1) (20’) When tossing a fair coin, the probability of head or tail is the same (50%).

(a) A fair coin is tossed until a head comes up for the first time. What’s the probability of this happening on (ending after) an odd number toss?

(b) You apply the following rules to a fair coin: In the first round, you flip it. Starting from the second round, (i) if it’s a “head” after the previous round, just simply turn it upside down (a “head” is guaranteed to become a “tail” in the next round); (ii) if it’s a “tail” after the previous round, toss it again for once (50-50 for being either head or tail). What is the expected probability of the coin being “head” after round K, where K is a sufficiently large number?

(Hint: K=1, \( p_H = 0.5 \); K=2, \( p_H = 0.25 \); K=3, \( p_H = 0.375 \);...; k=+inf, \( p_H = ? \)

When K is sufficiently large, the expected probability of the coin being “head” should be stable; i.e., \( p_H \) remains unchanged.)

2) (12’) CLRS Exercise 5.2-4 on page 122

3) (12’) CLRS Exercise 5.2-5 on page 122

4) (28’) CLRS 7-5 on page 188

5) (28’) CLRS 7-2 on page 186

Rules for ALL HWs (in addition to the statements in the syllabus):

You are encouraged to discuss the problem sets and study together in group, but when it comes to formulating/writing solutions you must work alone independently; i.e., you should be able to explain your answer clearly to anyone else. Note that this says discuss in group — copying homework solutions from another student, from the Internet, solution sets of friends who have taken this course or one similar to it previously, or other sources will be considered cheating and referred to the student attorney general. You must include a signed honor statement with each submission explicitly listing the people you worked with and stating that you completed the assignment in accordance with these rules.