Katholieke
Universiteit
Leuven

Departement Elektrotechniek
Afdeling ESAT/PSI
Kardinaal Mercierlaan 94
B-3001 Heverlee - Belgium

TECHNISCH RAPPORT - TECHNICAL REPORT

Some Geometric Insight in Self-Calibration and Critical Motion Sequences

Marc Pollefeys and Luc Van Gool

March 2000

Nr. KUL/ESAT/PSI/0001

Some Geometric Insight in Self-Calibration and Critical Motion Sequences

Marc Pollefeys and Luc Van Gool
ESAT-PSI
Kardinaal Mercierlaan 94
B-3001 Leuven
Belgium
Tel. ++32 16 32 1064 / FAX ++32 16 32 1723
Marc.Pollefeys@esat.kuleuven.ac.be
Luc.VanGool@esat.kuleuven.ac.be

Abstract

Self-calibration and critical motion sequences are problems which have received a lot of attention lately. The key concept for these problems is the absolute conic which appears as a natural calibration object always present in space. To observe it, however, self-calibration constraints are needed. In this paper a method to represent the rather abstract concept of the absolute conic through real geometric entities is proposed. The main benefit of this approach is that it allows to apply geometric intuition to understand problems related to self-calibration and critical motion sequences. The image of the absolute conic can be represented as an ellipse in the image. In this case self-calibration constraints are translated to simple geometric constraints on this ellipse. Several researchers have recently worked on critical motion sequences for self-calibration. The problem is however that the obtained results are hard to understand. Some cases are very hard to grasp intuitively and in an effort to do so the proposed method was developed. By mapping the problem to real geometric entities, even the most intricate cases obtained by the previous analyses (often through the help of automated solving tools) can be understood and visualized. Several of these cases are discussed explicitly in this paper. Several new insights are also presented.

Keywords: Critical Motion Sequences, Self-Calibration, Absolute Conic, Geometry

1 Introduction

Self-calibration has been an important research topic over the last few years. Since it became clear that in general projective reconstructions could be obtained from images acquired with an uncalibrated camera [3, 5], researchers have been looking for ways to upgrade these reconstructions to metric. This can be achieved by using additional scene, motion and/or calibration constraints.

One of the most interesting approaches consists of retrieving the metric properties of the scene by (only) imposing some constraints on the camera intrinsics. This is called self-calibration and has been an important research topic over the last few years. Many approaches have been proposed for unknown but constant intrinsic camera parameters (e.g. [4, 6, 17, 22]). Recently also more flexible approaches that can deal with varying intrinsic camera parameters, have been proposed [18, 9]. It was shown [18, 10, 12] that in principle the constraint of rectangular pixels is sufficient to allow for self-calibration.

Self-calibration is, however, not possible for all camera motions. There exist critical motion sequences (CMS). In these cases self-calibration is unable to achieve a metric reconstruction and some ambiguity remains on the final reconstruction. These CMS clearly depend on the constraints which are imposed for self-calibration. The extremes being the case with no constraints for which every motion is critical and the case were all intrinsic camera parameters are known for which almost no critical motion sequences exist [11].

For constant intrinsic camera parameters a complete analysis was provided by Sturm [19, 20]. An alternative approach was proposed by Ma et al. [15], but this approach is not complete since it only deals with CMS which form subgroups of the Euclidean group. Recently, some results were also given for varying intrinsic camera parameters [12].

Although most of these analyses are based on geometry, the solutions are obtained as solutions of systems of equations and can not always be understood intuitively. The problem is that the absolute conic is a purely virtual entity. In this case our geometric intuition is only partially valid. Although some critical motion sequences can be perfectly understood using our "real" geometric intuition, other cases can not. In general our geometric intuition can not be trusted when virtual entities are involved.

In this paper a method is proposed to translate problems of self-calibration and critical motion sequence analysis from virtual geometric entities to real geometric entities. This allows us to understand intuitively the results obtained in previous analyses of CMS [19, 20, 11, 12]. It is also very important for further analysis of the problem. As was pointed out in [11] automated theoretic tools are usefull, but are still far from solving everything. In addition, the solution is often useless without geometric interpretation.

The paper is organized as follows. In Section 2 some basic concepts are introduced. Section 3 first describes the concept of virtual cones, then the mapping from the virtual entities to an equivalent real representation is given. Section 4 discusses more in detail the image of the absolute conic. In Section 5 it is shown how the analysis of CMS can be understood based on the new representation. The paper is concluded by Section 6.

2 Background

Some familiarity with the projective formulation of vision geometry is assumed [2, 16]. A **perspective camera** is modeled through the projection equation $\mathtt{m} \sim \mathbf{P}\mathtt{M}$ where $\sim$ represents the equality up to a non-zero scale factor, $\mathtt{M} = [X\,Y\,Z\,1]^\top$ represents a 3D world point, $\mathtt{m} = [x\,y\,1]$ represents the corresponding 2D image point and $\mathbf{P}$ is a 3×4 projection matrix. In a metric or Euclidean frame $\mathbf{P}$ can be factorized as follows

$$\mathbf{P} = \mathbf{KR}[\mathbf{I}\,|\text{-}\mathtt{t}] \text{ where } \mathbf{K} = \begin{bmatrix} f_x & s & u_x \\ & f_y & u_y \\ & & 1 \end{bmatrix} \tag{1}$$

contains the intrinsic camera parameters, $\mathbf{R}$ is a rotation matrix representing the orientation and $\mathtt{t}$ is a 3-vector representing the position of the camera. The intrinsic camera parameters f_x and f_y represent the focal length measured in width resp. height of pixels, (u_x, u_y) represents the coordinates of the principal point and s is a term accounting for the skew. In general s can be assumed zero. In practice, the principal point is often close to the center of the image and the aspect ratio $\frac{f_y}{f_x}$ close to one.

A **quadric** is defined by the quadratic form $\mathtt{M}^\top \mathbf{Q}\mathtt{M} = 0$, where $\mathbf{Q}$ denotes a 4×4 symmetric matrix and $\mathtt{M}$ a homogeneous representation of a 3D-point. The dual concept is a quadric envelope defined by $\Pi^\top \mathbf{Q}^* \Pi = 0$ which contains the planes tangent to the quadric. For non-singular matrices $\mathbf{Q}^* \sim \mathbf{Q}^{-1}$ and the quadric is said proper. Quadrics with no real points are called virtual. Similar properties exist for the 2D concept which is called a conic. The image of a quadric is a conic and can be expressed in envelope form as

$$\mathbf{C}^* \sim \mathbf{PQ}^*\mathbf{P}^\top \tag{2}$$

Projective geometry only encodes cross-ratios and incidence. The affine structure (parallellism and ratios of parallel lengths) is encoded by defining the plane at infinity Π_∞. Euclidean structure (lengths and angles) is encoded by a proper virtual conic on Π_∞. The simplest way to represent this **absolute conic** is by its envelope, i.e. a 4×4 symmetric positive semidefinite rank 3 matrix Ω^*_∞. In a metric frame $\Omega^*_\infty = diag(1,1,1,0)$. The null-space of Ω^*_∞ is the plane at infinity Π_∞ and thus $\Omega^*_\infty \Pi_\infty = 0$. The similarities or **metric** transformations (i.e. Euclidean plus a global scalefactor) are exactly the transformations that leave the absolute conic unchanged. The affine transformations leave the plane at infinity unchanged. The following abreviations will be used repeatedly throughout the text (D)AC for (Dual) Absolute Conic and (D)IAC for (Dual) Image of the Absolute Conic.

The AC is the central concept for self-calibration. Localizing the AC in a projective frame allows to upgrade this frame to a metric one. Since it is invariant to rigid displacements, the IAC is only depending on the intrinsic calibration and not on the extrinsic parameters (i.e. camera pose). Constraints on the intrinsic camera parameters can thus be translated to constraints on the IAC. These can then be back-projected to constraints on the AC. In general it is then possible to single out the absolute conic by combining sufficient constraints from different views. It was shown that this was possible imposing only the rectangularity of pixels [18].

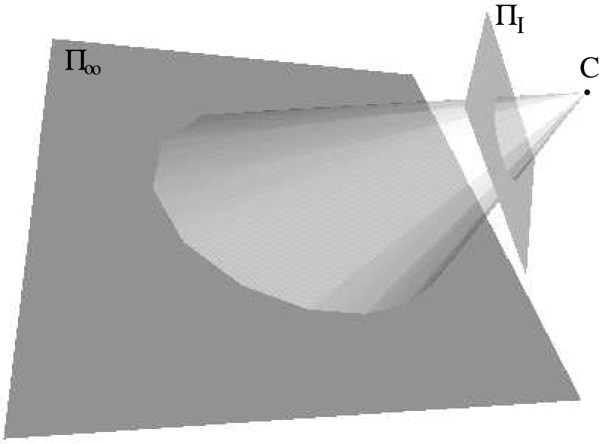

Figure 1: Projection cone of a conic

The self-calibration approach can be formulated as follows. If $\mathcal{P}$ represents the set of camera projection matrices for an image sequence and $\mathcal{K}$ represents the set of possible calibration matrices, then the AC, represented by its envelope Ω^*_∞, can be found as the proper virtual conic for which for every $\mathbf{P} \in \mathcal{P}$ there exists a $\mathbf{K} \in \mathcal{K}$ so that

$$\mathbf{P}\Omega^*_\infty\mathbf{P}^\top \sim \mathbf{K}\mathbf{K}^\top \ . \tag{3}$$

The problem is, however, that for a specific set of self-calibration constraint (i.e. constraints on the intrinsic camera parameters), not all motion sequences will yield a unique solution for the AC. In this case the motion sequence is termed **critical** with respect to the set of constraints. These sequences are called Critical Motion Sequences (CMS). In this case an ambiguity will persist on the metric frame.

3 Planar sections of virtual cones

The projection of the absolute conic on the image plane can be described by a projection cone. The center of this cone is the center of projection of the camera. The intersection with the plane at infinity is the AC and the intersection with the image plane is the IAC. In many papers this would be illustrated as in Figure 1.

The problem is however that this cone is a virtual cone. In this case our intuition is note reliable. Although in some cases these virtual entities exhibit the same properties as their real counterparts, in other cases our intuition can not be trusted.

For the analysis of self-calibration and CMS it is, however, important to have some intuition in the geometry of the problem. In this section it is shown that the intersection of virtual cones with planes can be mapped to an equivalent problem with real entities. In the latter case, however, geometric intuition can be used reliably. Before introducing this new representation, some general properties of virtual conics are given.

A virtual cone is described by a 4×4 positive semidefinite rank 3 matrix Γ. The center of the cone $\mathtt{C}$ corresponds to the nullspace of Γ, i.e. $\Gamma\mathtt{C} = 0$. The point-equation of the Euclidean standard form of such a cone is given by the following equation:

$$\mathtt{M}^\top \Gamma \mathtt{M} = 0 \text{ with } \Gamma = diag(\frac{1}{a^2}, \frac{1}{b^2}, \frac{1}{c^2}, 0) \tag{4}$$

with a, b, c strict positive scalefactors corresponding to the scaling of the cone along the main axes. Three different types of virtual cones exist. When a, b and c are equal the cone is an isotropic cone. The intersection of this cone with an arbitrary plane always results in a virtual (centered[1]) circle. Note that this behaviour can not be understood in terms of real cones (cfr. Figure 1). When only two of the three scalefactors are equal the cone is a circular cone. When all three scalefactors are different the cone is elliptic. For a specific proper virtual conic ϕ and a point $\mathtt{C}$ (not it the supporting plane of the conic), a unique virtual cone Γ can be determined so that it contains ϕ and has $\mathtt{C}$ as its center. The inverse, the fact that a specific cone Γ has a unique intersection ϕ with a plane, is obviously also the true.

The **mapping of the virtual ellipse** ϕ to the real ellipse $\mathcal{R}(\phi)$ is defined as follows:

$$\phi = \mathbf{H}^\top \mathrm{diag}(1,1,1)\mathbf{H} \leftrightarrow \mathcal{R}(\phi) = \mathbf{H}^\top \mathrm{diag}(1,1,-1)\mathbf{H} \tag{5}$$

where $\mathbf{H}$ represents an arbitrary 2D affine transformation.

Let us also define the **mapping of the virtual cone** Γ to the family of real ellipsoids $\mathcal{R}(\Gamma, \lambda)$:

$$\Gamma = \mathbf{T}^\top \mathrm{diag}(1,1,1,0)\mathbf{T} \leftrightarrow \mathcal{R}(\Gamma, \lambda) = \mathbf{T}^\top \mathrm{diag}(1,1,1,-\lambda^2)\mathbf{T} \tag{6}$$

where $\mathbf{T}$ represents an arbitrary 3D affine transformation and λ a scale factor which can vary. Let us also define the **ratio of intersection** R between a plane and an ellipsoid as 1 when the plane passes through the center of the ellipse, as 0 when it is tangent and linearly (with the distance between the planes) in between. This is illustrated in Figure 2

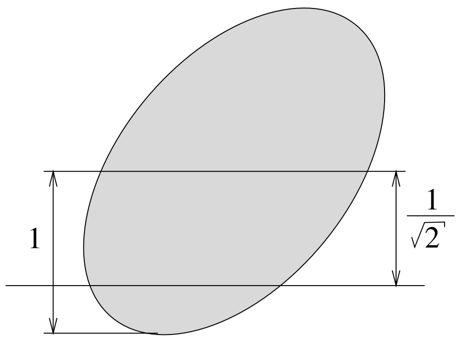

Figure 2: Illustration of the concept ratio of intersection ($R = \frac{1}{\sqrt{2}}$)

[1] By centered it is meant that the point of the plane closest to the center of the cone is the center of the circle.

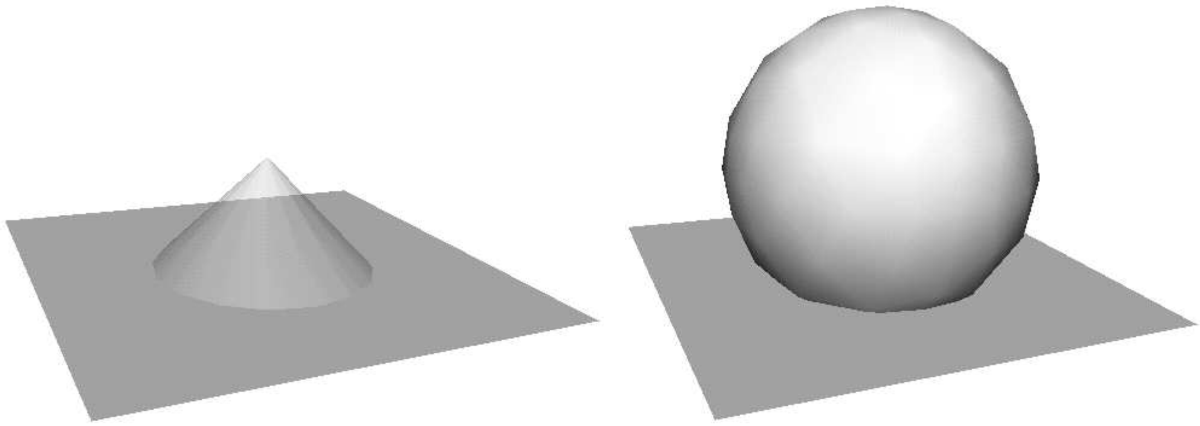

Figure 3: Traditional representation of virtual cone intersection with plane and proposed representation $\mathcal{R}(\Gamma, \lambda)$.

Theorem 1 *The intersection of the imaginary cone* Γ *with plane* Π *can be obtained as* ϕ *corresponding to* $\mathcal{R}(\phi)$ *which is the intersection of the ellipsoid* $\mathcal{R}(\Gamma, \lambda)$ *where* λ *is determined so that the ratio of intersection is* $\frac{1}{\sqrt{2}}$.

Proof: Let us first prove this for the simple case where $\Gamma : X^2+Y^2+Z^2 = 0$ and $\mathbf{\Pi} : Z = 1$:

$$\begin{array}{rcl} \Gamma : X^2 + Y^2 + Z^2 = 0 & \leftrightarrow & \mathcal{R}(\Gamma, \sqrt{2}) : X^2 + Y^2 + Z^2 = \sqrt{2}^2 \\ \phi : X^2 + Y^2 + 1 = 0 & \leftrightarrow & \mathcal{R}(\phi) : X^2 + Y^2 + 1 = \sqrt{2}^2 \end{array} \tag{7}$$

All other relative positions of planes and cones can be achieved by applying affine transformations to the geometric entities defined above. Since tangency, intersections and ratios of lengths along some line are affine invariants it follows that the mapping is valid for all imaginary cones and all planes.

Theorem 1 gives a way to represent the intersection of a virtual cone with a plane through real entities only. This allows the use of geometric intuition. An illustration of the idea is given in Figure 3.

4 The image of the absolute conic

As was seen in Section 2, one of the key concepts for self-calibration is the IAC. In this section we will discuss a geometric representation of this entity, both in the image and in 3D space. According to Equations (1) and (2) the IAC is given by

$$\omega_\infty^* \sim \mathbf{P}\Omega_\infty^*\mathbf{P}^\top \sim \mathbf{K}\mathbf{K}^\top \ . \tag{8}$$

The new representation, $\mathcal{R}(\omega_\infty)$, of the IAC is an ellipse. The center of this ellipse corresponds the principal point, for zero skew the axes are alligned with the image axis and for

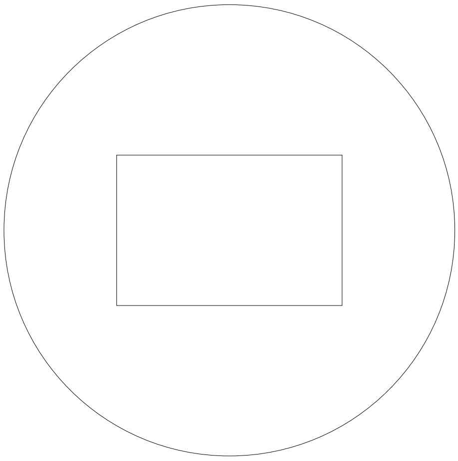

Figure 4: $\mathcal{R}(\omega_\infty)$ and image borders (for standard 35mm camera)

an aspect ratio of one $\mathcal{R}(\omega_\infty)$ is a circle. The inside of the ellipse corresponds to what is seen through a field-of-view of 90 degrees (45 degrees away from optical axis).

This representation is located in the image and is thus dependend on the way that the image plane is sampled. It is however also possible to represent the IAC in 3D as an ellipse located in the image plane. This representation is independent of the parametrization of the image plane and is always a circle with the center located at the point of the image plane closest to the center of projection and the radius equal to the distance between the center of projection and the image plane. This is illustrated in Figure 4. The rectangle represents the borders of the image for a standard 35mm photocamera.

When there is an ambiguity on the intrinsic camera parameters, an equivalent ambiguity exist on the IAC and thus also on $\mathcal{R}(\omega_\infty)$. For every potential IAC, represented by $\mathcal{R}(\phi)$, a corresponding virtual reprojection cone exist. As seen in Section 3 this cone can be represented by the family of ellipsoids $\mathcal{R}(\Gamma, \lambda)$. Depending on the constraints this cone can also have a constraint form. Fixed intrinsic camera parameters result in a fixed IAC and thus also in a virtual reprojection cone with a fixed shape. The position and orientation obviously depend on camera pose.

If some intrinsic camera parameters are known, this immediadely constraints the shape of the IAC and in some cases also the type of reprojection cone. The different possibilities are summarized in Table 1. The first column contains the intrinsic camera parameters which are assumed known. The second and third column give the corresponding type of IAC resp. reprojection cone. The last column indicates which camera orientations are possible for a specific reprojection cone keeping the constraints for the IAC satisfied. This will be useful for Section 5. Some restrictions on the AIC are illustrated in Figure 5 and Figure 6 for a few representative ambiguities.

known intrinsic camera parameters	type of $\mathcal{R}(\omega_\infty)$	type of $\mathcal{R}(\Gamma_\infty, \lambda)$	possible orientations
$\mathbf{f_x}, \mathbf{f_y}, \mathbf{u_x}, \mathbf{u_y}, \mathbf{s}$	centered circle (known radius)	spheres (isotropic cone)	any (∞^3)
$\frac{\mathbf{f_x}}{\mathbf{f_y}}, \mathbf{u_x}, \mathbf{u_y}, \mathbf{s}$	centered circle	circular ellipsoids (circular cone)	aligned with main axis (∞)
$\frac{\mathbf{f_x}}{\mathbf{f_y}}, \mathbf{s}$	circle	ellipsoids	($4 \times \infty$)
$\mathbf{u_x}, \mathbf{u_y}, \mathbf{s}$	centered ellipse (axis aligned with image)	ellipsoids	aligned with some axis (8)
$\mathbf{s}$	ellipse (axis aligned with image)	ellipsoids	any opt. axis ($4 \times \infty^2$)
/	ellipse	ellipsoids	any (∞^3)

Table 1: Possible types of $\mathcal{R}(\omega_\infty)$ and $\mathcal{R}(\Gamma_\infty, \lambda)$ with possible orientations for set of known intrinsic camera parameters.

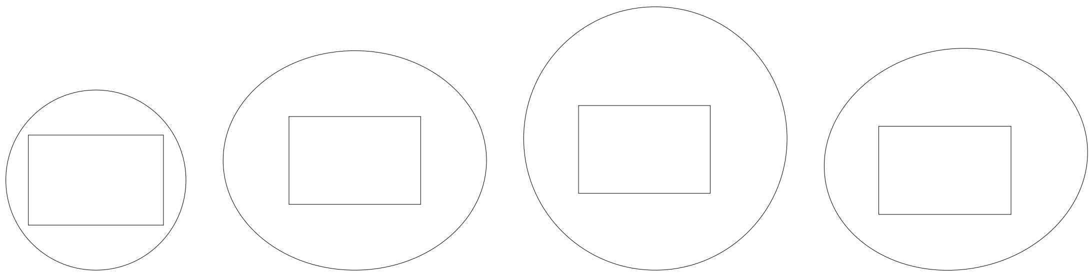

Figure 5: Representation of representative potential IAC together with image borders for different types of constraints in the image. Unknown focal length (left), unknown aspect ratio (middle-left), unknown principal point (middle-right) and fully uncalibrated (right).

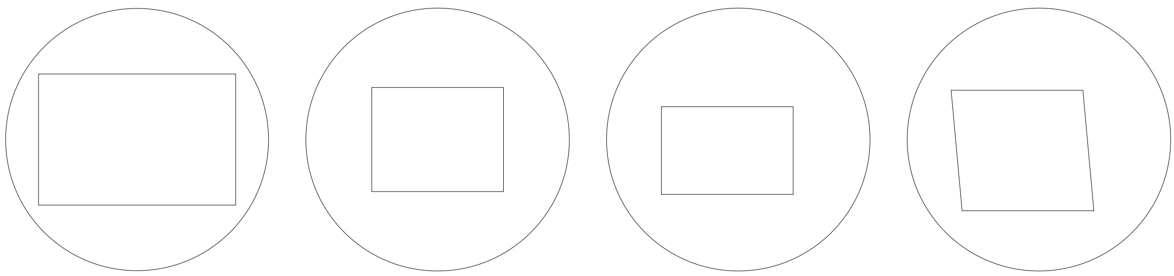

Figure 6: Same as Figure 5, but for the plane embedded in Euclidean 3D space. In this case the IAC is always a circle and the image borders are transformed depending on the intrinsic camera parameters.

5 Critical Motion Sequences

Depending on the motion sequence it can be that the self-calibration constraints do not provide a unique solution. This obviously also depends on which self-calibration constraints are used. In this section a general approach to the analysis of critical motion sequences is given. The goal is not so much to provide a catalogue of CMS, but mainly to be able to understand intuitively the results that have been obtained before. If a proper virtual conic satisfies all self-calibration constraints (see Equation (3)) it can be called a Potential Absolute Conic (PAC). To have a critical motion sequence there should be at least two PACs (the AC and another one).

The idea is to investigate which motion sequences results in a specific PAC. To analyse the problem in Euclidean space it is necessary to split it up in PACs in the plane at infinity and PACs out of the plane at infinity.

5.1 In the plane at infinity

In this case the problem is relatively simple. If the PAC is embedded in the plane at infinity the homography transforming the potential IAC to the PAC is simply the rotation matrix representing the orientation of the camera. Without loss of generality the world frame can be aligned with the first view, yielding the identity transformation for the first infinity homography. Therefore the PAC must itself satisfy the self-calibration constraints imposed on the IAC. The PAC can be seen as a 3D ellipsoid on which the rotations representing the different poses of the camera can act.

In this case there are no problems to understand –and even derive– intuitively all the possible cases. Therefore, no further attention will be paid to these cases in this paper.

5.2 Out of the plane at infinity

In this case the problem is more complicate. The problem can however be separated in two parts. First, for a specific PAC, the possible virtual projection cones are determined. This is equivalent to determining the possible positions for the camera. Then, for each cone the possible camera orientations have to be determined.

To get an intuitive handle on this problem the results of Section 3 can be applied. As was seen in Table 1, for some types of constraints there are constraints on the possible virtual cones. More precisely, this is the case for when all parameters are known, in which case the cone is an isotropic cone, and for the case that all parameters but the focal length are known, in which case the cone should be a circular one. For other constraints the type of the reprojection cones can not be determined beforehand. If all intrinsic camera parameters are fixed, then also the reprojection cone is fixed.

Note that when the cone is unrestricted, it is possible to draw the conclusion that very many CMS exist. This is made more explicit in the following proposition.

Proposition 1 *If the focal length and the aspect ratio or principal point are unrestricted,* *then for every possible proper virtual conic ϕ there exist a critical motion sequence which contains every point* $\mathtt{M}$.

No explicit proof is given here, but the proposition can be understood as follows. For every ϕ and a virtual projection cone with center $\mathtt{M}$ can be constructed. Since for the specified constraints the shape of the projection cone of a PAC is not restricted (as seen in Table 1), Γ is valid and several camera orientations will satisfy the self-calibration constraints. This proposition illustrates the complexity of the critical motion sequence problem when only a restricted number of constraints are at hand.

Before developing some specific examples, there are still a few general remarks to be made. It is important to note that a solution which involves $\mathcal{R}(\Gamma, \lambda)$ which are not spheres, suppose wrongly estimated intrinsic parameters. In fact the excentricity of $\mathcal{R}(\Gamma, \lambda)$ for the different views is a measure of the miss estimation. Inversely, if some bounds on the intrinsic camera parameters are available, this also bounds the possibility for potential $\mathcal{R}(\Gamma, \lambda)$ to deviate from spheres. This means that in the following examples the limiting position of the CMS for very elongated/flatened ellipsoids can immediately be discarded since they would involve unrealistic intrinsic camera parameters. Note also that for the orientation of the camera to deviate from the main axis of the ellipsoid a miss estimation of the principal point must be involved. In practice the feasible orientations can thus be restricted to cones with opening angles of a few degrees around the main axis of $\mathcal{R}(\Gamma, \lambda)$.

Another interesting remark is that some critical cases can be disambiguated, because they would involve violation of the ordering/cheirality constraints [7, 13]. This is for example the case for the first ambiguity that will be discussed in the following section.

5.3 Some examples

It is outside the scope of this paper to give a full analysis of the possible critical motion sequences. However, in this sectios a few examples are given. We refer to the work of Sturm [19, 20, 21], Kahl and Triggs [11] and Kahl [12] for more complete discussions.

A first simple case is the case where all intrinsic camera parameters are known. In this case the ellipsoids are spheres. In the plane at infinity, obviously, only the absolute conic itself satisfies this constraint. Outside of it only one conic is a potential absolute conic. This case is illustrated in Figure 7. Note that in this case the orientation of the camera is arbitrary.

A more complicated case occures when fixed but unknown intrinsic camera parameters are assumed. Here only the most obscure case of a finite ellipse as potential absolute conic will be described. Note that in the original paper of Sturm [19] the number of potential camera positions was restricted to 8. This error was however corrected in [20] where the possible camera locations were extended to a degree 12 curve. No effort, other than plotting a numerically evaluated example, was made to explain this motion. This illustrates the complexity of the problem and the importance to be able to understand intuitively the solutions.

Starting from an ellipse $\mathcal{R}(\phi)$ with axes length (a, b) and a vertex $\mathtt{C}$, $\mathcal{R}(\Gamma, \lambda)$ is uniquely

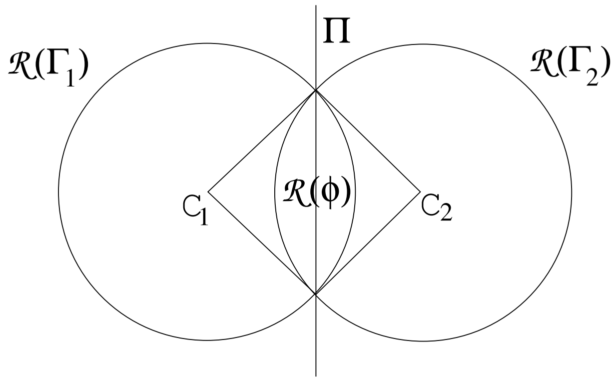

Figure 7: Potential absolute conic, represented by the circle $\mathcal{R}(\phi)$ in plane Π, for all intrinsic parameters known.

determined. Since the size does not matter, this family of ellipsoids is characterized by the ratio of the lengths (A, B, C) of the axes. Assume that $A > B > C$ and $a > b$, then the following inequality can be verified $\frac{C}{A} < \frac{b}{a} < 1$. It is now easy to show that besides the 8 symmetrical positions, 8 additional solutions can easily be found in the planes containing the main axes of the ellipse $\mathcal{R}(\phi)$ and which are orthogonal on it. Placing one of the axes corresponding to A, B and C parallel with a plane and rotating around it, will yield as intersection ellipses with aspect ratios[2] ranging from $\frac{C}{A}$ to $\frac{B}{A}$, from $\frac{C}{B}$ to $\frac{A}{B}$ respectively from $\frac{B}{C}$ to $\frac{A}{C}$. Note that if $\frac{B}{A} \neq \frac{C}{B}$ some values of aspect ratios between $\frac{A}{C}$ and $\frac{C}{A}$ can not be reached and others are obtained twice. Note however that if the aspect ratios r and $\frac{1}{r}$ are seen as equivalent (i.e. $(r, \frac{1}{r})$), then each value is obtained exactly twice. The two different cases are illustrated in Figure 8 and Figure 9. In the first case 4 poses for $\mathcal{R}(\Gamma, \lambda)$ are obtained in the two orthogonal planes. By mental interpolation it is possible to visualize a path transforming from one pose to the other. For this case the global set of possible poses is located on two bended circle-like shapes (one above and one under the plane). In the second case when in one plane 8 solutions can be found and none in the other, the global locus of the poses should consist of 4 circle-like shapes. In [20] only the first of these two cases was illustrated.

Another difficult case which was discussed by several authors [21, 12, 16], is the case where all intrinsic camera parameters are known except for the focal length which can vary freely. This case corresponds more or less to a calibrated zoom camera. As was seen in Table 1, $\mathcal{R}(\Gamma, \lambda)$ should have two main axes of equal length, in other words, cigar-like or `m&m`-like ellipsoids. Here also, we will concentrate on the most complex case. This is the case where the potential AC is an ellipse. In this case the locus of potential camera poses consist of a hyperbola and an ellipse. Reasoning with real ellipses and cones (without using the method proposed in this paper), only the hyperbola motion can be understood. In fact, in this case the wrong hyperbola would be visualized (namely one going through

[2] In this case the aspect ratio is defined as the division of the length of one axis by the other. This last axis is selected as the one that is parallel with the rotation axis.

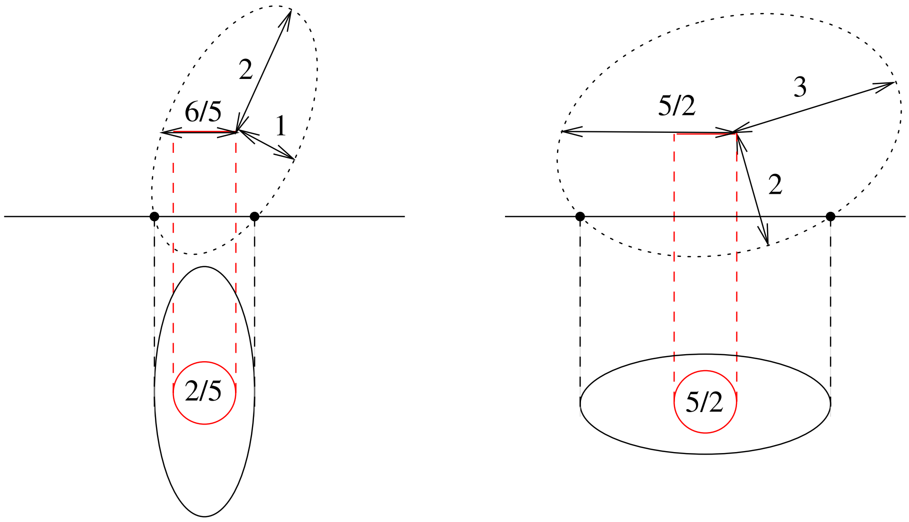

Figure 8: Ellipse $\mathcal{R}(\phi)$ with aspect ratio of $\frac{2}{5}$ and two possible poses for a $\mathcal{R}(\Gamma,\lambda)$ with axes ratio of (3,2,1). An estimation of the locus of the possible camera locations is also shown.

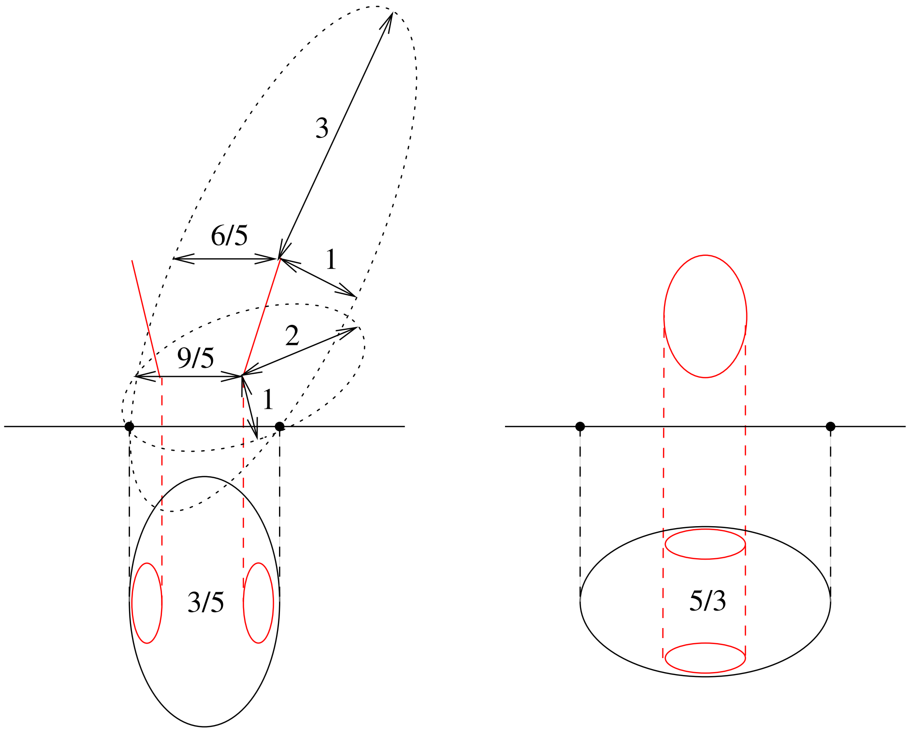

Figure 9: Ellipse $\mathcal{R}(\phi)$ with aspect ratio of $\frac{3}{5}$ and two possible poses for a $\mathcal{R}(\Gamma,\lambda)$ with axes ratio of (3,2,1). An estimation of the locus of the possible camera locations is also shown.

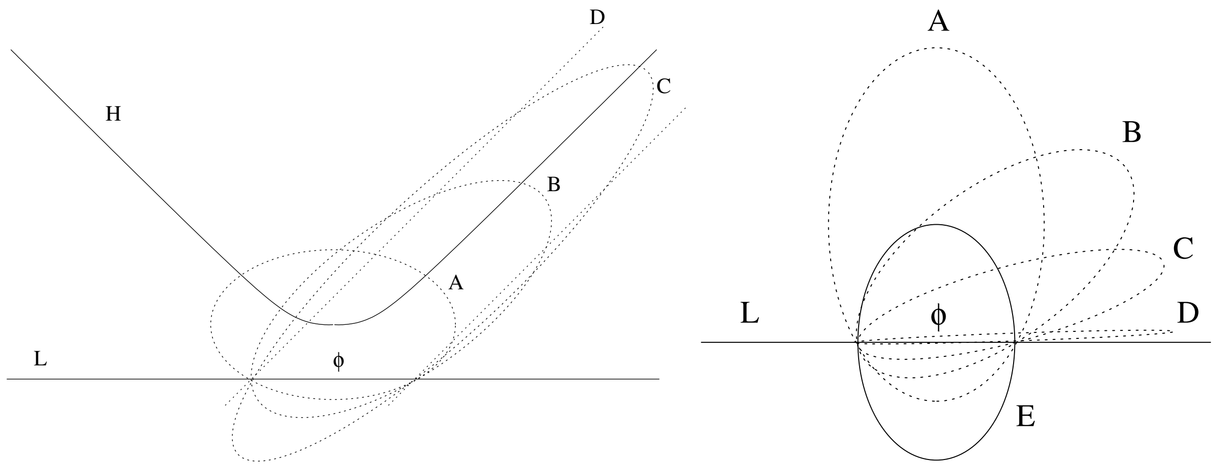

Figure 10: Illustration of critical hyperbolic and elliptic motion of a zoom camera.

the ellipse).

Making use of the proposed mapping $\mathcal{R}$, it is however simple to fully understand this case. This is illustrated in Figure 10. The potential absolute conic (represented by $\mathcal{R}(\phi)$) is the intersection of the circular cones (represented by the ellipsoids $\mathbf{A}, \mathbf{B}, \mathbf{C}, \mathbf{D}$) with the plane Π. It has the same excentricity as the ellipsoid $\mathbf{A}$ (in both cases). The plane containing $\mathbf{H}$, the plane containing $\mathbf{E}$ and the plane Π are mutually orthogonal. Note that for the ellipsoids associated with the hyperbola (left) the smallest eigenvalue is double while on for the ones associated with the ellipse (right) it is the largest one that is double. The factor λ is given by the ratio of the single eigenvalue (associated with the optical axis) and the double eigenvalue (associated with the image plane). This gives the ratio between the true focal length and the one obtained by considering ϕ as the absolute conic. Therefore, only the central part of $\mathbf{H}$ and the top and bottom parts of $\mathbf{E}$ can still yield realistic focal lengths. This also eliminates the possiblity to consider very elongated ellipses as potential absolute conics for this class of critical motions. This also eliminates the problems that were foreseen by Sturm [21] for the asymptotic similarity of the hyperbole with a straight forward motion. Figure 11 gives a 3D view of this case.

6 Conclusion

In this paper a method to represent the rather abstract concept of the absolute conic through real geometric entities was given. The main benefit of this approach is that it allows to apply geometric intuition to understand problems related to self-calibration and critical motion sequences. The image of the absolute conic can be represented as an ellipse in the image. In this case self-calibration constraints are translated to simple geometric constraints on this ellipse. The analysis of critical motion sequences can take advantage of this new representation. Although automated approaches for obtaining theoretic solutions have made progress, they are often still not able to solve complex problems without being

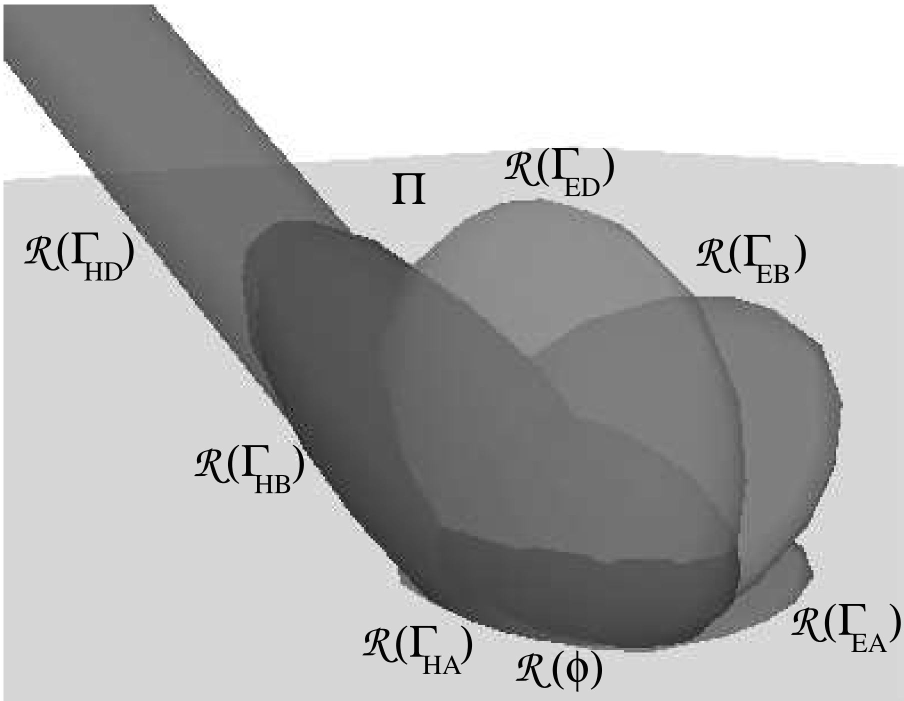

Figure 11: 3D view of critical hyperbolic and elliptic motion of a zoom camera.

guided. In this case intuition can come in very handy. In the last part of the paper some complex critical motion sequences which could not be understood through standard geometric intuition were analysed and illustrated with the presented method. This not only allows to understand these cases, but also allowed us to draw some interesting conclusions which would have been hard to get without a real geometric representation.

References

[1] S. Bougnoux, "From Projective to Euclidean Space under any practical situation, a criticism of self-calibration". *Proc. International Conference on Computer Vision*, Narosa Publishing House, New Delhi /Madras /Bombay /Calcutta /London, pp. 790-796, 1998.

[2] O. Faugeras, *Three-Dimensional Computer Vision: a Geometric Viewpoint*, MIT press, 1993.

[3] O. Faugeras, "What can be seen in three dimensions with an uncalibrated stereo rig", *Computer Vision - ECCV'92*, Lecture Notes in Computer Science, Vol. 588, Springer-Verlag, pp. 563-578, 1992.

[4] O. Faugeras, Q.-T. Luong and S. Maybank. "Camera self-calibration: Theory and experiments", *Computer Vision - ECCV'92*, Lecture Notes in Computer Science, Vol. 588, Springer-Verlag, pp. 321-334, 1992.

[5] R. Hartley, Rajiv Gupta and Tom Chang, "Stereo from Uncalibrated Cameras", Proc. CVPR'92, pp.761-764, 1992.

[6] R. Hartley, "Euclidean reconstruction from uncalibrated views", in : J.L. Mundy, A. Zisserman, and D. Forsyth (eds.), *Applications of Invariance in Computer Vision*, LNCS, Vol. 825, Springer-Verlag, pp. 237-256, 1994.

[7] R. Hartley, "Cheirality invariants", *Proc. D.A.R.P.A. Image Understanding Workshop*, pp. 743-753, 1993.

[8] A. Heyden and K. Åström, "Euclidean Reconstruction from Constant Intrinsic Parameters" *Proc. 13th International Conference on Pattern Recognition*, IEEE Computer Soc. Press, pp. 339-343, 1996.

[9] A. Heyden and K. Åström, "Euclidean Reconstruction from Image Sequences with Varying and Unknown Focal Length and Principal Point", *Proc. IEEE Conference on Computer Vision and Pattern Recognition*, IEEE Computer Soc. Press, pp. 438-443, 1997.

[10] A. Heyden and K. Åström, "Minimal Conditions on Intrinsic Parameters for Euclidean Reconstruction", *Proc. ACCV*, Hong Kong, 1998.

[11] F. Kahl and B. Triggs, "Critical Motions in Euclidean Structure from Motion", *Proc. CVPR*, 1999.

[12] F. Kahl, "Critical Motions and Ambiguous Euclidean Reconstructions in Auto-Calibration", *Proc. ICCV*, pp.469-475, 1999.

[13] S. Laveau and O. Faugeras, "Oriented Projective Geometry for Computer Vision", in : B. Buxton and R. Cipolla (eds.), *Computer Vision - ECCV'96*, Lecture Notes in Computer Science, Vol. 1064, Springer-Verlag, pp. 147-156, 1996.

[14] Q.-T. Luong and O. Faugeras, "Self Calibration of a moving camera from point correspondences and fundamental matrices", *Internation Journal of Computer Vision*, vol.22-3, 1997.

[15] Y. Ma, S. Soatto, J, Košecká and S. Sastry, "Euclidean Reconstruction and Reprojection Up to Subgroups", *Proc. ICCV*, pp.773-780, 1999.

[16] M. Pollefeys, *Self-calibration and metric 3D reconstruction from uncalibrated image sequences*, PhD. thesis, K.U.Leuven, 1999.

[17] M. Pollefeys and L. Van Gool, "Stratified self-calibration with the modulus constraint", *IEEE Transactions on Pattern Analysis and Machine Intelligence*, August 1999, Vol 21, No.8, pp.707-724.

[18] M. Pollefeys, R. Koch and L. Van Gool. "Self-Calibration and Metric Reconstruction in spite of Varying and Unknown Internal Camera Parameters", *International Journal of Computer Vision*, 32(1), 7-25, 1999.

[19] P. Sturm, "Critical Motion Sequences for Monocular Self-Calibration and Uncalibrated Euclidean Reconstruction", *Proc. CVPR*, IEEE Computer Soc. Press, pp. 1100-1105, 1997.

[20] P. Sturm, *Vision 3D non-calibrée: contributions à la reconstruction projective et études des mouvements critiques pour l'auto-calibrage*, Ph.D. Thesis, INP de Grenoble, France , 1997.

[21] P. Sturm, "Note 28: Critical Motion Sequences for Cameras with Free Focal Length", University of Reading, August 1998.

[22] B. Triggs, "The Absolute Quadric", *Proc. CVPR*, IEEE Computer Soc. Press, pp. 609-614, 1997.

[23] C. Zeller, *Calibration projective, affine et Euclidienne en vision par ordinateur et application a la perception tridimensionnelle*, Ph.D. Thesis, Ecole Polytechnique, France, 1996.