Homework solution for 5 February 2002

\[ V_{out} = V_{in} \frac{Z_c}{Z_l + R + Z_c} \]

The filter gain is defined as,

\[ A \equiv \frac{V_{out}}{V_{in}} \]

Therefore,

\[ A = \frac{1/sC}{sL + R + 1/sC} \]

\[ = \frac{1}{s^2LC + sRC + 1}, \quad \text{and for } s = j\omega, \]

\[ = \frac{1}{-\omega^2LC + j\omega RC + 1}, \]

\[ = \frac{(1 - \omega^2LC)/|A| - j\omega RC/|A|,}{\text{real}} \quad \text{and} \quad \text{imag} \]

where \(|A| = (1 - \omega^2LC)^2 + (\omega RC)^2\).

Inspecting the denominator of the third form above, notice that for \(\omega^2LC \ll 1\) and \(\omega RC \ll 1\), the first and second terms respectively are negligible, and in this regime \(A \approx 1\). Thus for \(L = 6.3 \times 10^{-6} \text{ [H]}, R = 16 \text{ [\Omega]}, \text{ and } C = 1.0 \times 10^{-7} \text{ [F]},\) the interesting behavior of \(A\) occurs for values of \(\omega\) near \(1/\sqrt{LC} = 1.26 \times 10^6\) and \(1/RC = 6.25 \times 10^5\) [rad/s].
The denominator of the second form above can be factored, giving

\[ A = \frac{1}{(s - s_1)(s - s_2)}, \]

where \([s_1, s_2] = -R/2L \pm \sqrt{(R/2L)^2 - 1/LC}\). The points \(s_1\) and \(s_2\) are known as \textit{poles} of the function \(A(s)\) because they represent locations on the complex plane where \(A(s) \to \infty\).

The plots look like,

(A Bode plot is a log log magnitude vs. frequency plot).