1. On slide 20 of lecture 16 we derived a parallel implementation of matrix multiplication with BSP cost

\[ C^{MM}(n, p) = \frac{2n^3}{p} + \left(\frac{2n^2}{\sqrt{p}}\right) \cdot g + 2 \cdot L \]

Can you design a way to improve the communication cost?

2. An alternate way to construct a bitonic sort for \( N = np \) elements with \( p = 2^k \) processors is to extend the compare-exchange operation to *sorted sequences*. For \( v, w \) sorted sequences of length \( n \) let \( CE_{\text{seq}}(v, w) = s, t \) with \( s = \text{merge}(v, w)[1:n] \) and \( t = \text{merge}(v, w)[n + 1:2n] \). Observe the \( 2n \) elements of \( s, t \) define a permutation of the \( 2n \) elements of \( v, w \) and \( s, t \) are partitioned, meaning any element in \( s \) is less than or equal to any element in \( t \). More formally, the transitive relation \( s \preceq t \equiv \forall_{1 \leq i, j \leq n} s_i \leq t_j \). Using this relation, design a comparison-based bitonic sort of \( N = np \) elements using \( p \) processors with asymptotically optimal work efficiency (in the sense of lecture 16, last slide). Show the complete BSP cost of your solution.