Exercise 2 Solutions
Due at 3:30 PM (on paper), February 10, 2009.

1. (25 pts) Consider the following grammar:
   
   \[ S \rightarrow aScB \mid A \mid b \]
   
   \[ A \rightarrow cA \mid c \]
   
   \[ B \rightarrow d \mid A \]

Which of the following sentences are in the language generated by this grammar? For those that are, use a parse tree to show a derivation.

   a. abcd  \hspace{1cm} \text{In the language}

   \[ S \]
   \[ a \rightarrow a \]
   \[ S \rightarrow aScB \]
   \[ c \rightarrow c \]
   \[ B \rightarrow d \]
   \[ b \rightarrow b \]

   b. acccbd  \hspace{1cm} \text{Not in the language}

   c. accbccc  \hspace{1cm} \text{Not in the language}

   d. acd  \hspace{1cm} \text{Not in the language}

   e. accc  \hspace{1cm} \text{In the language}

   \[ S \]
   \[ a \rightarrow a \]
   \[ S \rightarrow aScB \]
   \[ c \rightarrow c \]
   \[ A \rightarrow cA \]
   \[ A \rightarrow cA \]
   \[ B \rightarrow d \]
   \[ A \rightarrow cA \]
   \[ A \rightarrow cA \]
   \[ B \rightarrow d \]
   \[ c \rightarrow c \]
2. (20 pts) Show that the following grammar is ambiguous by drawing two different parse trees for the same string of your choosing.

\[ foo \rightarrow AAA \ bar \ qux \ | \ DDD \]
\[ qux \rightarrow CCC \ foo \ | \ \varepsilon \]
\[ bar \rightarrow BBB \ foo \]

Answers may vary, but here is a relatively short string that illustrates the ambiguity:

AAA BBB AAA BBB DDD CCC DDD

And the following are two different parse trees for that same string:

![Parse Trees](image)

Note that the general problem of even determining whether a grammar is ambiguous is undecidable. In other words, there is no algorithm to determine whether a grammar is ambiguous. Luckily for us humans, we have intuition to help us come up with a counterexample. The intuition behind the solution to this problem is that qux could be empty. So the idea is to have two qux’s next to one another, one empty and nonempty. Which one is the empty one and which one is the nonempty one? Therein lies the ambiguity.

3. (30 pts) For each of the following grammars, why is the grammar not LL(1)? Modify each one to fix the problem.

a. 

\[ foo \rightarrow AAA \ bar; \quad <- \text{start symbol} \]
\[ bar \rightarrow \bar{bar} \ BBB \]
\[ bar \rightarrow BBB \]
The problem here is left recursion in the rules for \textit{bar}. Fix by rewriting with right recursion instead:

\begin{align*}
  \text{foo} & \rightarrow \text{AAA bar} \\
  \text{bar} & \rightarrow \text{BBB bartail} \\
  \text{bartail} & \rightarrow \text{BBB bartail} \\
  \text{bartail} & \rightarrow ;
\end{align*}

Note: Initial version of this solution set had an error in the above problem. The current version is correct.

\textit{b.}

\begin{align*}
  \text{foo} & \rightarrow \text{AAA bar} \quad \text{<-- start symbol} \\
  \text{bar} & \rightarrow \text{BBB foo} \\
  \text{bar} & \rightarrow \text{BBB qux} \\
  \text{qux} & \rightarrow \text{CCC}
\end{align*}

The problem here is a common prefix (BBB) in the rules for \textit{bar}. Fix by factoring out the common prefix:

\begin{align*}
  \text{foo} & \rightarrow \text{AAA bar} \\
  \text{bar} & \rightarrow \text{BBB bartail} \\
  \text{bartail} & \rightarrow \text{foo} \mid \text{qux} \\
  \text{qux} & \rightarrow \text{CCC}
\end{align*}

\textbf{4. (25 pts)} Write a paragraph summarizing significant characteristics of the following scripting languages. Use Perl as a point of comparison. If you use other sources besides the book, cite them.

a. bash  

b. awk  

c. tcl  

d. Python  

e. Ruby

Answers may vary. Consult Chapter 13 of the PLP text.