Comp181 Spring 2002

Additional Context-Free Languages Exercise Solutions

1) Given the following languages, show that they are context-free by constructing context-free grammars that generate them:

a. $\{ ab^n cd^n f \}$

V = { a,b,c,d,f,S,A }

Σ = { a,b,c,d,f }

R = {

  S → aAf

  A → bAd | c

}

b. $\{ a^n b^m c^p : n \leq m+p \}$

V = { a,b,c,S,A }

Σ = { a,b,c }

R = {

  S → ε | Sc | aSc | Ac | aAc

  A → ε | Ab | aAb

}

c. $\{ wc^* w^R : w \in \{a,b\}^* \}$

V = { a,b,c,S,C }

Σ = { a,b,c }

R = {

  S → C | aSa | bSb

  C → ε | Cc

}

d. { $\{a,b\}^*$: the number of a's = the number of b's }

V = { S, A, B, a, b }

Σ = { a,b }

R = {

  S → ε

  S → SASBS

  S → SBSAS

  A → a

  B → b

}

2) Given the following grammar:

V = { a,b,c,(,),+,*,S,T }
Σ = { a,b,c,(,),+,* }
R = {
S → T + S | T
T → T * T | (S)
T → a | b | c
}

a. show a derivation for:
(All derivations are *leftmost derivations*)

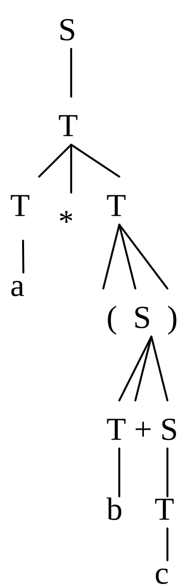
i. a*(b+c)
S ⇒ T ⇒ T*T ⇒ a*T ⇒ a*(S) ⇒ a*(T+S) ⇒ a*(b+S) ⇒ a*(b+T) ⇒ a*(b+c)

ii. a+(b*c)
S ⇒ T+S ⇒ a+S ⇒ a+T ⇒ a+(S) ⇒ a+(T) ⇒ a+(T*T) ⇒ a+(b*T) ⇒ a+(b*c)

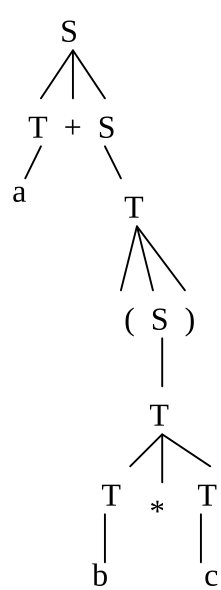
iii. ((a+b)*(b+c))
S ⇒ T ⇒ (S) ⇒ (T) ⇒ (T*T) ⇒ ((S)*T) ⇒ ((T+S)*T) ⇒ ((a+S)*T) ⇒
((a+T)*T) ⇒ ((a+b)*T) ⇒ ((a+b)*(S)) ⇒ ((a+b)*(T+S)) ⇒
((a+b)*(b+S)) ⇒ ((a+b)*(b+T)) ⇒ ((a+b)*(b+c))

give a parse tree for each of the above:

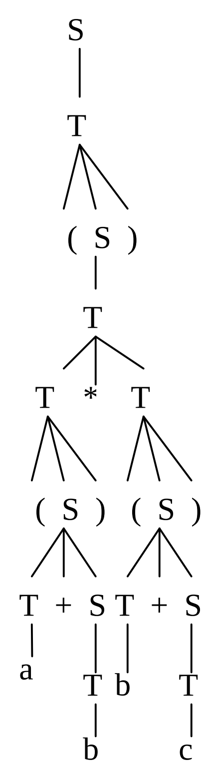
i. a*(b+c)



ii. a+(b*c)



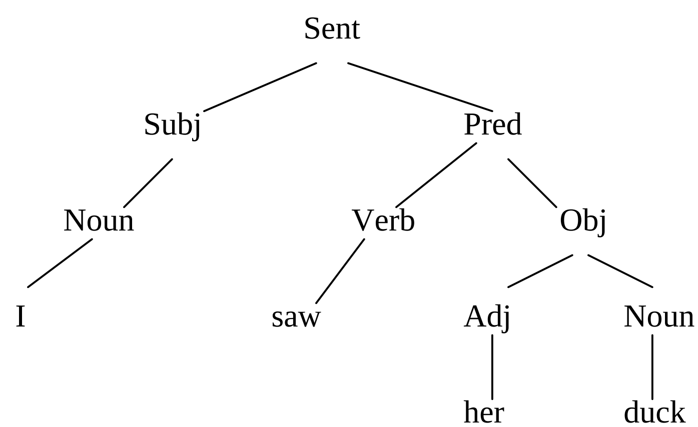
iii.((a+b)*(b+c))

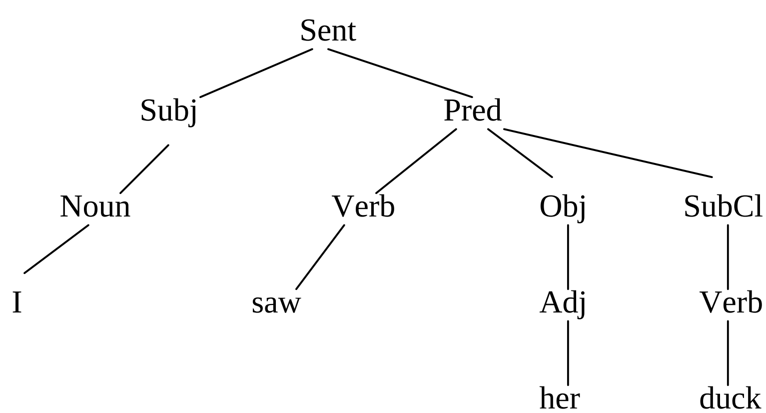


3) Given the following grammar:

V = { Sentence, Subject, Predicate, Noun, Verb, Object, SubordinateClause,
Adjective, her, I, duck, saw}
Σ = { saw, duck, I, her }
R = {
Sentence → Subject Predicate
Subject → Noun
Predicate → Verb Object | Verb Object SubordinateClause
Object → Adjective Noun | Noun
SubordinateClause → Verb
Adjective → her
Noun → I, her, duck
Verb → saw, duck
}

Show that the statement “I saw her duck” is ambiguous by constructing two non-equivalent parse trees.





4) Construct a PDA that recognizes the following grammars:

   a. { {a,b}* : the number of bs = twice the number of a's}

      K = { s,t,q,f }

      Σ = { a,b }

      Γ = { a,b,$ }

      F = { f }

      Δ = {

         ((s,**ε**,**ε**),(t,$))

         ((t,a,$),(t,aa$))

         ((t,a,a),(t,aaa))

         ((t,a,b),(q,**ε**))

         ((t,b,$),(t,b$))

         ((t,b,a),(t,**ε**))

         ((t,b,b),(t,bb))

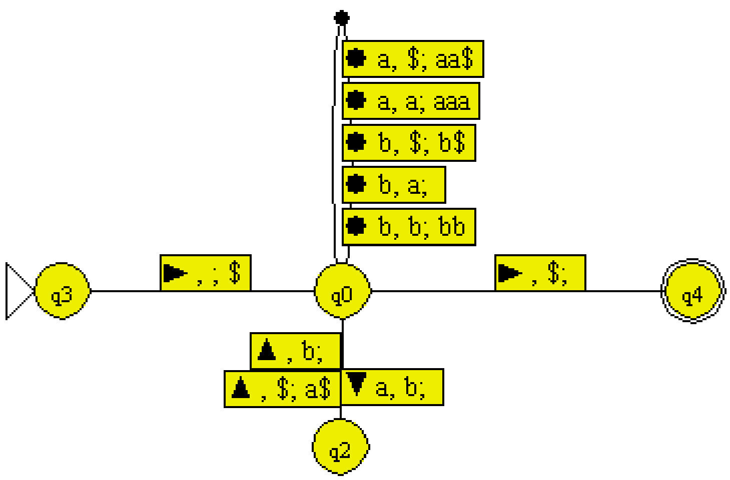
         ((q,**ε**,$),(t,a$))

         ((q,**ε**,b),(t,**ε**))

         ((t,**ε**,$),(f,**ε**))

      }

   …or as would be represented in JFLAP:



b. { {a,b}* : the number of a's ≠ the number of b's }

K = { s,q,f }

Σ = { a,b }

Γ = { a,b,$ }

F = { f }

Δ = {

((s,**ε**,**ε**),(q,$))

((q,a,$),(q,a$))

((q,a,a),(q,aa))

((q,a,b),(q,**ε**))

((q,b,$),(q,b$))

((q,b,a),(q,**ε**))

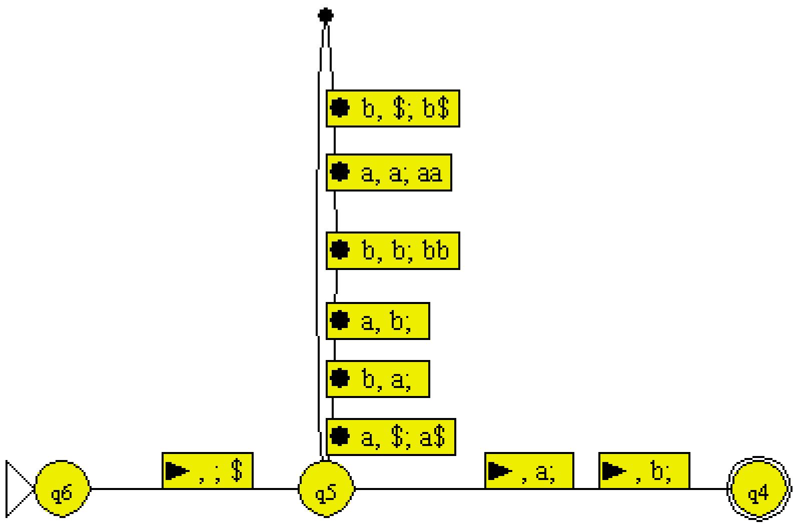
((q,b,b),(q,bb))

((q,**ε**,a),(f,**ε**))

((q,**ε**,b),(f,**ε**))

}

…or as would be represented in JFLAP:



5) Give an intuitive description of the following grammars, and construct a PDA that recognizes it:

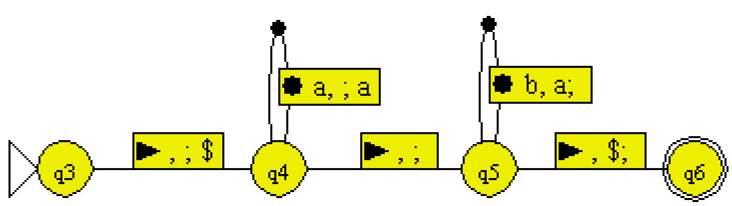
a.

V = { S, A, B, a, b }
Σ = { a,b }
R = {
S → **ε**
S → ASB
A → a
B → b
}

A string with n a’s followed by n b’s

K = { s,q,t,f }
Σ = { a,b }
Γ = { a,b,$ }
F = { f }
Δ = {
((s,**ε**,**ε**),(q,$))
((q,a,**ε**),(q,a))
((q,**ε**,**ε**),(t,**ε**))
((t,b,a),(t,**ε**))
((t,**ε**,$),(f,**ε**))
}

…or as would be represented in JFLAP:



b.

V = { S, A, B, a, b }
Σ = { a,b }
R = {
- S → **ε**
- S → SASBS
- S → SBSAS
- A → a
- B → b

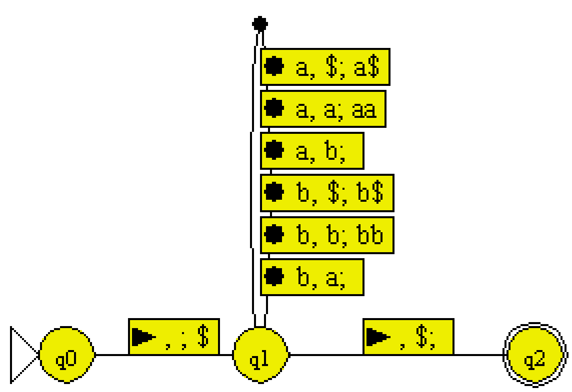
}

A string with an equal number of a's and b's

K = { s,q,f }
Σ = { a,b }
Γ = { a,b,$ }
F = { f }
Δ = {
- ((s,**ε**,**ε**),(q,$))
- ((q,a,$),(q,a$))
- ((q,a,a),(q,aa))
- ((q,a,b),(q,**ε**))
- ((q,b,$),(q,b$))
- ((q,b,a),(q,**ε**))
- ((q,b,b),(q,bb))
- ((q,**ε**,$),(f,**ε**))

}

…or as would be represented in JFLAP:



c.

$V = \{ S, S_1, S_2, A, B, a, b \}$

$\Sigma = \{ a,b,c \}$

R = {

$S \rightarrow \varepsilon$

$S \rightarrow S_1cS_2$

$S_1 \rightarrow \varepsilon$

$S_1 \rightarrow AS_1B$

$S_2 \rightarrow \varepsilon$

$S_2 \rightarrow S_2AS_2BS_2$

$S_2 \rightarrow S_2BS_2AS_2$

$A \rightarrow a$

$B \rightarrow b$

}

The string from 5a, a ‘c’, and then the string from 5b

K = { s,q,t,v,u,w,f }

$\Sigma = \{ a,b,c \}$

$\Gamma = \{ a,b,\$ \}$

F = { f }

Δ = {

((s,**ε**,**ε**),(q,$))

((q,a,**ε**),(q,a))

((q,**ε**,**ε**),(t,**ε**))

((t,b,a),(t,**ε**))

((t,**ε**,$),(v,**ε**))

((v,c,**ε**),(u,**ε**))

((u,**ε**,**ε**),(w,$))

((w,a,$),(w,a$))

((w,a,a),(w,aa))

((w,a,b),(w,**ε**))

((w,b,$),(w,b$))

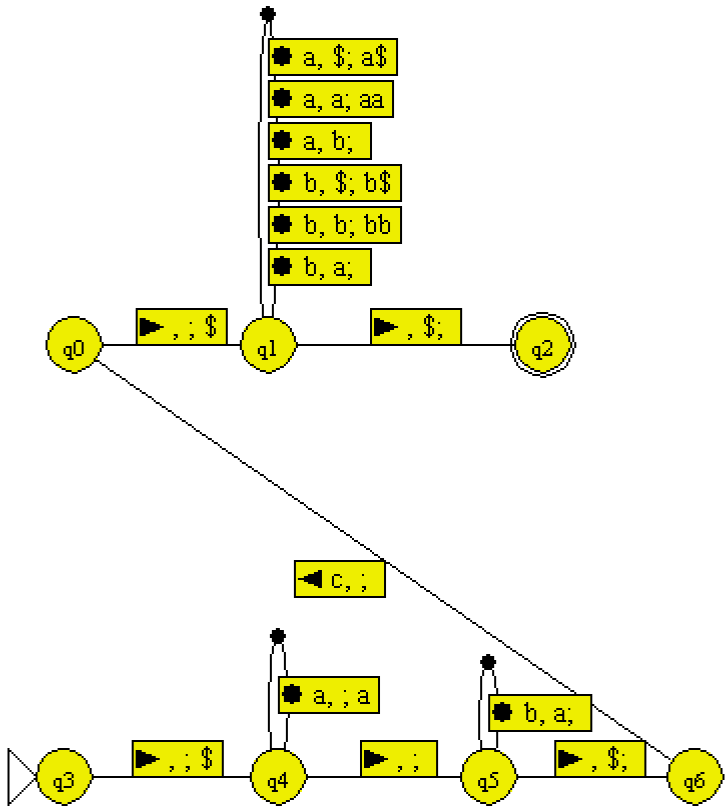
((w,b,a),(w,**ε**))

((w,b,b),(w,bb))

((w,**ε**,$),(f,**ε**))

}

…or as would be represented in JFLAP:



d.

V = { S, A, a, b }

Σ = { a,b }

R = {

S → **ε**

S → ASb

A → a | aa

}

A string with n a's followed by m b's where m ≤ n ≤ 2m

K = { s,q,t,f }

Σ = { a,b }

Γ = { a,b,$ }

F = { f }

Δ = {

((s,**ε**,**ε**),(q,$))

((q,a,**ε**),(q,a))

((q,**ε**,**ε**),(t,**ε**))

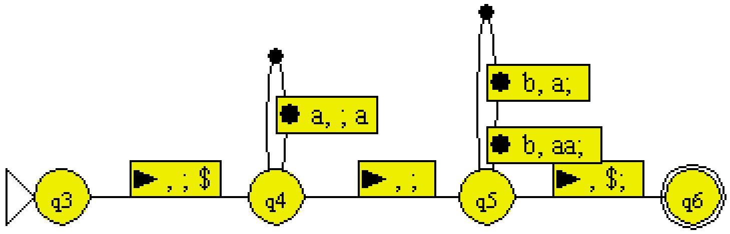
((t,b,a),(t,**ε**))

((t,b,aa),(t,**ε**))

((t,**ε**,$),(f,**ε**))

}

…or as would be represented in JFLAP:



6) Use the pumping lemma for context free grammars to show that the following is not a context free grammar:

L = { $\{a,b\}^n\{c,d\}^n$: the number of a's = the number of c's }

The pumping lemma for CFL's states that for an infinite context free language (like the one above), that any string with length larger than *m* must have a few properties:

1) S = uvwxy – that is, the string can be broken into five parts (though some of these parts can be empty).
2) |vwx| ≤ m – vwx can't be too big. Specifically, it cannot exceed a length larger than m
3) |vx| ≥ 1 – v and x cannot both be empty, but one of them can
4) $uv^iwx^iy \in L$ – we can repeat v and x an arbitrary number of times, and the string should still be part of the language.

Consider string $S = a^mb^mc^md^m$, which is definitely larger than *m* and which is a member of language L. Then vwx can at most span two characters (since vx cannot be larger than *m*) which makes it impossible to pump 'a' and maintain the property of having an equal number of 'c' characters. Likewise 'c' cannot be pumped. The remaining choices for vwx are therefore to pump solely 'b' or solely 'd' – both of which would violate the property of maintaining equal number of {a,b}'s and {c,d}'s.

■