

**Comp 455**

**Fall 2007**

**Homework 1**

(Due: Wednesday, September 12)

**Note:** Proofs will be graded based on both correctness and style.

1. [5 points] Prove by induction on  $n$  that  $\sum_{i=0}^n i = \frac{n(n+1)}{2}$ .
  
2. **(a)** [5 points] Construct a DFA  $M_1$  for which  $L(M_1)$  is the set of strings over  $0, 1$  with an odd number of 1's.  
**(b)** [5 points] Construct a DFA  $M_2$  for which  $L(M_2)$  is the set of strings over  $0, 1$  with no occurrence of 010 as substring.  
**(c)** [10 points] Construct a DFA  $M$  for which  $L(M) = L(M_1) \cap L(M_2)$ . Explain the method you used to construct  $M$ .
  
3. **(a)** [15 points] HMU, Exercise 2.2.5(b). Hint: Think about how a string of 0's and 1's evaluates as a binary number. The machine for this part may have a *lot* of states, so described it parametrically, without actually writing down a transition table or transition diagram. (By "parametrically", I mean that you can state something like  $\delta(q_i, 0) = ???$  for  $i$  in the range ???.)  
**(b)** [5 points] Give an NFA that accepts the same language as in part (a).  
**(c)** [5 points] HMU, Exercise 3.1.1(b).
  
4. Write regular expressions for each of the following languages over the alphabet  $0, 1$ .  
**(a)** [10 points] The set of all strings in which every pair of adjacent 0's appears before any pair of adjacent 1's.  
**(b)** [10 points] The set of all strings with an equal number of 0's and 1's such that no prefix has two more 0's than 1's nor two more 1's than 0's.
  
5. **(a)** [15 points] Construct a DFA that accepts strings over  $\{0, 1\}$  that, interpreted as their binary representation, are evenly divisible by 7.  
**(b)** [15 points] Formally prove that your DFA is correct.