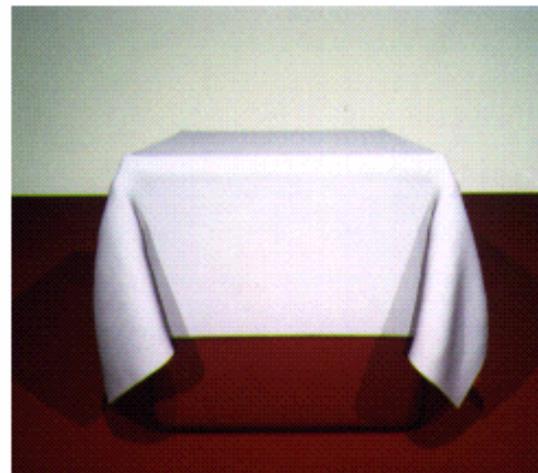
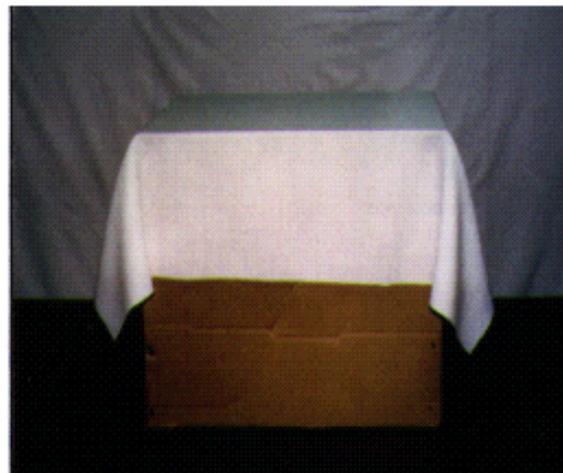
actual



virtual



100% Cotton Weave



100% Wool Weave

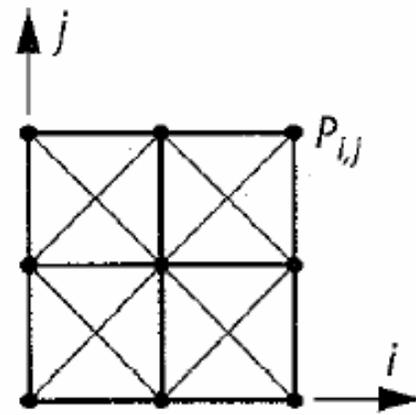
- Use spring-mass model to simulate cloth under constraint
- Use Newton's law of dynamics

$$F_{ext}(i, j) + F_{int}(i, j) = m a(i, j)$$

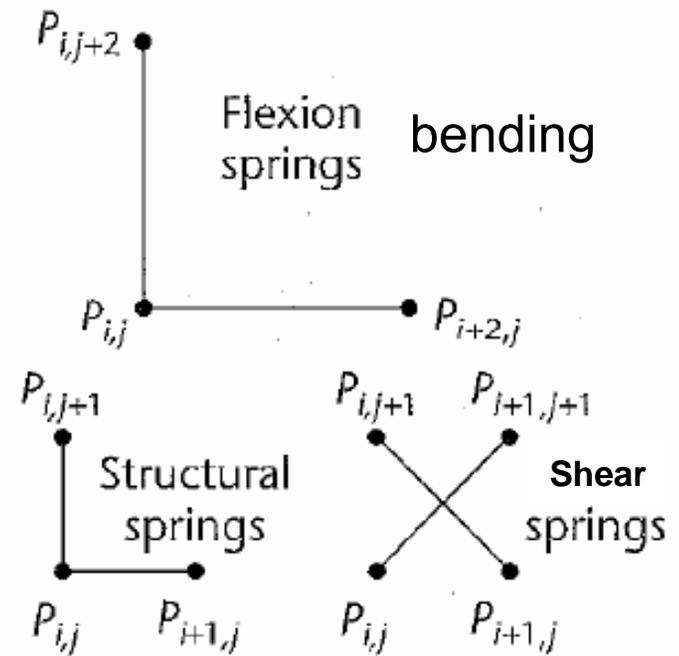
$F_{ext}(i, j)$ include gravity, damping, and viscosity

- Use Euler method to integrate

20 The mass-spring model.



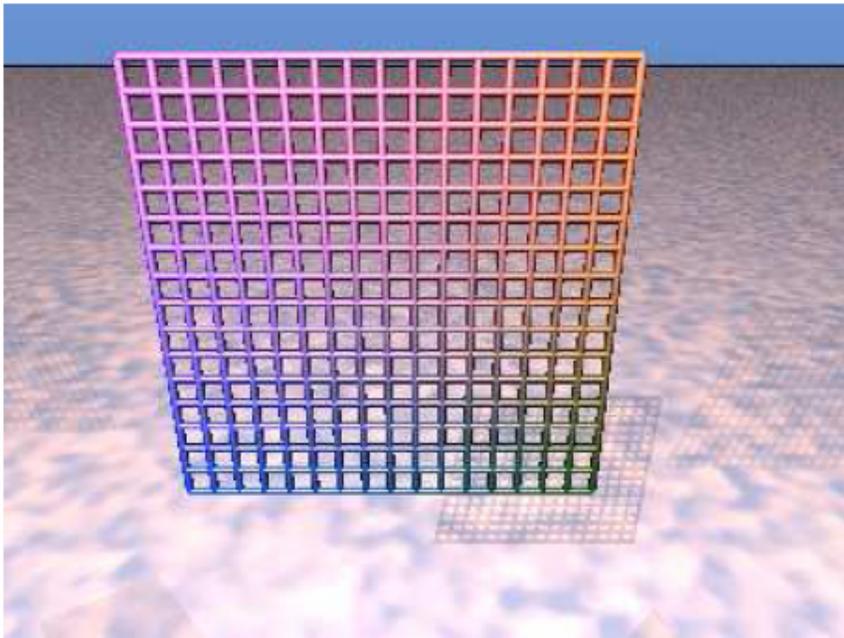
(a)



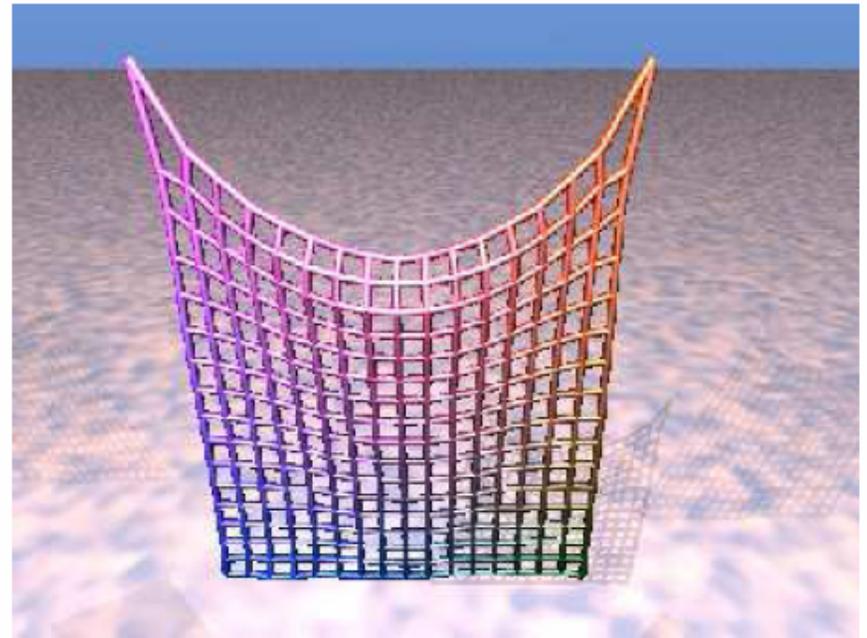
(b)

- Had unrealistic deformations at constraint points
- Solved by detecting the deformation rate in those spring around the constraint points to determine if any of them exceeded the predetermined threshold. If so, limit their elongation to 10%.
- Demonstration examples:
 - a cloth held by 2 corners, a flag in the wind
 - computation time: 1 minute to compute 1 second of animation for 17x17 mesh on a Sun Sparc 10 workstation; collision was not considered

Categories of Techniques :: Physical :: **Example 10: Provot 1995**



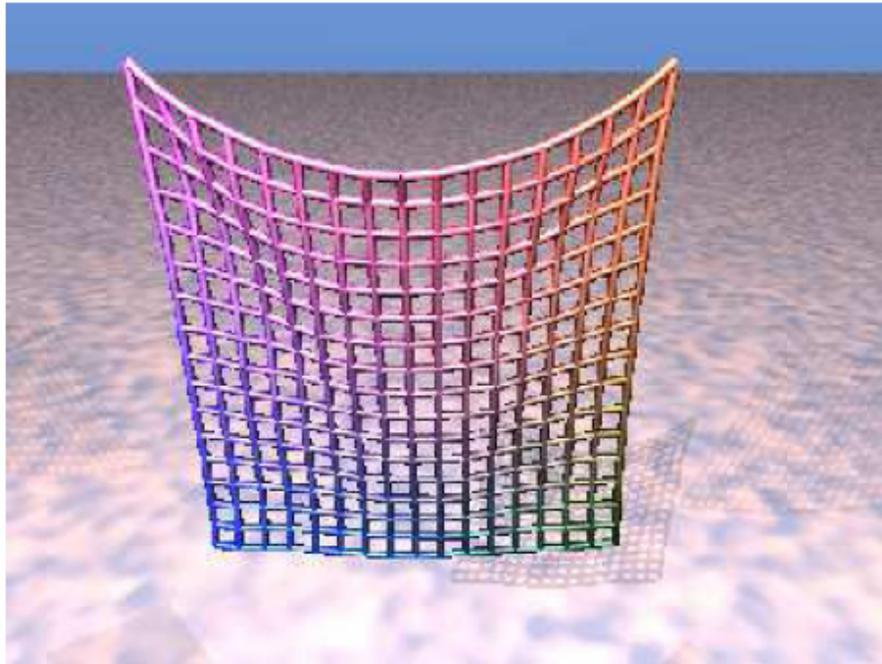
(a) Initial position



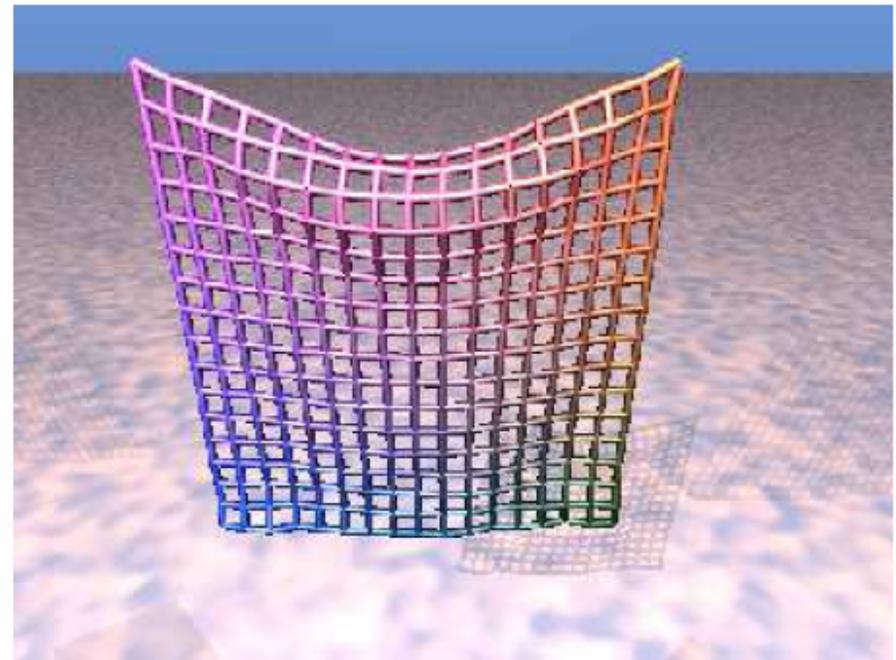
(b) After 200 iterations

Figure 2: Deformation of the elastic model of a sheet hanging by two adjacent corners.

Categories of Techniques :: Physical :: **Example 10: Provot 1995**

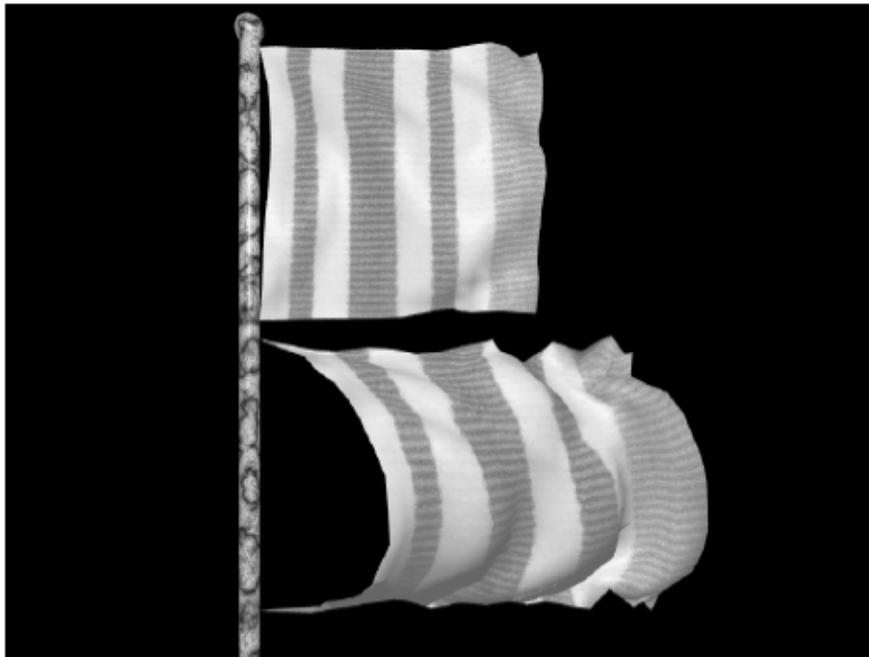


(a) Our method applied to “structural” springs

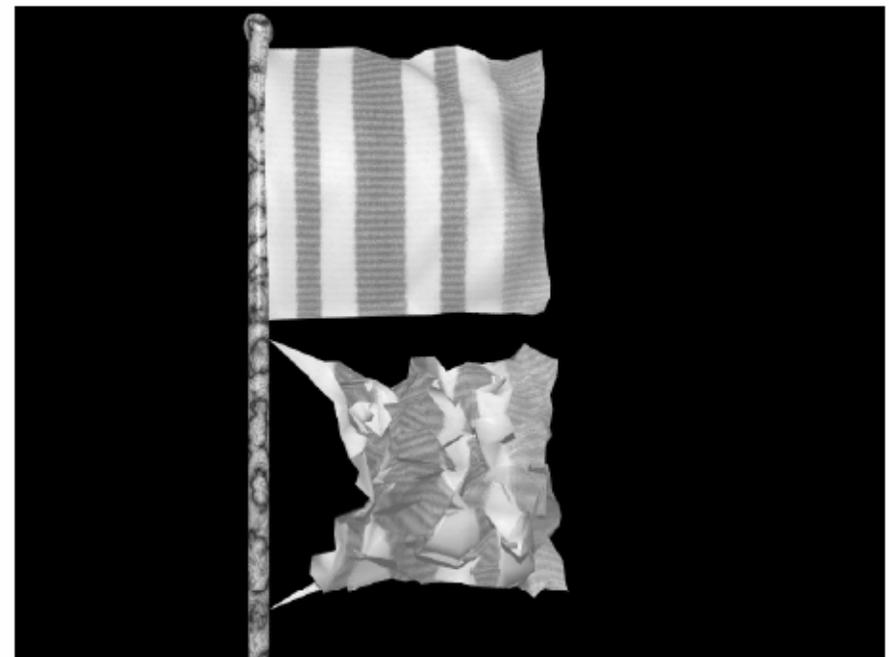


(b) Our method applied to “structural” and “shear” springs

Figure 4: Deformation of our model of a sheet hanging by two adjacent corners (after 200 iterations).



(a) Elastic flag and our “semi-rigid” flag when stiffness is low.



(b) Increasing stiffness: elastic flag becomes chaotic and “semi-rigid” flag remains stable.

Figure 6: Comparison for the case of a flag.

Categories of Techniques :: Physical :: **Example 10: Provot 1995**

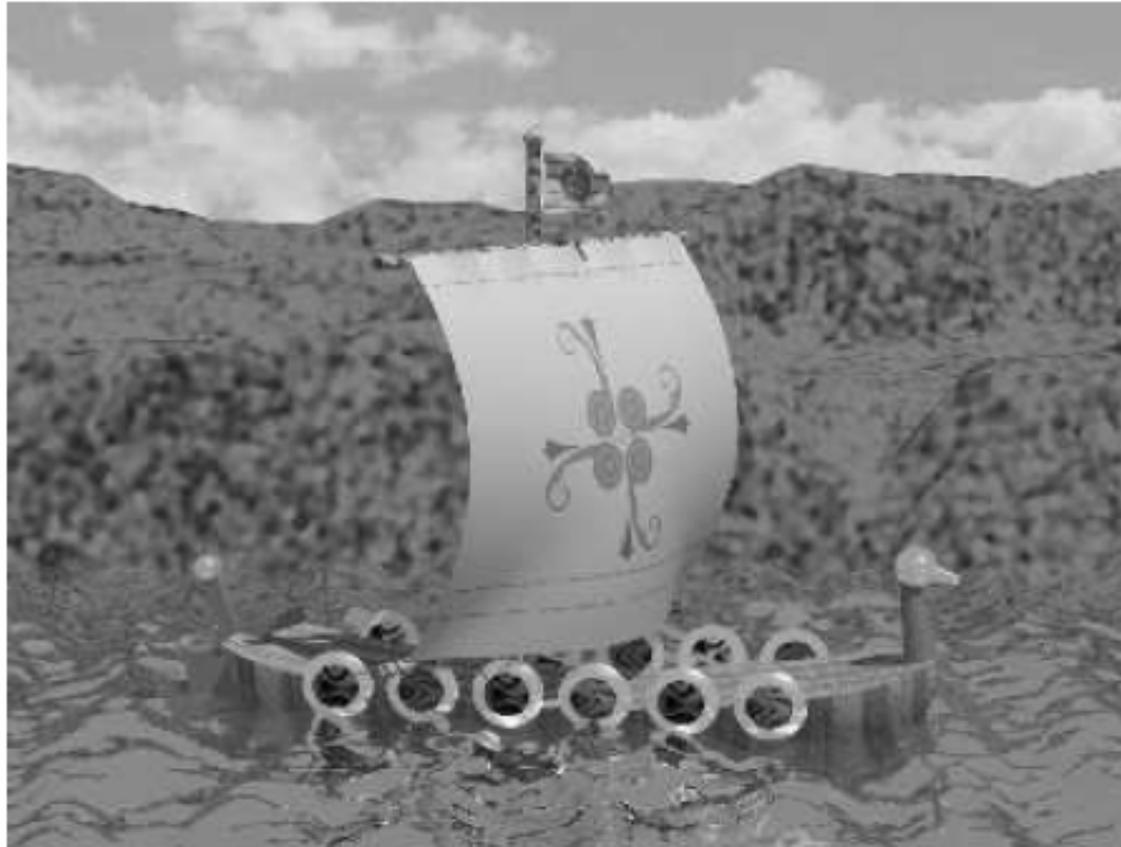


Figure 7: Wind blowing in a sail.

Categories of Techniques :: Physical :: **Example 11: Baraff and Witkin 1998**

- Energy-based
- Use of implicit method (Backward Euler) to integrate, allowing larger Δt
- Use of adaptive time stepping
 - Simulation is run with a parameter that indicates the maximum allowable step size (set by the user, and is always less than or equal to one frame.) (Most of their simulations involving human motions use a step size of 0.02 seconds.)
 - Whenever the simulator reduces the step size, after two successes with the reduced step size the simulator tries to increase the step size.
 - If the simulator fails at the larger step size, it reduces the size again, and waits for a longer period of time before retrying to increase the step size.
 - At its limit, the simulator will try increasing the step size every 40 steps; thus, if the user chooses too large a step, the simulator settles down to wasting only one out of every 40 steps in attempting too large a step.

Categories of Techniques :: Physical :: **Example 11: Baraff and Witkin 1998**

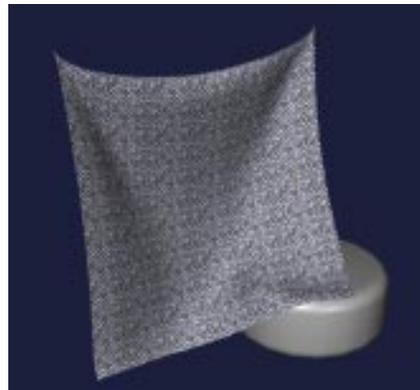
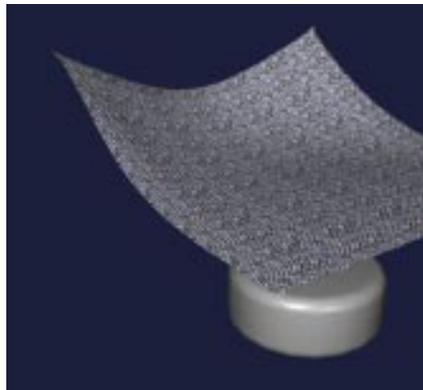
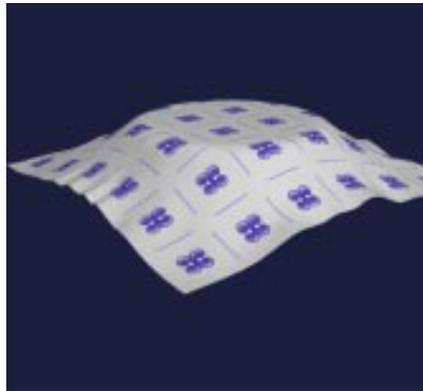


Figure 1 (top row): Cloth draping on cylinder; frames 8, 13 and 35.

Figure 2 (second row): Sheet with two fixed particles; frames 10, 29 and 67.

Categories of Techniques :: Physical :: **Example 11: Baraff and Witkin 1998**



Figure 3 (top row): Shirt on twisting figure; frames 1, 24 and 46.

Figure 4 (second row): Walking man; frames 30, 45 and 58.

Categories of Techniques :: Physical :: **Example 11: Baraff and Witkin 1998**



Figure 5 (top row): Dancer with short skirt; frames 110, 136 and 155.

Figure 6 (second row): Dancer with long skirt; frames 185, 215 and 236.

Categories of Techniques :: Physical :: **Example 11: Baraff and Witkin 1998**



Figure 7. Closeups from figures 4 and 6.

Categories of Techniques :: Physical :: Example 11: Baraff and Witkin 1998

| figure | no. vertices/no. triangles | | time/frame (CPU sec.) | step size min/max (ms) | total frames/ total steps | task breakdown percentage | | | | |
|--------|----------------------------|--------------|--------------------------|---------------------------|------------------------------|---------------------------|------|------|------|-----|
| | cloth | solid | | | | EVAL | CG | C/C | C/S | |
| 1 | 2,602/4,9442 | 322/640 | 2.23 | 16.5/33 | 75/80 | 25.7 | 50.4 | 18.3 | 1.4 | |
| 2 | 2,602/4,9442 | 322/640 | 3.06 | 16.5/33 | 75/80 | 17.9 | 63.6 | 15.3 | 0.2 | |
| 3 | 6,450/12,654 | 9,941/18,110 | 7.32 | 16.5/33 | 50/52 | 18.9 | 37.9 | 30.9 | 2.6 | |
| 4 | (shirt) | 6,450/12,654 | 9,941/18,110 | 14.5 | 2.5/20 | 430/748 | 16.7 | 29.9 | 46.1 | 2.2 |
| | (pants) | 8,757/17,352 | 9,941/18,110 | 38.5 | 0.625/20 | 430/1214 | 16.4 | 35.7 | 42.5 | 1.7 |
| 5 | (skirt) | 2,153/4,020 | 7,630/14,008 | 3.68 | 5/20 | 393/715 | 18.1 | 30.0 | 44.5 | 1.5 |
| | (blouse) | 5,108/10,016 | 7,630/14,008 | 16.7 | 5/20 | 393/701 | 11.2 | 26.0 | 57.7 | 1.3 |
| 6 | (skirt) | 4,530/8,844 | 7,630/14,008 | 10.2 | 10/20 | 393/670 | 20.1 | 36.8 | 29.7 | 2.6 |
| | (blouse) | 5,188/10,194 | 7,630/14,008 | 16.6 | 1.25/20 | 393/753 | 13.2 | 30.9 | 50.2 | 1.4 |

Table 1: System performance for simulations in figures 1–6. Minimum and maximum time steps are in milliseconds of simulation time. Time/frame indicates actual CPU time for each frame, averaged over the simulation. Percentages of total running time are given for four tasks: EVAL—forming the linear system of equation (18); CG—solving equation (18); C/C—cloth/cloth collision detection; and C/S—cloth/solid collision detection.

- Triangular mesh cloth model
- Energy-based

$$\ddot{x} = M^{-1} \left(-\frac{\partial E}{\partial x} + F \right)$$

x = geometric state of the cloth

M = mass distribution of the cloth

E = scalar function of x ; the cloth's internal energy

F = a function of x and \dot{x} ; other forces

(air drag, contact, constraint, internal damping) acting on the cloth

Types of Forces:

- Stretch
- Shear
- Bend
- Others: gravity, air-drag, user-controlled "mouse" force
- Damping

- Every particle has a changing position x_i
- Given a vector condition $C(x)$ which we want to be zero
- Associate an energy function E_C with C

$$E_C(x) = \frac{k}{2} C(x)^T C(x)$$

k = stiffness constant

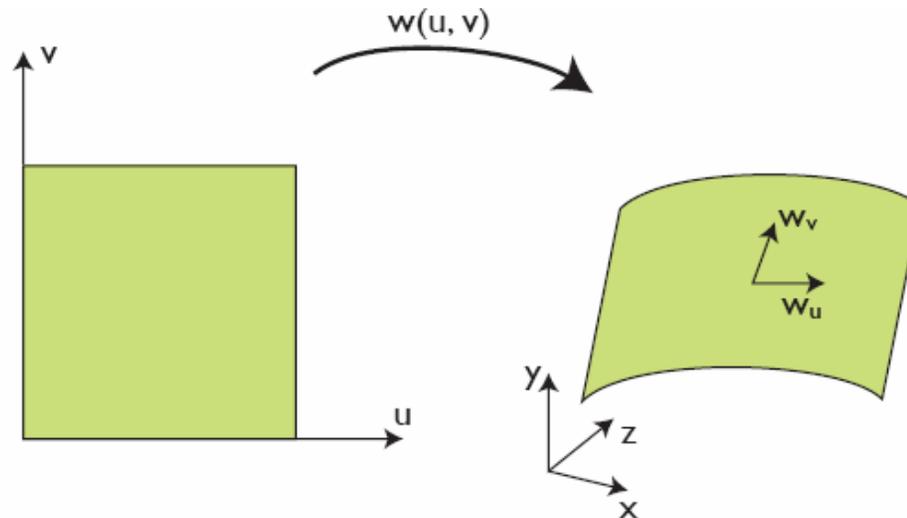
Categories of Techniques :: Physical :: Example 11: Baraff and Witkin 1998 ::
Finding the Governing Equations :: **Forces**

The force of each particle i that C depends on is:

$$\begin{aligned} f_i &= -\frac{\partial E_C}{\partial x_i} = \frac{\partial}{\partial x_i} \left(\frac{k}{2} C(x)^T C(x) \right) \\ &= k \frac{\partial C(x)}{\partial x_i} C(x) \end{aligned}$$

Premises:

- Every cloth particle has a changing position x_i in world space and a fixed plane coordinate (u_i, v_i)
- We have a continuous function $w(u, v)$ that maps from plane coordinates to world space.



Premises:

- Stretch at any point on the cloth surface can then be measured by:

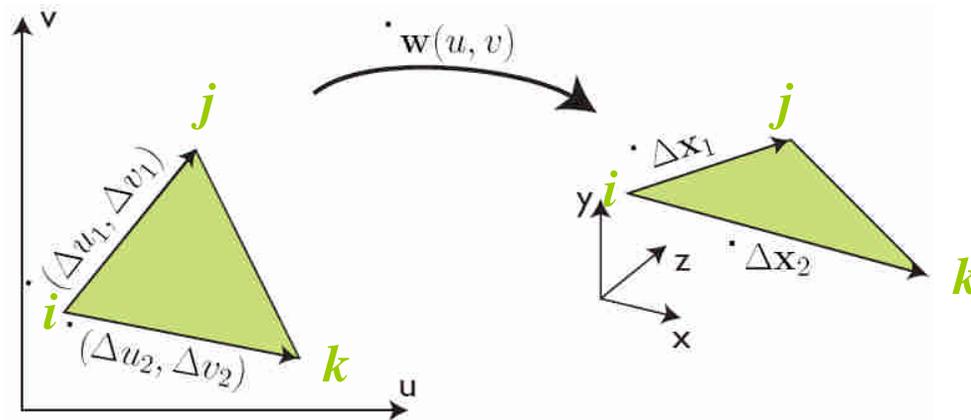
$$w_u = \frac{\partial w}{\partial u}$$

$$w_v = \frac{\partial w}{\partial v}$$

- But how to solve for w_u and w_v ? See next slide.

To solve for w_u and w_v :

When applied to a triangle:



$$\Delta x_1 = w_u \Delta u_1 + w_v \Delta v_1$$

$$\Delta x_2 = w_u \Delta u_2 + w_v \Delta v_2$$

To solve for w_u and w_v using linear approximation over each triangle:

$$\Delta x_1 = w_u \Delta u_1 + w_v \Delta v_1$$

$$\Delta x_2 = w_u \Delta u_2 + w_v \Delta v_2$$

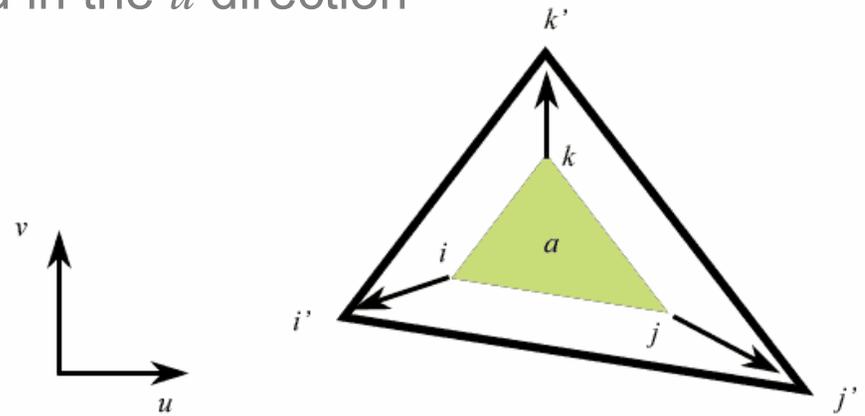
$$\Rightarrow (w_u \quad w_v) = (\Delta x_1 \quad \Delta x_2) \begin{pmatrix} \Delta u_1 & \Delta u_2 \\ \Delta v_1 & \Delta v_2 \end{pmatrix}^{-1}$$

Condition $C(x)$ used for the stretch energy:

$$C(x) = a \left(\begin{array}{l} \left\| w_u(x) \right\| - b_u \\ \left\| w_v(x) \right\| - b_v \end{array} \right)$$

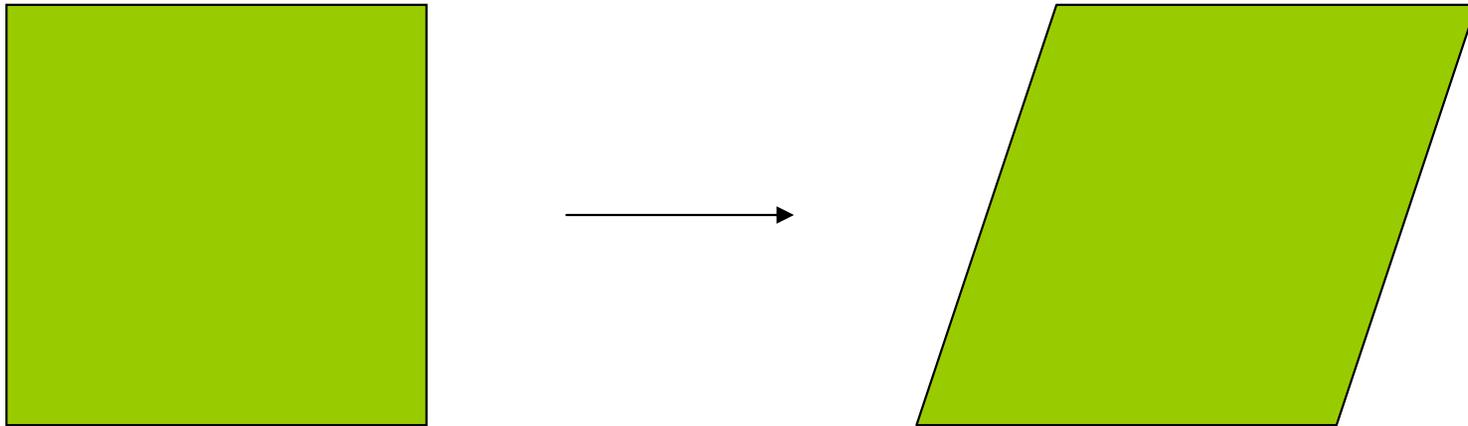
magnitude stretched in the u direction

magnitude stretched in the v direction



where $a =$ triangle's area in uv coordinates
 $b_u = b_v =$ rest length $= 1$

Categories of Techniques :: Physical :: Example 11: Baraff and Witkin 1998 ::
Finding the Governing Equations :: Forces :: **Shear**



Condition $C(x)$ used for the shear energy:

$$C(x) = a w_u(x)^T w_v(x)$$

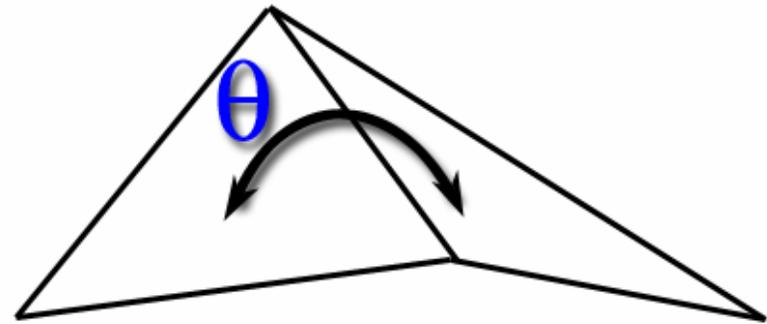
where $a =$ triangle's area in uv coordinates

the inner product $w_u(x)^T w_v(x) \approx$ *shearing angle*

Categories of Techniques :: Physical :: Example 11: Baraff and Witkin 1998 ::
Finding the Governing Equations :: Forces :: **Bending**

Condition $C(x)$ used for the bending energy:

$$C(x) = \theta$$



- Important both for realism and numerical stability
- Act in direction of corresponding elastic force
- Proportional to the velocity in that direction
- Damping force:

$$f_d = -k_d \frac{\partial C(x)}{\partial x} \dot{C}(x)$$

/ velocity

/ direction of force

- Constraints are imposed on individual cloth particles
- Can be automatically determined by the user (such as geometric attachment constraints on a particle)
- Can be contact constraints between a solid object and a particle

Basic Idea:

- Enforcing constraints by mass modification
- For example: zero acceleration along z-axis

$$\ddot{x} = \begin{pmatrix} \frac{1}{m_i} & 0 & 0 \\ 0 & \frac{1}{m_i} & 0 \\ 0 & 0 & 0 \end{pmatrix} f_i$$

- resultant sparse linear system is solved using conjugate gradient
- Integration: Backward Euler (implicit method)

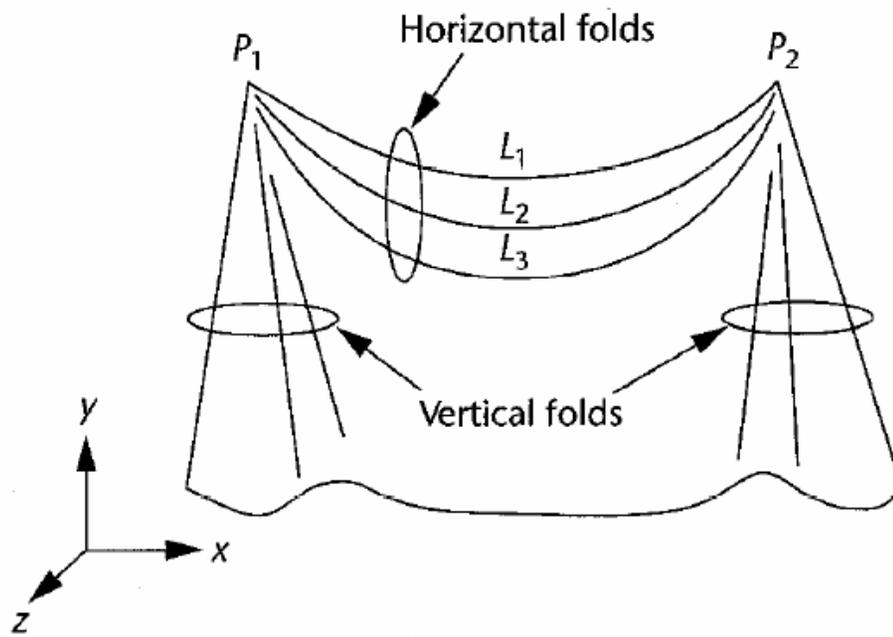
- **Collision detection:**
 - cloth-cloth:
 - particle-triangle intersection
 - edge-edge intersection
 - cloth-solid object
 - cloth particle against the faces of solid object
- **Collision Response:**
 - cloth-cloth: insert a strong damped spring force to push the cloth apart
 - cloth-solid:
 - When cloth contacts a solid, lock the particle onto the surface, if the relative tangential velocity is low.
 - If the tangential force exceeds some fraction of the normal force, allow the particle to slide on the surface.

Hybrid Techniques

- Geometrical + Physical
- In general, use the geometrical techniques to get a rough shape for the simulated cloth, and then use a physical technique to refine.

- Cloth is represented as a grid of spring-mass system
- Find energy minima using gradient descent method
- Then use singularity theory to characterize the resulting wrinkles. Use a sinusoidal curve to construct the surface.
- Demonstrated example: animation sequences of a sleeve on a bending arm; no interactions with the underlying object were considered

- To model folds in a hanging cloth
- Categorized folds into 2 types: horizontal and vertical



23 Two types of folds: horizontal and vertical.

- Use geometrical methods (catenaries) to model the horizontal folds
- Use relaxation process with constraints to model vertical folds

Geometrical vs. Physical Techniques

Summary :: Geometrical vs. Physical Methods

- Geometrical
 - Aim at modeling the shape and appearance of the cloth
 - Do not attempt to model the cloth properties faithfully
 - Do not have as intuitive control over parameters selection
 - Shorter computation time
- Physical
 - Take longer computation time because they usually involve a large set of differential equations or iterating the process many times to search for an energy minima
 - Allow intuitive control in selecting the parameter values, e.g. mass value corresponds to the weight of the physical cloth