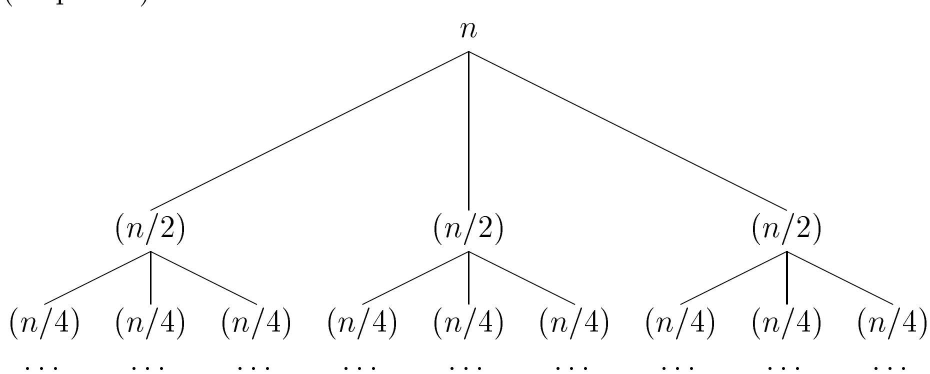
COMP 122
Algorithms and Analysis
Fall 2004
Mid Semester Exam
Thursday, Sept. 23, 2004
Closed Book - Closed Notes
Don't forget to write your name or ID and pledge on the exam sheet.
This exam has three pages.

1. (12 points) For each problem, write in the blank all elements F of the set $\{\Theta, O, o, \Omega, \omega\}$ such that the statement $f(x) = F(g(x))$ is a correct statement of the asymptotic relationship between f and g. Thus if $f(x) = \Omega(g(x))$ and $f(x) = \Theta(g(x))$ and $f(x) = O(g(x))$ are the only three valid asymptotic relationships between f and g, write Ω, Θ, O in the blank.

a). $f(x) = x^2 + 1, g(x) = 3x - 2.$ ____________________
b). $f(x) = x^2 + 5, g(x) = 3x^2 + 4x.$ ____________________
c). $f(x) = 2^x + 3x, g(x) = 3^x + 2x + 1.$ ____________________
d). $f(x) = 2x + 1, g(x) = 3\log^2 x + 2.$ ____________________
e). $f(x) = 2\log_2 x, g(x) = \log_3(2x).$ ____________________
f). $f(x) = \sqrt{x}, g(x) = 4\log x.$ ____________________

2. (12 points) Consider a recursion tree that looks like this:



a). What recurrence relation could generate this recursion tree? ____________________
b). How many levels would there be in this tree, as a function of n? ____________________
c). How many leaves would there be in this tree, as a function of n? ____________________

d). Solve the recurrence to obtain an asymptotic expression for $T(n)$ as a function of n. ____________________

3. (8 points) A fair *die* when tossed will give each of the values 1 through 6 with equal probability. The plural of die is *dice*.

a). Suppose three fair dice are tossed. What is the probability that they will all produce equal values? ____________________
b). Suppose two fair dice are tossed. What is the probability that the sum of their values will equal 3? ____________________
c). What is the expected value for a single toss of a fair die? ____________________
d). What is the expected value for the sum of three tosses of a fair die? ____________________

4. (10 points) Solve the recurrence $T(n) = 2T(n/2) + \Theta(n)$. Indicate which solution method you used. ____________________

5. (10 points) Solve the recurrence $T(n) = 3T(n/3) + \Theta(n^2)$. Indicate which solution method you used. ____________________

6. (4 points) What is the asymptotic expected time for quicksort? ____________________

7. (4 points) What is the asymptotic worst case time bound for quicksort? ____________________

8. (4 points) How long does it take to build a max-heap of n elements? ____________________

9. (10 points) Give an asymptotic estimate for the sum $1 + \frac{1}{2} + \frac{1}{3} + \ldots + \frac{1}{n}$. ____________________

10. (10 points) What is the sum of the series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\ldots$? ____________________

11. (4 points) What is the asymptotic worst case time bound for heapsort? ____________________

12. (4 points) What is the asymptotic expected time for heapsort? ____________________

13. (10 points) Suppose Algorithm X operates on linear arrays. Suppose that if the array has length one, then Algorithm X returns an answer with a constant amount of work. Otherwise, Algorithm X calls itself recursively four times on linear arrays that are 3/4 as long, and in doing so performs a linear amount of work creating the subproblems and combining their solutions. That is, the work performed in creating the subproblems and combining their solutions is proportional to the number of elements in the array. Write down a recurrence for the running time of Algorithm X but do not solve it.

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14. (10 points) EXTRA CREDIT: Compute $\sum_{j=0}^{\infty} \frac{j}{3^j}$. ____________________

15. (10 points) EXTRA CREDIT: Solve the recurrence relation $T(n) = T(\sqrt{n}) + \Theta(\log n)$ ____________________

16. (10 points) EXTRA CREDIT: Solve the recurrence relation $T(n) = T(n-1) + \Theta(n)$ ____________________