Physical Dimensions and Units

Physical behavior is quantitatively represented by formulas which model relationships between physical quantities. All meaningful physical quantities comprise products and/or quotients of a limited number of types of basic **dimensions**. All *mechanical* and *electrical* quantities can be composed of just four dimensional types, mass [M], length [L], time [T], and charge [Q].

For example, a physical equation representing velocity v in terms of distance d traveled in time t is, $v = d/t$. Obviously the dimensions of d and t are [L] and [T], respectively. The units of v are reckoned by a *dimensional* equation, $[\mathrm{L\,T^{-1}}] = [\mathrm{L}]/[\mathrm{T}]$, of the same algebraic form as the physical equation. It is sometimes convenient to interleave these two equations, as in the following example for the area A of a circle of radius r, $A[\mathrm{L}^2] = \pi[1] \times r^2[\mathrm{L}]^2$. The **dimensionless constant** π is dimensionally represented as [1].

Physically meaningful formulas may *never* contain sums of different dimensions. For example, one might argue that $FatCats[\mathrm{cat} - \mathrm{fish}] = 4[\mathrm{cat}] - 6[\mathrm{fish}]$ gives some very satisfied kitties, but from a physically quantitative viewpoint, it makes no sense whatsoever!

Dimensional types are quantified by **units**. For example, a length can be measured in centimeters, feet, furlongs, etc. These are all the same kind of dimension, namely [L], but the units differ. Units can be thought of as values assigned to dimensions in dimensional equations. Conversion between different units is done using dimensionless constants. For example, the distance $d[\mathrm{L}] = 1500[\mathrm{ft}]$ in miles is $d[\mathrm{mi}] = 1500[\mathrm{ft}] \cdot 1.89{\times}10^{-4}[\mathrm{mi/ft}] = 0.284[\mathrm{mi}]$, where $1.89{\times}10^{-4}[\mathrm{mi/ft}] = 1[\mathrm{mi}]/5280[\mathrm{ft}]$ is dimensionless since $[\mathrm{L/L}] = [1]$.

A universally accepted coherent system of units is the Système International d'Unités (SI). A useful elecrical/mechanical subset of the SI units is summarized in the table below.

Quantity	SI Units &	Abbrev.	dimensions	commonsymbol
Mass	kilogram	[kg]	[M]	m
Length	meter	[m]	[L]	(various)
Time	second	[s]	[T]	t
Frequency	Hertz	[Hz]	$[\mathrm{T^{-1}}]$	f
Force	Newton	[N]	$[\mathrm{M\,L\,T^{-2}}]$	F
Energy	Joule	[J]	$[\mathrm{M\,L^2\,T^{-2}}]$	E
Power	Watt	[W]	$[\mathrm{M\,L^2\,T^{-3}}]$	P
Electric charge	Coulomb	[C]	[Q]	q
Voltage	Volt	[V]	$[\mathrm{M\,L^2\,T^{-2}\,Q^{-1}}]$	v, V
Current	Ampere	[A]	$[\mathrm{T^{-1}\,Q}]$	i, I
Resistance	Ohm	[Ω]	$[\mathrm{M\,L^2\,T^{-1}\,Q^{-2}}]$	R
Inductance	Henry	[H]	$[\mathrm{M\,L^2\,Q^{-2}}]$	L
Conductance	Siemen	[S]	$[\mathrm{M^{-1}\,L^{-2}\,T\,Q^2}]$	G
Capacitance	Farad	[F]	$[\mathrm{M^{-1}\,L^{-2}\,T^2\,Q^2}]$	C

Many physical models relate the behavior of two quantities as fixed ratios known as **dimensional constants**. For example, the spring constant $k[\mathrm{M\,T^{-2}}] = -f[\mathrm{M\,L\,T^{-2}}]/x[\mathrm{L}]$ relates the force f exerted by a spring in opposition to being deflected a distance x.