COMP 633: Parallel Computing
Fall 2021
Written Assignment #2

Assigned: Mon Nov 1
Due: Tue Nov 23

1. **[10]** Let *H*[1:*n*] be an array of integer values in the range $1..k$, with that $1 \le k \le n$. We want to find the most frequently occurring value *m* in *H*, i.e. the mode of *H*. For example, with $n = 8, k = 4$, and $H = [1,3,1,3,3,4,3,1]$, we should find $m = 3$. For simplicity you can assume that the mode is unique.

(a) Verify that the sequential time complexity for this problem is $\Theta(n)$.

(b) Describe an *efficient* parallel BSP algorithm for this problem using *p* processors assuming the condition *n* = *kp* with $k \ge p$ and give its BSP cost. The input *H* is distributed evenly over processors, so that initially each processor holds *k* input values (assume *p* divides *n* evenly). The result *m* should be available in the first processor on termination.

2. **[10]** An alternative way to construct a bitonic sort for $N = np$ elements with $p = 2^k$ processors is to extend the compare-exchange operation to *sorted sequences*. For v, w sorted sequences of length n let $\mathrm{CE}_{\mathrm{seq}}(v,w) = s,t$ with $s = \mathrm{merge}(v,w)[1{:}n]$ and $t = \mathrm{merge}(v,w)[n+1{:}2n]$. Observe the $2n$ elements of s,t define a permutation of the $2n$ elements of v,w and that s,t are partitioned, meaning any element in s is less than or equal to any element in t. More formally, the transitive relation $s \preccurlyeq t \equiv \forall_{1 \le i,j \le n}\, s_i \le t_j$.

Using this relation, design a comparison-based bitonic sort of $N = np$ elements using p processors with asymptotically optimal work efficiency (in the sense of lecture 16, last slide). Show the complete BSP cost of your solution.