COMP 181
Models of Languages and Computation
Fall 2005
Final Exam
Tuesday, December 13, 2005
Closed Book - Closed Notes
This exam has five pages.
Don't forget to write your name or ID and pledge on the exam sheet.

1. (Each part is worth 2 points) Fill in the blanks with one of the following:

a) finite
b) regular but not finite
c) deterministic context free but not regular
d) context free but not deterministic context free
e) recursive (that is, decidable) but not context free
f) recursively enumerable (that is, partially decidable) but not recursive
g) not recursively enumerable

Also recall that if i is an integer, then i can be interpreted as a binary number, that is, as a bit string, and this bit string can be read as a character string in ASCII or some other system, and then if this character string describes a Turing machine, then T_i is the Turing machine described by this character string, else T_i is some fixed Turing machine. The point is that T_i is some Turing machine that can be computed from i, and every Turing machine can be represented as T_i for some i.

1.1) The language $\{0^n c 0^m d 1^n : m, n \geq 0\}$ is ________
1.2) The language $\{x \in \{0,1\}^* : x$ does not contain the substring $101010\}$ is ________
1.3) The language $\{x \in \{0,1\}^* : x$ has odd length$\}$ is ________
1.4) The language $\{w \in \{a,b,c,d\}^* : w$ has the same number of a's, b's, c's, and d's$\}$ is ________
1.5) The language $\{(i,j) :$ Turing machine T_i halts on input $j\}$ is ________
1.6) The language $\{(i,j) :$ Turing machine T_i does not halt on input $j\}$ is ________
1.7) The language $\{1^m c 1^n : m \geq n\}$ is ________

1.8) The language $\{10, 11, 00, 01, 100, 101\}$ is ________

1.9) The language $\{x \in \{0,1\}^* : x$ contains unequal numbers of ones and zeroes$\}$ is ________

2. (Each part is worth 2 points) Fill in the blanks with one of the following:

a) finite
b) regular but not necessarily finite
c) deterministic context free but not necessarily regular
d) context free but not necessarily deterministic context free
e) recursive (that is, decidable) but not necessarily context free
f) recursively enumerable (that is, partially decidable) but not necessarily recursive
g) not recursively enumerable

2.1) If L is L_1^* where L_1 is context-free then L is ________

2.2) If L is the intersection of two regular languages then L is ________

2.3) If there is a Turing machine T with two halting states y and n and for strings (words) in L, T halts in state y and for strings (words) not in L, T halts in state n then L is ________

2.4) If there is a Turing machine T which halts for strings in L and does not halt for strings not in L then L is ________

2.5) If L is a language represented by a regular expression then L is ________

2.6) If $L\$$ is the language accepted by a deterministic push-down automaton then L is ________

2.7) If L is the language accepted by a deterministic finite automaton then L is ________

2.8) If L is the language generated by a context-free grammar then L is ________

2.9) If L is the union of two context-free languages then L is ________

2.10) If L is the language accepted by an arbitrary push-down automaton then L is ________

3. (4 points) True or false:

a) Nondeterministic Turing machines can decide some languages that deterministic Turing machines cannot decide.

b) General Turing machines can decide some languages that Turing machines that never move to the left cannot decide.
c) If τ is a reduction from L_1 to L_2 then τ is a computable (recursive) function.
d) If language L_1 is undecidable and there is a reduction from language L_2 to L_1 then L_2 is undecidable.

4. (4 points) For each of the following problems, answer the two questions: Is the problem decidable (recursive)? Is the problem partially decidable (recursively enumerable)? Thus there are two answers to give for each part:

a) Given a Turing machine T and a string w, to determine whether T will halt when started on the input w.
b) To determine whether Bush will win the presidential election next year.
c) Given a push-down automaton and an input string, to determine whether the automaton will accept the string.
d) Given a Turing machine T, to determine whether T will halt when started on a blank tape.

5. (4 points) Suppose L is a language and the relation $\approx_L$ has n equivalence classes. Let M be a finite automaton. True or false:

a) L is a regular language.
b) If $L(M) = L$ and M is deterministic then M has at least n states.
c) If $L(M) = L$ and M is nondeterministic then M has at least n
d) L is a context-free language.
states.

6. (10 points) Consider the language $L = \{w \in \{a,b,c\}^* : w$ has the same number of a's, b's, and c's$\}$. This language contains the words abc, $aabcbc$, $cabacbabc$ et cetera. Consider the following theorem and proof:

Theorem: L is not context free.

Proof: We show that in the context free language game, B (you) can always win. Suppose A (the opponent) picks the integer n and B picks the word $(abc)^n$ (that is, abc or $abcabc$ or $abcabcabc$ ... where abc is repeated n times) that has length larger than n. Then A picks u, v, x, y, and z where v and y are not both e and $uvxyz = (abc)^n$ and B picks $i = 2$. The word uv^2xy^2z is not in L because the numbers of a's, b's and c's are not the same. Therefore B always wins, and L is not context free.

(a) Is the theorem correct? Justify your answer.

(b) Is the proof correct? Justify your answer.

7. (8 points) What does the deterministic Turing machine $M = (K, \Sigma, \delta, s, H)$ do, where $K = \{s, t, h\}$, Σ includes $\{a, b, \sqcup\}$ and possibly other symbols, $H = \{h\}$, and δ includes the following rules, along with possibly other rules:

$\delta(s, \sqcup) = (s, \rightarrow)$
$\delta(s, a) = (s, \rightarrow)$
$\delta(s, b) = (t, \rightarrow)$
$\delta(t, \sqcup) = (t, \rightarrow)$
$\delta(t, a) = (h, \sqcup)$
$\delta(t, b) = (t, \rightarrow)$

Here $\sqcup$ represents a blank. State in words what happens when M is started with the read write head scanning a blank and a string in the set $\{a, b\}^*$ to the right of the scanned blank, followed by infinitely many blanks on the tape. What will M write on the tape? Under what conditions will M halt?

8. (10 points) Consider the context-free grammar $(\{S, A, a, b\}, \{a, b\}, R, S)$ where the rules R are as follows:

$$\begin{array}{rr} \multicolumn{2}{c}{S \rightarrow a} \\ S \rightarrow Aa & S \rightarrow Ba \\ A \rightarrow b & B \rightarrow b \\ A \rightarrow Ab & B \rightarrow Bb \end{array}$$

Is this grammar ambiguous? Justify your answer.

9. (12 points) Let L be $\{i : T_i \text{ does not halt on input } i\}$ where T_i is as defined in question 1. Show that there is no Turing machine T such that T halts on input i if i is in L, and T does not halt on input i otherwise. (Hint: Since T is a Turing machine, T is T_j for some integer j.)