Do ambiguous reconstructions always give ambiguous images?

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Abstract

In many cases self-calibration is not able to yield a unique solution for the 3D reconstruction of a scene. This is due to the occurrence of critical motion sequences. If this is the case, an ambiguity is left on the reconstruction. In this paper it is derived under which conditions correct novel views can be generated from ambiguous reconstructions. The problem is first approached from a theoretical point of view. It is proven that novel views are correct as long as the inclusion of the new view in the sequence yields the same ambiguity on the reconstruction. The problem is therefore much related to the problem of critical motion sequences since the virtual camera can be arbitrarily moved within the smallest critical motion set that contains the recovered camera motion without distortions becoming visible. Based on these result a practical measure for the expected ambiguity on a novel view based on the recovered structure and motion is derived. As an application a viewer was built that indicates if a specific novel view can be trusted or not by changing the background color.

1. Introduction

One of the important applications in computer vision is to retrieve the 3D structure of a scene from a collection of images. However, depending on the available knowledge and the images at hand, it is not always possible to obtain a unique solution for this problem. One well known ambiguity is when the observed features and the projection centers of the camera are all located on a special type of surface, called a critical surface [11]. Another well-known ambiguity is that when totally uncalibrated cameras are used, it is only possible to recover the structure of the scene up to an arbitrary projective transformation [1, 3]. It is possible to reduce this ambiguity by imposing constraints on the intrinsic parameters of the camera. This is in general understood as **self-calibration**. In recent years many different methods were proposed. Some are based on the assumptions that the intrinsics do not change during acquisition [2, 4, 13, 18]. Other method relax the constraint for constant intrinsics but require the knowledge of one or more intrinsic parameters [5, 7, 14]. It was proven that for sufficiently general motion the knowledge of one intrinsic camera parameter is sufficient to allow for successful self-calibration [14, 8].

However, in practice the motion of the camera is often restricted and there remains an ambiguity on the reconstruction. This is known as the problem of **critical motion sequences** (CMS). It was introduced by Sturm [15] and further studied in [9, 10, 17, 12]. Depending on the constraints available for self-calibration different classes of motions can be identified as critical. For each of these classes a specific ambiguity remains on the reconstruction.

It depends on the application if some ambiguity is acceptable or not. There are two main classes of applications for 3D reconstructions from images. The first one consists of metrology applications and in most cases no ambiguity can be tolerated. The second type of applications consists of visualization. In this case the goal is to generate novel views based on original images. Over the last years this second type of applications has received more and more attention. Image-based modeling of 3D objects or scenes has become a major topic in both computer vision and computer graphics. Considering this application the important point is not the correctness of the reconstruction, but the correctness of the novel views that are generated from it. This problem was already partially addressed in [10], but only theoretically, for constant intrinsics and using a more restricted case by case analysis.

In this paper a general theorem is derived that allows to determine which motion a virtual camera can undergo to generate unambiguous novel views given the recovered (ambiguous) motion of the original camera. Further on, a practical algorithm is presented that allows to characterize the ambiguity on a novel view. This was used in a number of synthetic experiments to verify the validity of the theorem on some restricted motion sequences and to derive some more insight into this problem. This algorithm was also included into a 3D viewer that tells the user in how far he can trust a specific view based on the poses of the original cameras (and the applicable constraints on the intrinsics). This could for example be used to optimize fly-throughs of virtual worlds containing visual 3D reconstructions.

2. Background

Some familiarity with the projective formulation of vision geometry is assumed [6]. A **perspective camera** is modeled through the projection equation $\mathtt{m} \sim \mathbf{P}\mathtt{M}$ where $\sim$ represents the equality up to a non-zero scale factor, $\mathtt{M} = [X\,Y\,Z\,1]^\top$ represents a 3D world point, $\mathtt{m} = [x\,y\,1]^\top$ represents the corresponding 2D image point and $\mathbf{P}$ is a 3×4 projection matrix. In a metric or Euclidean frame $\mathbf{P}$ can be factorized as follows

$$\mathbf{P} = \mathbf{K}\mathbf{R}^\top[\,\mathbf{I}\,|\text{-}\mathtt{t}] \text{ where } \mathbf{K} = \begin{bmatrix} f & s & u \\ & rf & v \\ & & 1 \end{bmatrix} \quad (1)$$

contains the intrinsic camera parameters, $\mathbf{R}$ is a rotation matrix representing the orientation and $\mathtt{t}$ is a 3-vector representing the position of the camera. The intrinsic camera parameters f represent the focal length measured in width of pixels, r is the aspect ratio of pixels, (u, v) represents the coordinates of the principal point and s is a term accounting for the skew. In general s can be assumed zero. In practice, the principal point is often close to the center of the image and the aspect ratio r close to one.

Projective geometry only encodes cross-ratios and incidence. The affine structure (parallelism and ratios of parallel lengths) is encoded by defining the plane at infinity Π_∞. Euclidean structure (lengths and angles) is encoded by a proper virtual conic on Π_∞. The simplest way to represent this **absolute conic** is by its envelope, i.e. a disc-quadric represented by a 4×4 symmetric positive semidefinite rank-3 matrix Ω^*. In a metric frame $\Omega^* = diag(1,1,1,0)$. The null-space of Ω^* is the plane at infinity Π_∞ and thus $\Omega^*\Pi_\infty = 0$. The similarities or **metric** transformations (i.e. Euclidean plus a global scale-factor) are exactly the transformations that leave the absolute conic unchanged. The following abbreviations will be used repeatedly throughout the text AC for Absolute Conic and IAC for Image of the Absolute Conic.

The AC is the central concept for self-calibration. Localizing the AC in a projective frame allows to upgrade this frame to a metric one. Since it is invariant to rigid displacements, the IAC is only depending on the intrinsic calibration and not on the extrinsic parameters (i.e. camera pose). Constraints on the intrinsic camera parameters can thus be translated to constraints on the IAC. These can then be back-projected to constraints on the AC. In general it is then possible to single out the absolute conic by combining sufficient constraints from different views, i.e. at least 8 equations are needed. It was shown that this was possible imposing only the rectangularity of pixels [14].

The self-calibration approach can be formulated as follows. If $\mathcal{P}$ represents the set of camera projection matrices for an image sequence, then the AC, represented by its envelope Ω^*, can be found as the proper virtual conic for which for every $\mathbf{P} \in \mathcal{P}$ there exists a valid $\mathbf{K}$ satisfying the self-calibration constraints so that

$$\mathbf{P}\Omega^*\mathbf{P}^\top \sim \mathbf{K}\mathbf{K}^\top \ . \quad (2)$$

The problem is, however, that for a specific set of self-calibration constraints, not all motion sequences will yield a unique solution for the AC. In this case there is more than one potential absolute conic and the motion sequence is termed **critical** with respect to the set of constraints. The set of potential absolute conic is defined as $C(\mathcal{P})$. In this case an ambiguity will persist on the reconstruction.

3. Theoretical analysis

The classification of all possible CMS for a specific set of self-calibration constraints can be used to avoid critical motions when acquiring an image sequence on which one intends to use self-calibration. In some cases, however, an uncalibrated image sequence is available from which a metric reconstruction of the recorded scene is expected. In this case, it is not always clear, at first, what can be achieved nor if the motion sequence is critical or not.

It can be shown that the recovered set of cameras also has to satisfy the self-calibration constraints. This result is also valid for CMS, where the recovered motion sequence would be in the same CMS class as the original sequence. In [16] a proof was given for the case of constant intrinsic camera parameters. Here a simpler and more general proof based on the disc quadric representation is given. It is valid for all possible types of self-calibration constraints.

Lemma 1 (Conjugacy of self-calibration solutions) *Let $\mathcal{P}$ be a camera sequence and let $\Phi^* \in C(\mathcal{P})$. Let $\mathbf{T}$ be any projective transformation mapping Φ^* to Ω^* and let $\tilde{\mathcal{P}}$ be the transformed sequence. Then $\Omega^* \in C(\tilde{\mathcal{P}})$ and for each $\Psi \in C(\mathcal{P})$ it follows that the transformed $\mathbf{T}\Psi^*\mathbf{T}^\top \in C(\tilde{\mathcal{P}})$.*

Proof: Since $\Phi^* \in C(\mathcal{P})$, it follows that for every $\mathbf{P} \in \mathcal{P}$ the $\tilde{\mathbf{K}}$ obtained from

$$\tilde{\mathbf{K}}\tilde{\mathbf{K}}^\top \sim \mathbf{P}\Phi^*\mathbf{P}^\top$$

must satisfy the self-calibration constraints. Since $\Phi^* \sim \mathbf{T}^{-1}\Omega^*\mathbf{T}^{-\top}$ and $\tilde{\mathbf{P}} = \mathbf{P}\mathbf{T}^{-1}$, one gets

$$\tilde{\mathbf{K}}\tilde{\mathbf{K}}^\top \sim \tilde{\mathbf{P}}\Omega^*\tilde{\mathbf{P}}^\top$$

and therefore $\Omega^* \in C(\mathcal{P})$. The second claim is trivially proven since $(\mathbf{K}\mathbf{K}^\top \sim)\ \mathbf{P}\Psi^*\mathbf{P}^\top = \tilde{\mathbf{P}}(\mathbf{T}\Psi^*\mathbf{T}^\top)\tilde{\mathbf{P}}^\top$ q.e.d.

From this lemma one can conclude that any reconstruction (i.e. structure and motion) being a solution to the self-calibration problem allows us to identify the ambiguity on the reconstruction. In that case more specific algorithms can be called or additional constraints can be brought in to reduce the ambiguity [19]. In addition, the values for the intrinsic camera parameters are not arbitrary. The pixels are always rectangular and most often close to squares. The principal point is in general close to the center of the image and even the focal length can not take on arbitrary values. On top of that one can impose that all the visible points must be located in front of the camera (see [5]). This means that even for CMS one will in general be able to do much better than what could be expected from the CMS analysis.

Nevertheless, in some cases there will be an ambiguity on the structure and motion reconstruction. An important question is then: *What can still be done with an ambiguous reconstruction?* An interesting answer is given by the next theorem (a similar result was recently presented by Ma [10]).

Theorem 1 (correctness of novel views within CMS) *Let $\mathcal{P}$ represent a CMS. Let Φ^* be an arbitrary element of $C(\mathcal{P})$ and let $\mathbf{T}$ be an arbitrary projective transformation mapping Φ^* to Ω^*. Let $\tilde{\mathcal{P}}$ be the transformed sequence. Let $\tilde{\mathbf{P}}_V$ represent a novel camera projection matrix for which $C(\tilde{\mathcal{P}} \cup \tilde{\mathbf{P}}_V) = C(\tilde{\mathcal{P}})$. Then the corresponding projection matrix $\mathbf{P}_V = \tilde{\mathbf{P}}_V \mathbf{T}$ satisfies the self-calibration constraints.*

Proof: From $\Omega^* \in C(\mathcal{P})$, it follows that $\mathbf{T}\Omega^*\mathbf{T}^\top \in C(\tilde{\mathcal{P}})$. Since it is assumed that $C(\tilde{\mathcal{P}} \cup \tilde{\mathbf{P}}_V) = C(\tilde{\mathcal{P}})$, it follows that Lemma 1 can be applied to the sequence $\tilde{\mathcal{P}} \cup \tilde{\mathbf{P}}_V$, with the dual quadric $\mathbf{T}\Omega^*\mathbf{T}^\top$ and the transformation $\mathbf{T}^{-1}$. q.e.d.

This theorem allows us to conclude that it is possible to generate correct new views (i.e. with a virtual camera for which the equivalent camera in the real world satisfies the self-calibration constraints), even starting from an ambiguous reconstruction. In this case, we should, however, restrict the motion of the virtual camera to the type of the CMS recovered in the reconstruction. For example, if a model was acquired by a camera with constant intrinsic parameters performing a planar motion on the ground plane and thus rotating around vertical axes, then we should not move the camera outside this plane nor rotate around non-vertical axes. But, if we restrict our virtual camera to this critical motion, then all these motions will correspond to Euclidean motions in the real world and no distortion will be present in the images (except, of course, for modeling errors). Note that the *recovered* camera parameters should be used (i.e. the ones obtained by factorizing $\tilde{\mathbf{P}}$ in $\tilde{\mathbf{K}}[\tilde{\mathbf{R}}^\top | \text{-}\tilde{\mathbf{R}}^\top \tilde{\mathbf{t}}]$.), except when some parameters are unconstraint in which case these parameters are allowed to be varied. In fact, this result is related to the more general rule that for the generation of new views interpolation is more desirable than extrapolation.

4. A practical approach

Self-calibration consists in general of estimating the position of the absolute conic through the minimization of a least-squares function

$$\mathbf{F}(\Omega) = \mathbf{f}(\Omega)^\top \mathbf{f}(\Omega) = \sum_i f_i(\Omega)^2 \ . \tag{3}$$

The uncertainty ellipsoids around the estimate $\tilde{\Omega}$ are given by $\Delta\Omega^\top \mathbf{J}^\top \mathbf{J} \Delta\Omega = k^2$ with $\Delta\Omega$ an 8-vector representing variations around $\tilde{\Omega}$, the matrix $(J_{ij}) = \left(\frac{\partial f_i}{\partial \Omega_j}\right)$ is the Jacobian of $f(\Omega)$ and k represents a certain level of certitude. The axes of this ellipsoids are given by the eigenvectors of $\mathbf{E}_\Omega = \mathbf{J}^\top \mathbf{J}$ and the lengths of the half-axes are given by $\sqrt{\lambda_j}$ with λ_j the eigenvalues of $\mathbf{E}_\Omega$.

These ellipsoids characterize the ambiguity on the 3D reconstruction. However, in our case the goal is to characterize the ambiguity on a specific view. Therefore, the uncertainty ellipsoids have to be projected in that view. It turns out that it is simpler to deal with the dual entities:

$$\mathbf{E}^*_\omega = \left(\frac{\partial \omega}{\partial \Omega}\right) \mathbf{E}^*_\Omega \left(\frac{\partial \omega}{\partial \Omega}\right)^\top \tag{4}$$

Note that $\mathbf{E}^*_\Omega = \mathbf{E}_\Omega^{-1}$ if $\mathbf{E}_\Omega$ is of full rank. If $\mathbf{E}_\Omega$ is not of full rank, a good approximation is obtained by changing the singular values smaller than a small preset number (e.g. 10^{-6}) to that number.

The ellipsoids corresponding to $\mathbf{E}_\omega$ give us the ambiguity on the image of the absolute conic. This corresponds to the variations in intrinsic camera parameters that could occur due to the ambiguity on the 3D reconstruction. This is not yet what we are looking for, since -as was shown in Theorem 1- these two effects could cancel each other out.

In fact $\mathbf{E}_\Omega$ could be seen as the equivalent of $C(S_P)$ in Theorem 1 since it contains the acceptable absolute conics. When a virtual camera is set up to generate a novel view it should clearly satisfy the self-calibration constraints with respect to $\tilde{\Omega}$. The question is how well it would for other elements of $C(S_P)$. In other words one would like to verify if $C(S_P \cup M) = C(S_P)$. A local approximation for $C(S_P \cup M)$ can be obtained by including constraints for the novel view in the self-calibration minimization criterion $\mathbf{F}(\Omega)$. This yields $\mathbf{E}_{\Omega N} = \mathbf{J}_N^\top \mathbf{J}_N$.

Since the purpose is to investigate the ambiguity in the novel view, this comparison can be carried out on the projection of these sets in this view. The simplest approach to compare two ellipsoids is to apply a transformation so that one of the ellipsoids is transformed to unity. Applying the same transformation to the other ellipsoid yields an ellipsoid that can be seen as the ratio of the two other ellipsoids. Thus if $\mathbf{E}_{\omega N} = \mathbf{C}^\top \mathbf{C}$ (obtain $\mathbf{C}$ through Cholesky factorization), the comparing ellipsoid $\mathbf{E}_{\frac{\omega}{\omega N}} = \mathbf{C}^{-\top} \mathbf{E}_\omega \mathbf{C}^{-1}$.

Figure 1: Example of experimental setup. A sequence of 10 views consisting of orbital motion (type D) observed from an arbitrary viewpoint (type G). The viewpoints are illustrated by little pyramids.

The eigenvectors of $\mathbf{E}_{\frac{\omega}{\omega N}}$ yield the main directions of uncertainty, the square root of the eigenvalues yields the relative uncertainty. As a global measure of ambiguity on a view

$$\alpha = \sum(\sqrt{\lambda_{\frac{\omega}{\omega N}}} - 1)^2 \quad (5)$$

can be used (with $\lambda_{\frac{\omega}{\omega N}}$ the eigenvalues of $\mathbf{E}_{\frac{\omega}{\omega N}}$). Because there is a direct mapping between ω and the intrinsic camera parameters, it is also possible to get an estimate of the relative uncertainty of these parameters. It is thus possible to get an idea of what type of distortion can be expected in the novel views.

5. Synthetic experiments

This approach was applied to determine the dimensionality of the ambiguity on novel views from 3D reconstructions obtained from restricted motion sequences. The dimensionality can be estimated by counting the number of very large eigenvalues of $\mathbf{E}_{\frac{\omega}{\omega N}}$ for synthetic experiments. In Table 1 and Table 2 a few typical motion sequences were analyzed. These different type of motions (i.e. pure translations, pure rotations, planar motion, orbital motion, forward motion, translations together with rotations along the optical axis and general motions) are illustrated in Figure 2. The rows correspond to the type of motion that was used for the reconstruction (10 views) and the columns to the motion for the novel view. The setup is illustrated in Figure 1. The number on the left of the vertical line corresponds to the dimensionality of the ambiguity on the 3D structure and should thus always be larger or equal to the ambiguity on a specific view. In general observation from an arbitrary viewpoint yields the same ambiguity as in 3D space, except for constant intrinsic parameters and pure translations or forward motion (Table 2) where a single extra view would not be sufficient to guarantee unambiguous 3D reconstruction. The number in the table gives the dimensionality of the ambiguity. Notice that as predicted by the theorem the diagonal elements are all zero.

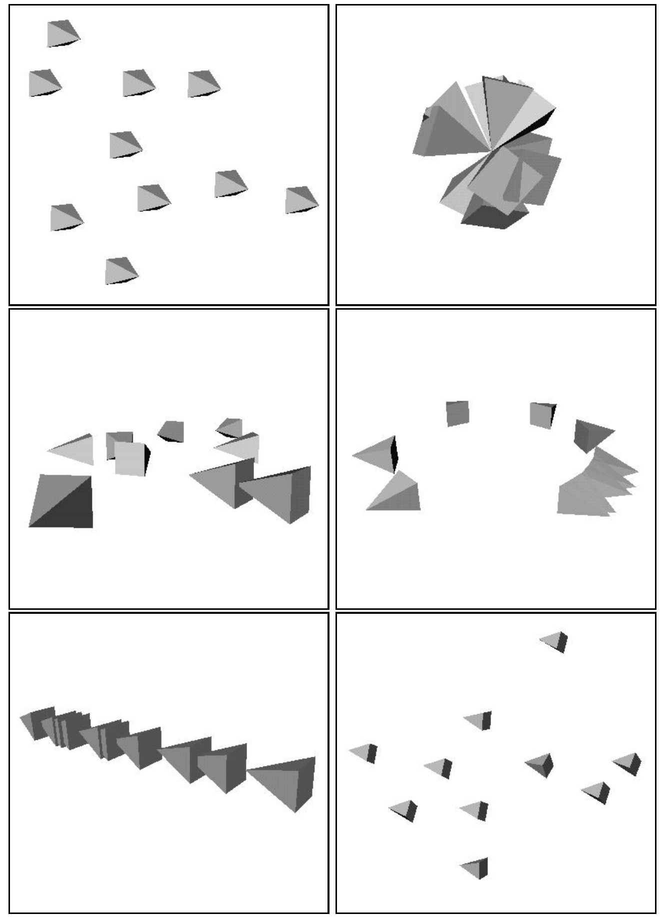

Figure 2: Some restricted motion sequences (pure translation, pure rotation, planar motion, orbital motion, straight forward, translation and rotation along optical axis).

6. A special viewer

As was already mentioned the presented approach can also be used to determine the expected ambiguity for specific views. We have incorporated the ambiguity evaluation algorithm into a 3D viewer. While viewing a 3D reconstruction, the user is informed of the expected ambiguity on the view. Our implementation of this viewer shifts the background color from green to red, depending on the ambiguity. We used $(\beta, \sqrt{1-\beta^2}, 0)$ with $\beta = 1 - e^{-\sqrt{\alpha}}$ as RGB-values for the background color.

For testing purposes we have used the *castle* sequence (see Figures 3). As was already mentioned in [12] self-calibration leaves a large scaling ambiguity along the optical axis for the central viewpoint when assuming all parameters known except the focal length. For illustration purposes the 3D model was scaled by a factor of 2 along this

motion type		A	B	C	D	E	F	G
A pure translation	1	**0**	1	1	1	0	0	1
B pure rotation	3	3	**0**	3	3	2	3	3
C planar motion	0	0	0	**0**	0	0	0	0
D orbital motion	0	0	0	0	**0**	0	0	0
E forward motion	2	1	1	2	2	**0**	1	2
F transl.& rot.opt.ax.	1	0	1	1	1	0	**0**	1
G general	0	0	0	0	0	0	0	**0**

Table 1: Ambiguities for novel views when all intrinsic parameters but the focal length are known

motion type		A	B	C	D	E	F	G
A pure translation	5	**0**	4	4	4	0	4	4
B pure rotation	3	3	**0**	3	3	2	3	3
C planar motion	1	0	1	**0**	0	0	1	1
D orbital motion	2	1	2	1	**0**	1	2	2
E forward motion	5	0	4	4	4	**0**	4	4
F transl.& rot.opt.ax.	1	0	1	1	1	0	**0**	1
G general	0	0	0	0	0	0	0	**0**

Table 2: Ambiguities for novel views when all intrinsic parameters are constant (but unknown)

ambiguity (i.e. this distortion would only cause a minimal increase in residual for the self-calibration cost function), so that the reader can visually verify the predictions of the viewer.

A few views are shown in Figure 4 and Figures 5. It should be clear that even some views very far away from the originally recorded images can be rendered without risk of ambiguity (green or lightviews), while some others that are less far away are showing a lot of ambiguity (red or dark views).

7. Conclusion

In many practical cases the motion of the camera is not sufficiently general to allow for the unambiguous computation of the metric structure and motion. The question that was addressed in this paper is what can still be achieved in terms of generating novel views. It was proven that novel views are correct as long as the inclusion of the new pose in the motion sequence yields the same ambiguity on the reconstruction. Based on this a practical approach was derived that determines the expected ambiguity from novel views. This was used successfully on synthetic data to determine the level of ambiguity for different types of restricted motion sequences and included into a 3D viewer to visualize the expected level of distortion of the rendered views. There could be a number of interesting applications for the presented work. It can extend the range of applications within reach for self-calibration, especially in the area of modeling for visualization. It could for example also be used to automatically optimize a fly-through in virtual environment containing 3D models obtained from image sequences.

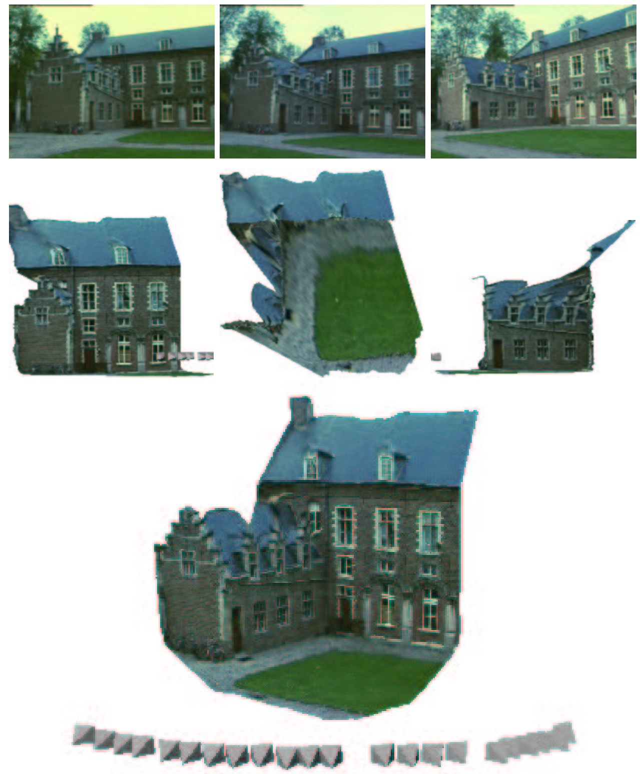

Figure 3: The *castle* sequence. Some images (top), 3 orthographic views (middle) and a general view (bottom).

Acknowledgments

Marc Pollefeys is a post-doctoral fellow of the Fund for Scientific Research - Flanders (Belgium). The financial support of the FWO project G.0223.01 and the ITEA BEYOND project are also gratefully acknowledged.

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Figure 4: Different views of the castle with estimated ambiguities of 0.5, 1, 1.5, 2 and 3.

Figure 5: A few *good* views and a few *bad* views. The estimate ambiguities are 0.1, 0.2, 1 and 6, 8 respectively.

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