COMP 633 - Parallel Computing

Lecture 5
Sep 2, 2021

PRAM (4)
The relative power of different *PRAM models*

Topics

- Comparison of PRAM models
  - relative performance of EREW, CREW, and CRCW models
    - separated by lower bounds for key problems
    - related by simulation results
- Restricted CR/CW models
- Abstract computational complexity classes
  - inherently sequential problems?
- Work-Time and PRAM model
  - advantages
  - disadvantages

Comparison of PRAM memory models

- Some specific problems illustrate asymptotic advantage of CR and CW
  - Assume p processor PRAM (so not WT model)

  - Copy problem
    - given scalar y in shared memory, create vector R[1:p] with R[i] = y for $1 \leq i \leq p$
      EREW: $T_C(p) = \Theta(\lg p)$
      CREW, CRCW: $T_C(p) = \Theta(1)$

  - Maximum problem (comparison-based)
    - given X[1:p] in shared memory, find maximum value m in X
      EREW, CREW: $T_C(p) = \Theta(\lg p)$
      arbitrary CRCW: $T_C(p) = \Theta(\lg \lg p)$
      max-combining CRCW: $T_C(p) = \Theta(1)$

- These problems illustrate that
  EREW ‹ CREW ‹ arbitrary CRCW ‹ combining CRCW

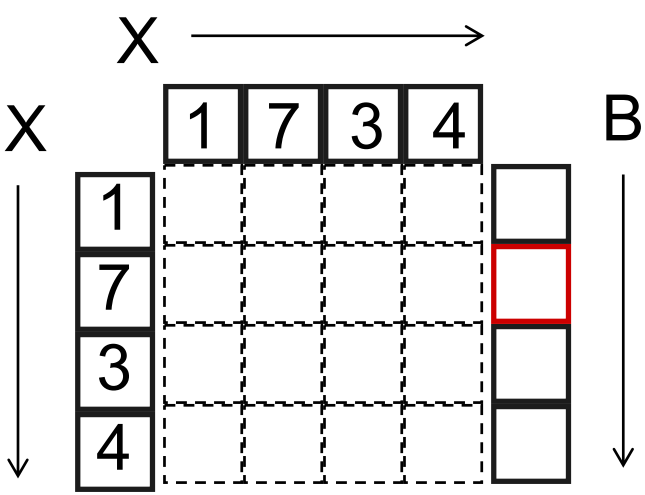
  where A ‹ B means that some problem can be solved asymptotically faster using model B than using model A

Power of concurrent reads: Copy problem

- EREW Upper bound: $O(\lg p)$ time
  - how?
- EREW Lower bound: $\Omega(\lg p)$ time
  - consider a step consisting of an EREW PRAM read and write operation.
    - step 1
      - at the start of step 1, only one copy of the value y exists
      - Only one processor can read it and write it back to a new location
    - step 2
      - at the start of step 2, two copies of the value exist
      - two processors can each read one value simultaneously and write it back simultaneously to distinct locations
    - step ($\lg p$)
      - at most $2^{(\lg p)-1} = p/2$ copies exist, $p/2$ processors copy them simultaneously yielding p copies.
    - therefore $\Omega(\lg p)$ steps are needed for p processors to read a single value
- CREW upper and lower bound: $\Theta(1)$ time
  - trivial

Power of concurrent writes: Maximum problem

- Find maximum *m* of sequence $X = \langle x_1, \ldots, x_n \rangle$
  - EREW algorithm:
    - S(n) = Θ(lg *n*) , W(n) = Θ(*n*)
  - CRCW algorithm
    - S(n) =
    - W(n) =



CRCW fast maximum - WT formulation

Input: `X[1:n]`

Output: $m = \max_{i \in 1:n} X[i]$

Auxiliary: `B[1:n]`

```
forall i in 1 : n do
   B[i] := 1
enddo

forall (i,j) in {1..n} × {1..n} do
   if X[i] < X[j] then
        B[i] := 0
   endif
enddo

forall i in 1 : n do
   if B[i] = 1 then
      m := X[i]
   endif
enddo
```

Work-efficient CRCW Maximum

1. Apply CRCW fast max to inputs organized into a very shallow tree

   place values at leaves of doubly-logarithmic depth tree

   - assume $n = 2^{2^k}$ for some $k > 0$, so $k = \lg \lg n$
   - branching factor at level $1 \leq i \leq k$ is $2^{2^{k-i}}$ (add last level k+1 with bf 2)
   - apply fast maximum in parallel to all nodes in a level
     - total work at each level is O(n), number of levels is O(lg lg n)
   - $S_1(n) = O(\lg \lg n)$, $W_1(n) = O(n \lg \lg n)$ better but still not work efficient
2. In parallel apply sequential max to groups of (lg lg n) elements of input
   - let $m = n / (\lg \lg n)$
   - $S_2(n) = O(\lg \lg n)$, $W_2(n) = O(mS_2(n)) = O(n)$
3. Cascade (1) and (2) to construct work-efficient fast maximum
   - $S(n) = S_1(m)+S_2(n) = O(\lg \lg n)$, $W(n) = W_1(m)+ W_2(n) = O(n)$
4. CRCW PRAM-level *upper* bound for maximum
   - Brent's theorem with n = p: $T_C(p,p) = O(p/p + \lg \lg p) = O(\lg \lg p)$

CRCW PRAM-level *lower* bound for maximum: $\Omega(\lg \lg p)$ [hard]

Quantifying relative power of CR/CW

- Lower bound: from example problems
  - $\Omega(\lg p)$ slowdown of p-processor EREW PRAM over p-processor CREW or CRCW PRAM
  - guaranteed for some problems

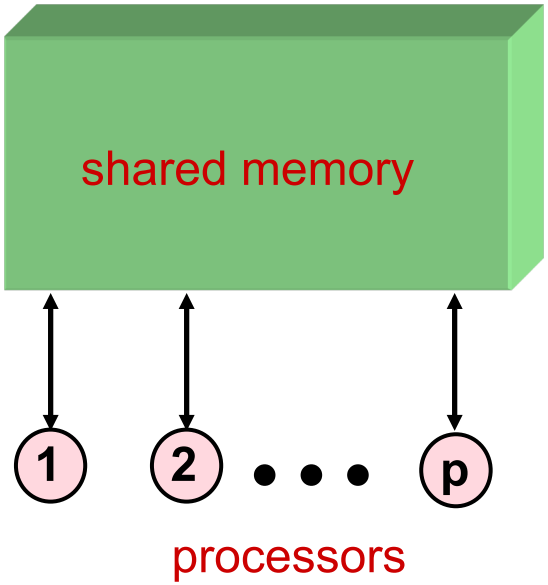
- Upper bound: by simulation argument
  - $O(\lg p)$ slowdown in p-proc EREW PRAM simulation of p-proc CRCW PRAM
    - Depends on existence of EREW algorithm to sort p values in $O(\lg p)$ steps using p processors [Cole]
  - To simulate a single CR step using EREW PRAM
    - sort all addresses being read
    - identify unique addresses
    - EREW read of unique addresses
    - replicate (COPY) results to match number of reads (copy)
  - To simulate a single CW step using EREW PRAM
    - sort all (addr, new value) pairs
    - implement CW resolution strategy using parallel prefix
    - EREW write of surviving (addr, value) pairs

PRAM shared memory system

- PRAM model
  - assumes access latency is constant, regardless of value of p
  - includes CR and/or CW
- Physically impossible
  - processors and memory occupy finite volume *p* and *m*
    - speed of light dictates increasing latency

$$L = \Omega\left((p+m)^{1/3}\right)$$

  - CR / CW must be reduced to ER / EW
    - requires $\Omega(\lg p)$ time in general case



Restricted CR / CW models

- unbounded CR and CW are expensive to implement
  - multi-stage combining and expansion network with lg p depth
    - NYU Ultracomputer, SB-PRAM
- restricted models with more efficient implementations
  - QRQW - queued read / queued write
    - cost of reference proportional to number of concurrent readers / writers
  - single-value broadcast
    - fundamental for SIMD operation
    - (simple) custom network
  - concurrent write of single shared location (bit)
    - fundamental for SIMD operation
      - O(1) detection if any processor is enabled
    - (simple) custom network: write combining via logical OR
    - example: maximum problem in bit model

Bit-serial CRCW maximum algorithm

- $S(n) = O(b)$
- $W(n) = O(bn)$

Bit-serial CRCW fast maximum

Input: `X[1:n]` *b*-bit unsigned int

Output: `m =` $\max_{i \in 1:n} X[i]$

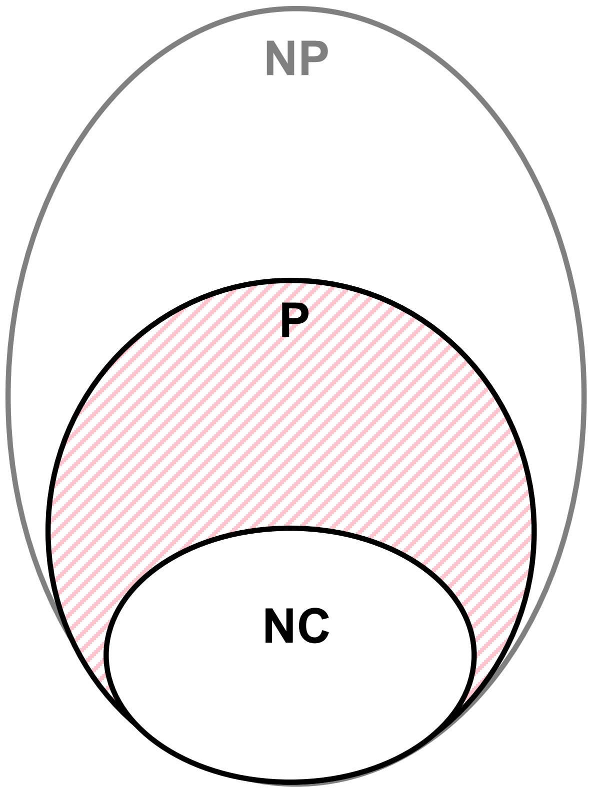
Auxiliary: `B[1:n]` single-bit values

`c` single-bit CRCW value

```
forall i in 1 : n do
    B[i] := 1
enddo
m := 0
for k := b-1 downto 0 do
    c := 0
    forall i in 1 : n do
        if (B[i] & (X[i] bit k)) then
            c := 1
        endif
        B[i] := B[i] & (c == (X[i] bit k))
    enddo
    (m bit k) := c
enddo
```

Relation of (P)RAM complexity classes

- Complexity class P
  - problems with polynomial time complexity on RAM
    - $W(n) = S(n) = O(n^{O(1)})$ in W-T model
- Complexity class NC
  - problems in P with fast parallel algorithms
    - $W(n) = O(n^{O(1)})$ polynomial work
    - $S(n) = O(\lg^{O(1)} n)$ poly-logarithmic step complexity
  - very coarse form of work-efficiency
- (P – NC)
  - "inherently sequential" problems

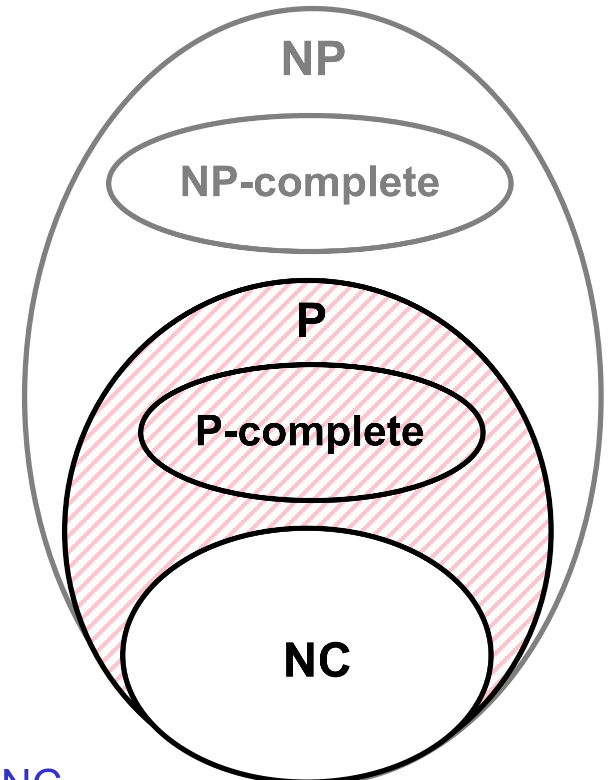


Inherently Sequential Problems

- Polynomial Time complete (P-complete) problems
  - H is P-complete if
    - $H \in P$
    - for all $A \in P$, A is log-space (RAM) reducible to H
  - P-complete problem
    - Circuit Value Problem (CVP)
  - P-complete by reduction to CVP
    - maximum flow in network, CFG parsing, (predicate logic) unification

- Can we find a fast parallel algorithm for a P-complete problem?
  - $H \in NC$ and H is P-complete implies P = NC
  - no luck yet

- Conjecture: $(P - NC) \neq \emptyset$
  - if true
    - there exist “inherently sequential” problems
    - of limited consequence due to coarse definition of NC



W-T and PRAM models - conclusions

- Strengths of W-T and PRAM models
  - Ignore memory access costs
  - Source-level complexity metrics simplify analysis
  - Widely developed body of techniques
  - W-T programs are simple and expressive

- Liabilities of W-T and PRAM models
  - memory access cost is not constant in real life
    - already true with RAM model
      - random memory access time is $\Omega(|mem|^{1/3})$ in 3D space with speed of light restrictions
      - this is the reason for cache memories in modern processors
    - even less accurate for PRAM model
      - random memory access time is $\Omega((p+|mem|)^{1/3})$
      - CR / CW implementations require $\Omega(\lg p)$ time with present technologies
      - switching and bandwidth issues complicate situation further

W-T and PRAM model - conclusions

- Liabilities of W-T and PRAM models
  - Source-level complexity metrics oversimplify analysis
    - given two implementations
      - efficient sequential algorithm S on sequential computer
      - work-efficient and fast parallel algorithm C on PRAM-like parallel computer
    - for sufficiently large n, there exists p such that $T_c(n,p) < T_s(n)$
      - parallel algorithm is guaranteed to run faster
    - but p (and n) may be impractically large
      - p is not a truly scalable parameter in practice
  - Widely developed body of unrealistic techniques
    - extensive use of asymptotically efficient but impractical building blocks
      - fast and efficient sorting, efficient pointer jumping, etc.
  - W-T programs may not be able to fully express efficient implementations
    - homework problem 1