Basics of electric circuit theory

The **circuit** concept is an abstraction that radically simplifies the behavior of the underlying E&M. A circuit is a *topological* construct of arcs and nodes. Each arc represents a two-terminal **component** carrying a **current** between nodes. Each node represents a **voltage** which is shared in common by all the component terminals connected to it.

Voltage V is actually a *difference* in electric potential between nodes. It is *always* relative, but is often considered in reference to a hypothetical "**ground**" node at zero potential so that the concept of "the voltage on a node" makes sense. The voltage across a component is the voltage between the nodes to which it is connected. Sometimes called the *electromotive force* (EMF), voltage characterizes the force motivating charge to flow through the component.

Current $I = dQ/dt$ is the rate of flow of charge Q through a component. Current is physically carried by *conduction electrons* in the component. Each electron carries a charge of $-1.60{\times}10^{-19}$ [C].

A component represents the behavior of the voltage across and the current conveyed between its terminals. Two basic kinds of components are **sources** and **impedances**.

There are two kinds of sources, **voltage sources** that generate a voltage irrespective of the current in them, and **current sources** that generate a current irrespective of the voltage across them. Sources may be *independent*, either constant or varying as a fixed function of time; or they may be *dependent* on other circuit parameters.

Impedances Z relate the voltage across them with the current through them by a behavior called **Ohm's Law**:

$$V = IZ.$$

Linear circuit theory assumes linear, time-independent (**LTI**) impedances. Three kinds of LTI impedance components (discussed elsewhere) are commonly used, **resistors** R, **capacitors** C, and **inductors** L.

Two simple behavioral rules, **Kirchoff's Laws**, completely characterize the circuits just described (known as lumped constant LTI circuits):

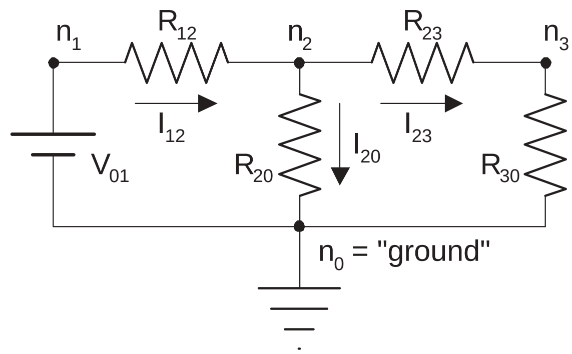
$$\sum_{k=1}^{n} V_k = 0, \qquad \text{(KVL)}$$

$$\sum_{k=1}^{n} I_k = 0. \qquad \text{(KCL)}$$

Kirchoff's voltage law (KVL) says the sum of voltages across components taken around a closed loop is zero. This is trivially apparent: since the first and last nodes of a closed loop are the same node, the voltage "between" them must vanish.

Kirchoff's current law (KCL) says that current leaving a node must balance the current entering the node. The underlying physical principle is the *conservation of charge*, i,e, charge cannot be created (or destroyed).

Example – calculate circuit currents



We are given the circuit topology, voltage source V_{01}, and resistors R_{12}, R_{20}, R_{23}, and R_{30}. There are two non-trivial current nodes, namely n_0 and n_2, which are linearly dependent, so we choose n_0. Using KCL we write,

$$I_{12} - I_{20} - I_{23} = 0$$

In all, there are three topological loops, namely (n_0, n_1, n_2, n_0), (n_0, n_2, n_3, n_0), and $(n_0, n_1, n_2, n_3, n_0)$. Only two of these are linearly independent, so we arbitarily choose the first two. Recognizing that current emanates from the positive source terminal while it passes through a resistor from the positive to the negative terminal we use KVL to write,

$$V_{01} - I_{12}R_{12} - I_{20}R_{20} = 0$$
$$I_{20}R_{20} - I_{23}R_{23} - I_{23}R_{30} = 0$$

Solving the above three equations for the three currents we get,

$$I_{12} = V_{01}\frac{R_{30} + R_{23} + R_{20}}{R_{20}(R_{30} + R_{23} + R_{12}) + R_{12}(R_{30} + R_{23})}$$
$$I_{20} = V_{01}\frac{R_{30} + R_{23}}{R_{20}(R_{30} + R_{23} + R_{12}) + R_{12}(R_{30} + R_{23})}$$
$$I_{23} = V_{01}\frac{R_{20}}{R_{20}(R_{30} + R_{23} + R_{12}) + R_{12}(R_{30} + R_{23})}$$

Notice we could have specified two more currents I_{01} and I_{30} and added two more KCL equations, $I_{01} - I_{12} = 0$ and $I_{23} - I_{30} = 0$, but these were trivially apparent.