1) (10’) CLRS 3-1a
2) (18’) CLRS 3-2
3) (32’) CLRS 3-4 (You don’t have to formally prove these true, just identify those that are true and provide a counter example for those that aren’t).
4) (10’) Prove the following summation by induction:

\[ \sum_{k=1}^{n} kk! = (n + 1)! - 1 \]

5) (15’) The Carnival Coin Game: You are running a game booth at your local village carnival. In your game you lay out an array of \( n \) (\( n>0 \)) coins on a table. In this game, you and your customer alternately pick coins from the table, either 1 or 2 at a time. If your customer can make you pick up the last coin, he wins and walks away with all the coins. You graciously allow your customer to go first. Being an enterprising sort, you want to arrange the game so that you will always win.

(a) (5’) What constraint(s) for \( n \) do you need to guarantee a win every time?

(b) (5’) Describe a correct algorithm (strategy).

(c) (5’) What is the loop invariant for the algorithm?

6) (15’) The Max Subvector Problem: Given an array \( a[1...n] \) of numeric values (can be positive, zero and negative) determine the maximum value of sums to all subvectors \( a[i...j] \) (\( 1 \leq i \leq j \leq n \)). Show that:

(a) (5’) Maximum Subvector is of \( \Omega (n) \)

(b) (5’) Maximum Subvector is of \( O(n^3) \)

(c) (5’) Maximum Subvector is of \( \Theta (n) \)

\( (Hint: \ How \ to \ treat \ a \ subvector \ with \ negative \ sum \ during \ the \ search \ process?) \)
Rules for ALL HWs (in addition to the statements in the syllabus):

You are encouraged to discuss the problem sets and study together in group, but when it comes to formulating/writing solutions you must work alone independently; i.e., you should be able to explain your answer clearly to anyone else. Note that this says discuss in group — copying homework solutions from another student, from the Internet, solution sets of friends who have taken this course or one similar to it previously, or other sources will be considered cheating and referred to the student attorney general. You must include a signed honor statement with each submission explicitly listing the people you worked with and stating that you completed the assignment in accordance with these rules.