COMP 520: Compilers
Sample Solutions – Written Assignment 2

(Note: the rules applied in these solutions can be found in lecture 4, slides 6 – 19)

1. For the following grammars, find the Nullable, Starters, and Followers sets for the nonterminals and justify whether or not the grammar meets the LL(1) condition.

<u>Grammar (a)</u> S ::= A B **c**
A ::= **a** B | ε
B ::= **b** | ε

First determine the grammar properties by finding the fixpoint of the relevant equations.

1. Nullable
   $N_0(S) = N_0(A) = N_0(B) = \text{ false}$

   $N_{i+1}(S) = N_i(A) \wedge N_i(B) \wedge N(c) = N_i(A) \wedge N_i(B) \wedge \text{false} = \text{false}$
   $N_{i+1}(A) = \big(N(a) \wedge N_i(B)\big) \vee N(\varepsilon) = \big(\text{false} \wedge N_i(B)\big) \vee \text{ true} = \text{ true}$
   $N_{i+1}(B) = N(b) \vee N(\varepsilon) = \text{ false} \vee \text{true} = \text{ true}$

   No iteration is needed since the equations are at the fixpoint solution

2. Starters
   $ST_0(S) = \{\,\}, \quad ST_0(A) = ST_0(B) = \{\varepsilon\}$

   $ST_{i+1}(S) = ST_i(A) \oplus ST_i(B) \oplus ST(c) = (ST_i(A) - \{\varepsilon\}) \cup (ST_i(B) - \{\varepsilon\}) \cup \{c\}$
   $ST_{i+1}(A) = (ST(a) \oplus ST_i(B)\,) \cup \text{ST}(\varepsilon) = \{\text{a}\} \cup \{\varepsilon\} = \{a, \varepsilon\}$ (fixpoint)
   $ST_{i+1}(B) = ST(b) \cup \text{ST}(\varepsilon) = \{\text{b}\} \cup \{\varepsilon\} = \{b, \varepsilon\}$ (fixpoint)

   Iterate: $ST_1(S) = \{c\}$, $ST_2(S) = \{a, b, c\}$ (fixpoint)

3. Followers
   $FL_0(S) = \{\,\}$ S not on RHS of any rule
   $FL_0(A) = ST(Bc) - \{\varepsilon\} = \big(\{b, \varepsilon\} \oplus ST(c)\big) = \{b, c\}$ A on RHS rule 1
   $FL_0(B) = (ST(c) \cup ST(\varepsilon)) - \{\varepsilon\} = \{\text{c}\}$ B on RHS rule 1,2

   Since A ::= **a** B in rule 2 is of the form A ::= **a** Bγ, where γ is nullable, we must add the followers of A into the followers of B:

   $FL_{i+1}(B) = FL_i(B) \cup FL_i(A) = \{\text{c}\} \cup \{b, c\} = \{b, c\}$

   As $FL_1(B)$ is fully defined, no further iteration is needed and $FL(B) = FL_1(B) = \{b, c\}$.

   Finally we need to add ε to $FL(N)$ for any nonterminal N that can appear as the last symbol in a derivation starting from S. As this is an augmented grammar in which (a) the start symbol S does not appear on the RHS of any rule and (b) the rule for S ends in a terminal (c in this case), we can be sure only $FL(S)$ includes $\{\varepsilon\}$. So $FL(S) = \{\varepsilon\}$.

In summary we have

	Nullable	*Starters*	*Followers*
S	F	{a, b, c}	{ε}
A	T	{a, ε}	{b, c}
B	T	{b, ε}	{b, c}

To determine whether the LL(1) condition is met for this grammar, inspect all choice points to determine if they define the predict function unambiguously. There are only two choice points: the alternatives in the second and third rules.

- A ::= aB | ε

  Predict(aB) = Starters(aB) ⊕ Followers(A) = {a}
  Predict(ε) = Starters(ε) ⊕ Followers(A) = {b, c}
  The predict sets are disjoint, so the LL(1) condition is met.

- B ::= b | ε

  Predict(b) = Starters(b) ⊕ Followers(B) = { b }
  Predict(ε) = Starters(ε) ⊕ Followers(B) = {b, c}
  The predict sets are **not** disjoint, so the LL(1) condition is **not** met.

Since the LL(1) condition is not met by at least one choice point in the grammar, this grammar is **not** LL(1).

Grammar (b)
S ::= A $
A ::= **a** A **x** | B
B ::= **b** B **x** | D
D ::= **d** D **x** | **f**

First determine the grammar properties by finding the fixpoint of the relevant equations.

1. Nullable
   $N_0(S) = N_0(A) = N_0(B) = N_0(D) = \text{false}$

   $N_{i+1}(S) = N_i(A) \wedge \text{N}(\$) = \text{false}$
   $N_{i+1}(A) = N(\mathbf{a}\ A\ \mathbf{x}) \vee N_i(B) = \ N_i(B)$
   $N_{i+1}(B) = N(\mathbf{b}\ B\ \mathbf{x}) \vee N_i(D) = \ N_i(D)$
   $N_{i+1}(D) = N(\mathbf{d}\ D\ \mathbf{x})\ \vee N(\mathbf{f}) = \text{false}$

   After two iterations we have a fixpoint $N(S) = \ N(A) = N(B) = N(D) = \text{false}$

2. Starters
   $ST_0(S) = ST_0(A) = ST_0(B) = ST_0(D) = \{\,\}$ since no nullable NTs in this grammar

   $ST_{i+1}(S) = ST_i(A) \oplus ST(\$) = \ ST_i(A)$
   $ST_{i+1}(A) = ST(\mathbf{a}\ A\ \mathbf{x})\ \cup\ ST_i(B) = \ \{\mathbf{a}\} \cup ST_i(B)$
   $ST_{i+1}(B) = ST(\mathbf{b}\ B\ \mathbf{x})\ \cup\ ST_i(D) = \ \{\mathbf{b}\} \cup ST_i(D)$
   $ST_{i+1}(D) = ST(\mathbf{d}\ D\ \mathbf{x})\ \cup\ ST(\mathbf{f}) = \ \{\mathbf{d}, \mathbf{f}\}$

Iterate to fixpoint:
$ST(S) = \{a, b, d, f\}$, $ST(A) = \{a, b, d, f\}$, $ST(B) = \{b, d, f\}$, $ST(D) = \{d, f\}$

3. Followers

$FL_0(S) = \{\,\}$	S not on RHS of any rule
$FL_0(A) = \{\mathbf{x}, \$\}$	A on RHS rule 2 and rule 1
$FL_0(B) = \{\mathbf{x}\}$	B on RHS rule 3, and at end of rule 2 RHS
$FL_0(D) = \{\mathbf{x}\}$	D on RHS rule4 and at end of rule 3 RHS

In this grammar B and D occur at the RHS of a rules 2 and 3, respectively. Thus we have to add the follow of their respective LHS, i.e.

$FL_{i+1}(B) = FL_i(B) \cup FL_i(A)$
$FL_{i+1}(D) = FL_i(D) \cup FL_i(B)$

and iterate to fixpoint. This yields $FL(B) = FL(D) = \{\mathbf{x}, \$\}$.

Finally we need to add ε to $FL(N)$ for any nonterminal N that can appear as the last symbol in a derivation starting from S. As this is an augmented grammar, we can be sure only $FL(S)$ includes $\{\varepsilon\}$. So $FL(S) = \{\varepsilon\}$.

In summary we have

	Nullable	*Starters*	*Followers*
S	F	{**a**, **b**, **d**, **f**}	{ε}
A	F	{**a**, **b**, **d**, **f**}	$\{\mathbf{x}, \$\}$
B	F	{**b**, **d**, **f**}	$\{\mathbf{x}, \$\}$
D	F	{**d**, **f**}	$\{\mathbf{x}, \$\}$

To determine whether the LL(1) condition is met for this grammar, inspect all choice points to determine if they define the predict function unambiguously. There are three choice points: the

- A ::= **a** A **x** | B
  Predict(**a** A **x**) = {a}
  Predict(B) = ST(B) = {**b**, **d**, **f** }
  The predict sets are disjoint, so the LL(1) condition is met.
- B ::= **b** B **x** | D
  Predict(**b** B **x**) = { **b** }
  Predict(D) = Starters(D) = {**d**, **f**}
  The predict sets are disjoint, so the LL(1) condition is met.
- D ::= **d** D **x** | **f**
  Predict(**d** D **x**) = { **d** }
  Predict(**f**) = { **f**}
  The predict sets are disjoint, so the LL(1) condition is met.

Since the LL(1) condition is met at all choice points in the grammar, this grammar is LL(1).

<u>Grammar (c)</u> S ::= A **c**
A ::= **a** A* **b** | **b**

The Nullable and Starters functions are straightforward to compute. In this grammar A appears on the RHS in rule 1 where it is followed by c, and it appears on the RHS in rule 2 where it is either followed by b or by $ST(A)$ (since A* can yield consecutive occurrences of A). So we have

$FL_0(S) = \{\,\}$
$FL_0(A) = (\{c\} \cup ST(A^*b)) - \{\varepsilon\} = \{c\} \cup \{a, b\} \cup \{b\} = \{a, b, c\}$

Since every occurrence of A in a rule is followed by a terminal, there are no other followers to include. Since this grammar is in augmented form, $FL(S) = \{\varepsilon\}$. In summary:

	Nullable	*Starters*	*Followers*
S	F	{a, b}	{ε}
A	F	{a, b}	{a, b, c}

LL(1) Condition: The first rule has *no* choice points. There are *two* choice points in the second rule:

- **a** A* **b** | **b**
  Predict(**a** A* **b**) = { **a** } and Predict(**b**) = { **b** }
  The predict sets are disjoint, so the LL(1) condition is met at this choice point.

- A* **b**
  First we need to check that A is not nullable - this is the case from the table above. Second we need to be able to predict repetition or termination of the Kleene star. For repetition: Predict(A) = Starters(A) = {a,b}. To predict end of the repetition: Predict(b) = {b}. The predict sets are <u>not</u> disjoint, so the LL(1) condition is <u>not met</u> at this choice point.

Since the LL(1) condition is not met by at least one choice point in the grammar, this grammar is **not** LL(1).