COMP 520: Compilers

Sample Solution – Supplementary Problem for WA2

(Note: the rules applied in these solutions can be found in lecture 4, slides 6 – 19)

For the grammar below, determine the Nullable, Starter, and Follow sets for the grammar nonterminals. Next, identify all choice points in the grammar and for each choice point determine the Predict sets. Then answer whether the grammar meets the LL(1) condition.

\[
\begin{align*}
S & ::= A \ \$
\end{align*}
\]

\[
\begin{align*}
A & ::= a \ | \ A \ x \ | \ B \\
B & ::= b \ | \ B \ x \ | \ D \\
D & ::= d \ | \ D \ x \ | \ f
\end{align*}
\]

First determine the grammar properties by finding the fixpoint of the relevant equations.

1. Nullable

\[
\begin{align*}
N_0(S) &= N_0(A) = N_0(B) = N_0(D) = false \\
N_{i+1}(S) &= N_i(A) \land N(\$) = false \\
N_{i+1}(A) &= N(a \ A \ x) \lor N_i(B) = N_i(B) \\
N_{i+1}(B) &= N(b \ B \ x) \lor N_i(D) = N_i(D) \\
N_{i+1}(D) &= N(d \ D \ x) \lor N(f) = false
\end{align*}
\]

After two iterations we have a fixpoint

\[
\begin{align*}
N(S) &= N(A) = N(B) = N(D) = false
\end{align*}
\]

2. Starters

\[
\begin{align*}
ST_0(S) &= ST_0(A) = ST_0(B) = ST_0(D) = \{ \} \quad \text{since no nullable NT in this grammar} \\
ST_{i+1}(S) &= ST_i(A) \oplus ST(\$) = ST_i(A) \\
ST_{i+1}(A) &= ST(a \ A \ x) \cup ST_i(B) = \{a\} \cup ST_i(B) \\
ST_{i+1}(B) &= ST(b \ B \ x) \cup ST_i(D) = \{b\} \cup ST_i(D) \\
ST_{i+1}(D) &= ST(d \ D \ x) \cup ST(f) = \{d, f\}
\end{align*}
\]

Iterate to fixpoint:

\[
\begin{align*}
ST(S) &= \{a, b, d, f\}, \quad ST(A) = \{a, b, d, f\}, \quad ST(B) = \{b, d, f\}, \quad ST(D) = \{d, f\}
\end{align*}
\]

3. Followers

\[
\begin{align*}
FL_0(S) &= \{ \} \quad \text{S not on RHS of any rule} \\
FL_0(A) &= \{x, \$\} \quad \text{A on RHS rule 2 and rule 1} \\
FL_0(B) &= \{x\} \quad \text{B on RHS rule 3, and at end of rule 2} \\
FL_0(D) &= \{x\} \quad \text{D on RHS rule 43 and at end of rule 3}
\end{align*}
\]

Where NT X occur on as the last symbol on the RHS in a rule for NT Y, the FL(X) = FL(Y). This is the case for B and D in rules 2 and 3, respectively. So we have add

\[
\begin{align*}
FL_{i+1}(B) &= FL_i(B) \cup FL_i(A) \\
FL_{i+1}(D) &= FL_i(D) \cup FL_i(B)
\end{align*}
\]

and iterate to fixpoint. This yields \(FL(B) = FL(D) = \{x, \$\} \).
Finally we need to add \( \varepsilon \) to \( FL(N) \) for any nonterminal \( N \) that can appear as the last symbol in a derivation starting from \( S \). As this is an augmented grammar in which (a) the start symbol \( S \) does not appear on the RHS of any rule and (b) the rule for \( S \) ends in a terminal (\( c \) in this case), we can be sure only \( FL(S) \) includes \( \{ \varepsilon \} \). So \( FL(S) = \{ \varepsilon \} \).

In summary we have

\[
\begin{array}{|c|c|c|c|}
\hline
& \text{Nullable} & \text{Starters} & \text{Followers} \\
\hline
S & F & \{a, b, d, f\} & \{\varepsilon\} \\
A & F & \{a, b, d, f\} & \{x, $\} \\
B & F & \{b, d, f\} & \{x, $\} \\
D & F & \{d, f\} & \{x, $\} \\
\hline
\end{array}
\]

To determine whether the LL(1) condition is met for this grammar, inspect all choice points to determine if they define the predict function unambiguously. There are three choice points: the

- **A ::= a A x | B**
  
  \[
  \begin{align*}
  \text{Predict}(a A x) &= \{a\} \\
  \text{Predict}(B) &= \text{ST}(B) = \{b, d, f\} \\
  \end{align*} 
  \]
  
  The predict sets are disjoint, so the LL(1) condition is met.

- **B ::= b B x | D**
  
  \[
  \begin{align*}
  \text{Predict}(b B x) &= \{b\} \\
  \text{Predict}(D) &= \text{Starters}(D) = \{d, f\} \\
  \end{align*} 
  \]
  
  The predict sets are disjoint, so the LL(1) condition is met.

- **D ::= d D x | f**
  
  \[
  \begin{align*}
  \text{Predict}(d D x) &= \{d\} \\
  \text{Predict}(f) &= \{f\} \\
  \end{align*} 
  \]
  
  The predict sets are disjoint, so the LL(1) condition is met.

Since the LL(1) condition is met at all choice points in the grammar, this grammar is LL(1).