

COMP 455 (old 181)  
Models of Languages and Computation  
Spring 2012  
Rules of Inference for Sets

To show $\{a_1, a_2, \dots, a_n\} \subseteq A$ : show $a_1 \in A$ and show $a_2 \in A$ and ... and show $a_n \in A$	To show $\{a_1, a_2, \dots, a_n\} \not\subseteq A$ : show $a_1 \notin A$ or show $a_2 \notin A$ or ... or show $a_n \notin A$
To show $\{a\} \subseteq A$ : show $a \in A$	To show $\{a\} \not\subseteq A$ : show $a \notin A$
To show $\phi \subseteq A$ : succeed	To show $\phi \not\subseteq A$ : fail
To show $x \in \{a_1, a_2, \dots, a_n\}$ : show $x = a_1$ or show $x = a_2$ or ... or show $x = a_n$	To show $x \notin \{a_1, a_2, \dots, a_n\}$ : show $x \neq a_1$ and show $x \neq a_2$ and ... and show $x \neq a_n$
To show $a_i \in \{a_1, a_2, \dots, a_n\}$ : succeed	To show $a_i \notin \{a_1, a_2, \dots, a_n\}$ : fail
To show $x \in \{a\}$ : show $x = a$	To show $x \notin \{a\}$ : show $x \neq a$
To show $x \in \phi$ : fail	To show $x \notin \phi$ : succeed
To show $A = B$ : show $A \subseteq B$ and show $B \subseteq A$	To show $A \neq B$ : show $A \not\subseteq B$ or show $B \not\subseteq A$
To show $x \in A \cap B$ : show $x \in A$ and show $x \in B$	To show $x \notin A \cap B$ : show $x \notin A$ or show $x \notin B$

To show  $x \in A \cup B$  :  
show  $x \in A$  or  
show  $x \in B$

To show  $x \notin A \cup B$  :  
show  $x \notin A$  and  
show  $x \notin B$

To show  $x = x$  :  
succeed

To show  $x \neq x$  :  
fail

To show  $x \in 2^A$  :  
show  $x \subseteq A$

To show  $x \notin 2^A$  :  
show  $x \not\subseteq A$

To show  $x \in A - B$  :  
show  $x \in A$  and  
show  $x \notin B$

To show  $x \notin A - B$  :  
show  $x \notin A$  or  
show  $x \in B$

To show  $x \in \cup\{a_1, a_2, \dots, a_n\}$  :  
show  $x \in a_1$  or  
show  $x \in a_2$  or ... or  
show  $x \in a_n$

To show  $x \notin \cup\{a_1, a_2, \dots, a_n\}$  :  
show  $x \notin a_1$  and  
show  $x \notin a_2$  and ... and  
show  $x \notin a_n$

To show  $x \in \cup\phi$  :  
fail

To show  $x \notin \cup\phi$  :  
succeed

To show  $x \in \cap\{a_1, a_2, \dots, a_n\}$  :  
show  $x \in a_1$  and  
show  $x \in a_2$  and ... and  
show  $x \in a_n$

To show  $x \notin \cap\{a_1, a_2, \dots, a_n\}$  :  
show  $x \notin a_1$  or  
show  $x \notin a_2$  or ... or  
show  $x \notin a_n$

To show  $x \in \cap\phi$  :  
succeed

To show  $x \notin \cap\phi$  :  
fail

To show  $A \subseteq B$  if  $A$  is not given explicitly:

Let  $x$  be a new variable.

Show  $x \in B$  using the assumption that  $x \in A$ .

If  $x \in A$  and  $A \subseteq B$  then  $x \in B$ .