Passive LTI circuit components: Impedances and Admittances

These ideal circuit components are linear, time independent (LTI), and do not generate electrical power i.e., they are *passive*. There are three kinds, **resistors**, **capacitors**, and **inductors**. Resistors dissipate electrical energy, turning it into heat, while capacitors and inductors store and exchange electrical energy without dissipation.

The physics underlying the behavior of these components is expressed as linear first-order differential equations in time. In linear circuit theory, it is usually more convenient to express the behavior in terms of frequency. The **Laplace transform** $\mathcal{L}()$, discussed elsewhere, provides the means to transform from *time domain* to *frequency domain*.

Here it is sufficient to understand that $\mathcal{L}\big(a + bx + c(dx/dt)\big) = a + bx + (sc)x$, where the independent variables are time t and frequency s, respectively; and that $s = (\sigma + j\omega)$ is a complex number, where $j = \sqrt{-1}$. Thus in frequency or s-domain, we need only work with algebraic equations.

In Time domain, the behaviors of inductors, resistors, and capacitors are respectively,

$$V = L\frac{dI}{dt}, \qquad V = IR, \qquad I = C\frac{dV}{dt};$$

which in frequency domain become,

$$v = i(sL), \qquad v = iR, \qquad i = v(sC),$$

where $v = \mathcal{L}(V)$ and $i = \mathcal{L}(I)$.

The **impedance** Z and **admittance** Y of a component are defined by **Ohm's Law**,

$$v = iZ, \qquad \text{(OR)} \qquad i = vY,$$

where a voltage v impressed across the component motivates a current i through it. For a given voltage, a larger value of Z *impedes* the passage of current.

where a current i passing through the component develops a voltage v across it. For a given voltage, a larger value of Y *admits* passage of more current.

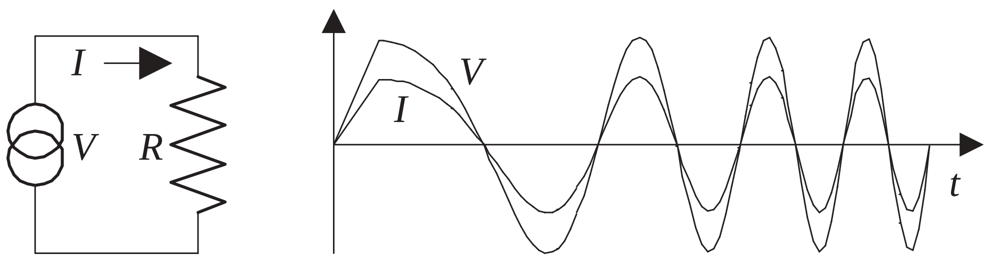
Accordingly, the impedances Z of inductors, resistors, and capacitors are respectively,

$$sL, \qquad R, \qquad \frac{1}{sC}.$$

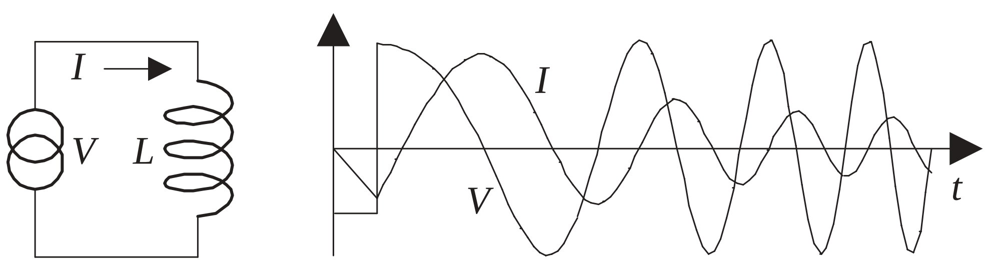
Y and Z are reciprocal representations of the same physical property, **immittance**, that relates voltage to current in a component. In general, immittances are complex scalars $Z = R + jX = 1/Y$ or equivalently $Y = G + jB = 1/Z$. We call $R, X, G,$ and B the **resistance**, **reactance**, **conductance**, and **susceptance**, respectively.

Examples of component behaviors

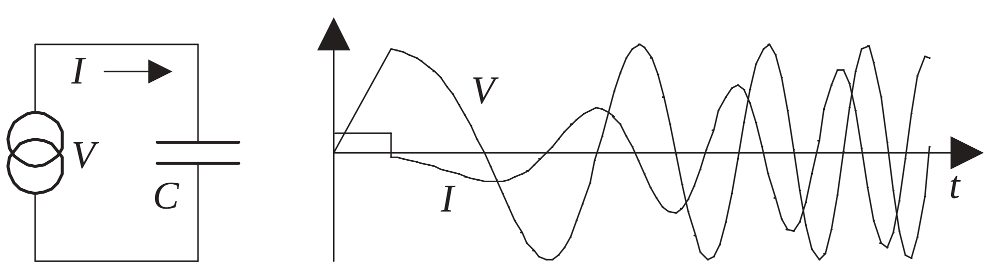
Time domain plots of current and voltage are shown for a resistor, an inductor, and a capacitor. The voltage source is controlled; the current depends entirely on the voltage and the impedance of the component.



Resistor: the voltage is ramped up linearly, then varies sinusoidally with increasing frequency. The current is proportional to the voltage at all times.



Inductor: the voltage is held constant, then switched to a sinusoid with increasing frequency. The current is the time integral of the voltage, at first a linear ramp, then varying sinusoidally lagging the phase of the voltage by 90°, and amplitude decreasing inversely with frequency.



Capacitor: the voltage is ramped up linearly, then varies sinusoidally with increasing frequency. The current is the time derivative of the voltage, at first a constant value, then varying sinusoidally leading the phase of the voltage by 90°, and amplitude increasing linearly with frequency.